Homework2/2003 – z-transform, inst. spectrum, filters

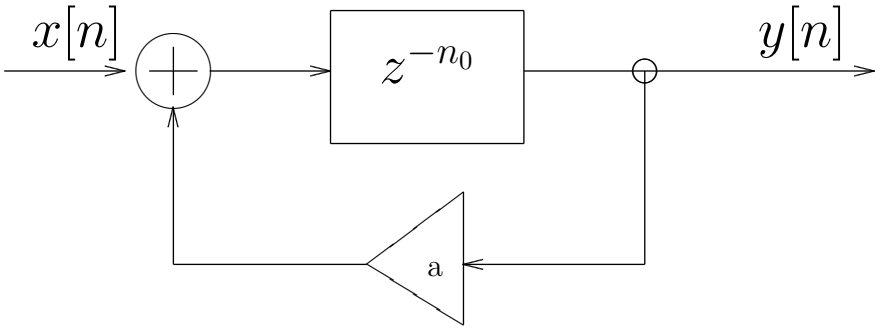
1. The instantaneous spectrum $X(e^{j\theta}, n)$ of a signal

$$x(r) = \begin{cases} sin(\theta_0 \cdot n) & \text{for} \quad r = 10, \ \ldots, 49 \\ 0 & \text{otherwise} \end{cases}$$

   is computed using rectangular window $g(k)$ of length $K = 10$.

   (a) How does the mainlobe width of $X(e^{j\theta}, n)$ change with n (make approximate sketch).
   (b) What is the minimum width of the mainlobe (write an expression)
   (c) If we change K, or the window type – how will the mainlobe change?
   (d) Sketch the value of $X(e^{j\theta}, n)$ versus n with $\theta = theta_0$.

2. Analyze an IIR filter with graph as follows For plots, assume $a = 0.5$, $n_0 = 2$



   (a) find $h[n]$, $H(z)$
   (b) Plot zeros and poles of the filter
   (c) How the filter stability depends on n_0 and a?
   (d) Plot filter amplitude characteristics
   (e) Find filter response for $x(n) = 1 + cos(n\pi)$

3. Calculate the z-transform and its region of convergence for the following series:

   (a) $\delta[n]$
   (b) $u[n] - u[n - n_0]$
   (c) $u[n] \cdot 0.4^n$
   (d) $u[-n] \cdot -0.4^n$

4. Calculate the impulse response, assuming phase equal to zero:

   (a) an ideal lowpass filter with cutoff frequency of θ_b
   (b) an ideal highpass filter with cutoff frequency of $\pi - \theta_b$
   (c) an ideal bandpass filter with passband of $\theta_c \pm \theta_b$

   Then, assume $\theta_b = pi/4$ and try to design a filter with rectangular window method, with order of 7. Try to calculate the filter characteristics by hand, then check with a computer.

Additional problems...

...may be found in *Oppenheim and Schafer with Buck*:

- z-transform: basic problems from Chapter 3 ("The z-transform")
- filters: basic problems from Chapter 5 ("Transform analysis of LTI systems")
- filter design: first few basic problems from Chapter 7 ("Filter design techniques")
- instantaneous spectrum: problems 10.9, 10.13 – 10.20 (problems from Chapter 10 "Fourier analysis of signals using the DFT" that have "TDFT" or "time-dependent FT" inside)

note: if yout have other editions of O. and S., the chapter numbering will be different, but contents are similar.