Multidimensional signals

- Analogue K-D signal $x_a(t_1,\ t_2,\ t_3,\ \ldots,\ t_K)$, t_k not necessarily time.
- Discretization (sampling) $\longrightarrow x(n_1,\ n_2,\ n_3,\ \ldots,\ n_K)$, *some signals are already discrete!*
  $\longrightarrow$ multidimensional discrete signal = multidimensional number series
- sampling periods $T_{sk} \longrightarrow$ sampling frequencies $f_{sk} = \frac{1}{T_{sk}}$ not equal;
  if t_k is spatial, $f_s k$ is *spatial frequency*
- Examples
  - 2-D picture
  - linear antenna array (t_1 – discrete or continuous space, t_2 – continuous time)
  - radar signal

Image – a 2-D signal

- $T_{s1},\ T_{s2}$ – pixel dimensions; $f_{s1},\ f_{s2}$ – resolution (dpi, lines/mm)
- Fourier spectrum:

$$X(e^{j\theta_1},\ e^{j\theta_2}) = \sum_{n_1=-\infty}^{+\infty} \sum_{n_2=-\infty}^{+\infty} x(n_1,\ n_2) e^{-jn\theta_1}\ e^{-jn\theta_2}$$

$\theta_k = \omega_k \cdot T_{sk} = \frac{2\pi f_k}{f_{sk}}$ – normalized angular frequency

- finite picture $\longrightarrow$ represented by a discrete spectrum

$$X(k,l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m,\ n) e^{-j2\pi km/M}\ e^{-j2\pi ln/N}$$

- reconstruction by a discrete Fourier series

$$x(m,n) = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} X(k\ ,l) e^{j2\pi km/M}\ e^{j2\pi ln/N}$$

2-D LTI systems

Linear and *(Time = shift in* n_k*)* Invariant (we extend the definition for 1-D systems)

- allows for analysis by impulse response
  (unit impulse: $\delta(m,n) = 1$ if $m = n = 0, = 1$ otherwise)
  impulse response is sometimes called Point Spread Function – PSF
- causality – meaningful if one of dimensions is time-related
- 2-D convolution (linear filtering)

$$y(m,n) = \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} x(i,\ j) \cdot h(m-i,\ n-j)$$

- if $h(m,\ n) = h_1(m) \cdot h_2(n)$, we may decompose 2-D filtering into 2x(1-D) (important for long impulse responses)