

## Homework1 – LTI systems, FT of DT signals

1. Determine whether the system has the following properties: stability, causality, linearity, time-invariance, memorylessness.

$$T(x[n]) = a \cdot x[n]$$

$$T(x[n]) = z[n] \cdot x[n], \quad z[n] \text{ given}$$

$$T(x[n]) = x[n - n_0]$$

$$T(x[n]) = ax[n] + b$$

Present your reasoning!

2. An LTI system is described by its impulse response  $h[n]$ . For input  $x[n]$  it produces output  $y[n]$ .

$$h[n] \text{ is nonzero only for } N_0 \leq n \leq N_1$$

$$x[n] \text{ is nonzero only for } N_2 \leq n \leq N_3$$

$$y[n] \text{ is nonzero only for } N_4 \leq n \leq N_5$$

Express  $N_4$  and  $N_5$  in terms of  $N_0, N_1, N_2, N_3$ .

3. Let  $x[n]$  be a periodic sequence with period  $N_1$ . Thus  $x[n]$  is also periodic for period  $N_3 = 3N_1$ . We may compute  $X_1[k]$  –  $N_1$ -point DFT of  $x[n]$  and  $X_3[k]$  –  $N_3$ -point DFT of  $x[n]$ .

- express  $X_3$  in terms of  $X_1$
- invent an example with  $N - 1 = 2$  and calculate  $X_1$  and  $X_3$  by hand.

4. Let  $x[n]$  be a finite length sequence of length  $N$ . Let us define two sequences of length  $N_2 = 2N$ :

$$x_1(n) = \begin{cases} x(n) & 0 \leq n \leq N - 1 \\ 0 & \text{otherwise} \end{cases}$$

$$x_2(n) = \begin{cases} x(n) & 0 \leq n \leq N - 1 \\ -x(n - N) & N \leq n \leq 2N - 1 \end{cases}$$

$X_1, X_2, X_3$  denote DFT's of respective  $x$ 's.

- How to compute  $X[k]$  from  $X_1[k]$  ?
- How to compute  $X_2[k]$  from  $X_1[k]$  ?

5.  $x[n]$  – real, finite length sequence.

$$X(e^{j\omega}) = \mathcal{F}(x[n])$$

$$X[k] = \text{DFT}(x[n])$$

$$\Im\{X[k]\} = 0$$

Prove or reject:  $\Im\{X(e^{j\omega})\} = 0$

(notation:  $\Im$  denotes imaginary part operator).