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For the short problems, try to write the answer in the provided space. Put your calculations on additional sheet. Solve long problems on the additional sheet, marked with your name.

1. A DT system is described as follows:

$$T(x[n]) = \sum_{k=0}^3 x(n+k)$$

(a) (2 p.) is  $T$

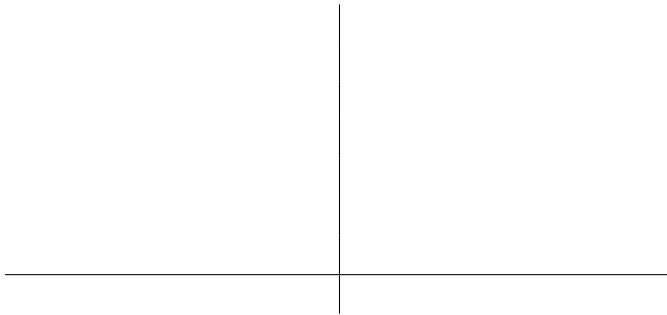
- stable yes:  or no:  explain why: .....
- causal yes:  or no:  explain why: .....
- linear yes:  or no:  explain why: .....

(b) (2 p.) find the impulse response of the system  $T$  .....

(c) (2 p.) find the step response .....

2. (6 p.) Let  $x[n]$  be a signal obtained by sampling a continuous-time cosine wave  $x(t)$  with sampling period of  $100 \mu\text{s}$ . The frequency of  $x[n]$  is equal to  $\theta = 0.125 \cdot \pi$

- (a) Calculate the frequency of  $x(t)$  and period  $K$  of the  $x[n]$ .
- (b) Sketch the absolute value of DFT  $X(k)$  for the transform size equal to  $2 \cdot K$ .
- (c) Label the frequency axes carefully with index  $k$  and with  $\theta$  values.



3. (10 p.) A filter is described by a difference equation

$$y(n) + 0.5y(n - 1) = x(n)$$

where  $x(n)$  denotes the input signal and  $y(n)$  the output signal.

- (a) Find the transfer function  $H(z)$  of the filter
- (b) Find zeros and poles
- (c) Sketch the graph of a simple implementation of the filter
- (d) Find the output for a discrete input signal defined as  $x(n) = 1 + (-1)^n$
- (e) Find the output for a discrete input signal defined as  $x(n) = \delta(n - 5)$

4. A causal bandpass FIR filter of the order 7 was designed from windowed Inverse Fourier Transform of the zero-phase ideal filter characteristics. A rectangular window was used. Ideal filter passband was from  $\theta_l = \frac{\pi}{4}$  to  $\theta_h = \frac{\pi}{2}$ .

- (a) (2 p.) Plot the phase characteristics of the resulting filter. Find the group delay.
- (b) (3 p.) Sketch the *approximate* amplitude characteristics of the resulting filter (exact calculations aren't required). Show and name the artefacts from the method nonideality.

5. "Tricky questions": to obtain a full score, remember to present your reasoning.

- (a) (3 p.) Let  $x(n)$  be a limited energy signal with non-zero samples only for  $0 < n < N - 1$ . What is the difference between Fourier transform and DFT of this signal ?
- (b) (3 p.) A filter  $y(n) = x(n) + x(n - 1)$  ( $x$ -input,  $y$ -output) filters a white noise signal  $\xi(n)$  with zero mean and standard deviation  $\sigma_\xi = 1$ . Find the mean value of the output signal  $\eta(n)$ .

Answer:  calculation: .....

- (c) (2 p.) In a Kaiser window - what depends on the  $\beta$  parameter?

Answer: .....

- (d) (2 p.) The inverse Fourier transform of limited energy signal is calculated by  summation  integration (choose answer, and explain why: .....

- (e) (4 p.) (fill in the answers) If we calculate the instantaneous spectrum of a signal using a window of length  $K$ ,

- i. the frequency resolution is proportional to .....
- ii. the time resolution is proportional to .....

- (f) (4 p.) What does a digital signal processor MAC instruction?

Answer: .....

Why is this operation implemented as one instruction?

Answer: .....

- (g) (2 p.) Calculate the  $\mathcal{Z}$  transform of a signal  $x[n] = \delta[n] - \delta[n - 1] + \delta[n + 1]$

Answer: .....

- (h) (3 p.) How many complex number multiplications do we need to compute a DFT of a finite-time signal of 8 samples,

- i. using the definition formula  calculation: .....
- ii. using the FFT method  calculation: .....

- (i) (5 p.) An analog voice phone is usually designed to transmit the tones in the range of 0-4 kHz. What should be the sampling frequency, if we want to convert the analog lines to digital?

Answer:  calculation: .....

What should the designer keep in mind if the actual microphone passes the 0-10 kHz band?

Answer: .....

- (j) (3 p.) How many nonzero samples will be in the convolution of two 15-point square impulses?

Answer:  calculation: .....