# EDISP
# (English) Digital Signal Processing

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# General information

**“Credits”** 2h/week lecture + 2h/week $\approx 7 \cdot 4h$ **laboratory** exercises in groups $\leq 12$ persons.

**Next lecture** Tue, 14:15-16 $\longrightarrow$ *the lab schedule will be announced then!!*

**Textbook** A. V. Oppenheim, R. W. Schafer, *Discrete-Time Signal Processing*, Prentice-Hall 1989 (or II ed, 1999; also acceptable previous editions entitled *Digital Signal Processing*).

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**Homeworks** Announced as a preparation for the tests.

**Exams** Two short tests within lecture hours (TBA with the lab schedule) and a final exam during the winter exam session (TBA).

**Scoring:**

| | | | |
|---|---|---|---|
| 2x10% | = | 20% | tests |
| 6x5% | = | 30% | lab + entry test (lab 0 – not scored) |
| | | 50% | final exam |
| 2x2% | = | 4% | extra for homeworks |

# Signal classification

Continuous or Discrete **amplitude** and **time**.

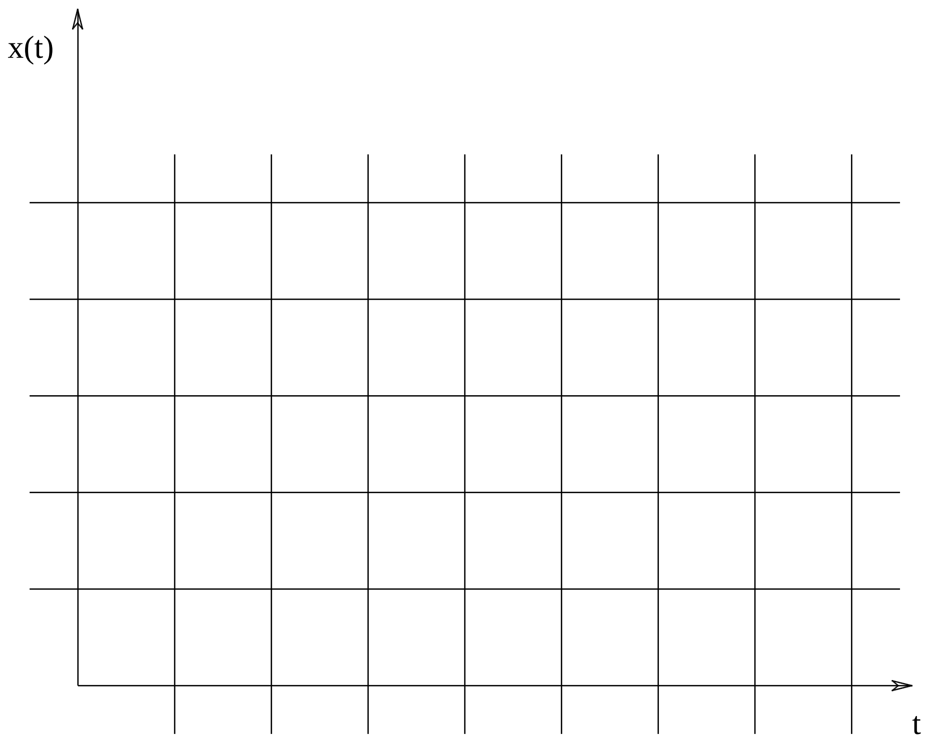
**CA-CT** $\longrightarrow$ “analog” signals

**DA-CT** $\longrightarrow$

**CA-DT** $\longrightarrow$ CCD, SC, SAW devices

**DA-DT** $\longrightarrow$ digital devices

We’ll speak mainly about DT properties; only in some subject DA will be of importance.



# DT signal representations

DT signal $\longleftarrow\longrightarrow$ a number sequence

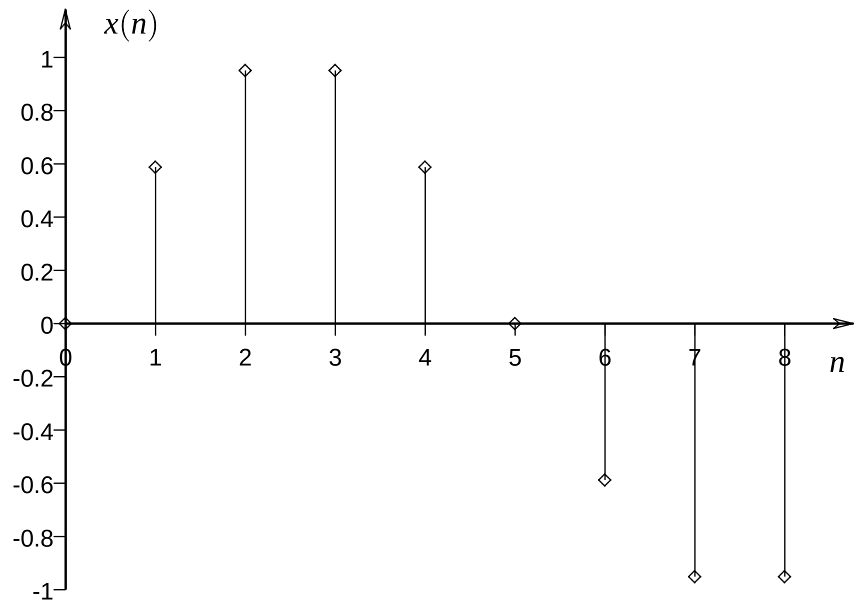
$$x[n] = \{x(n)\}$$

$x[n]$ is a number sequence (or ...)

$x(n)$ is a $n-$th sample

$\longrightarrow$ $x(n)$ is *undefined* for $n \notin \mathrm{I}$

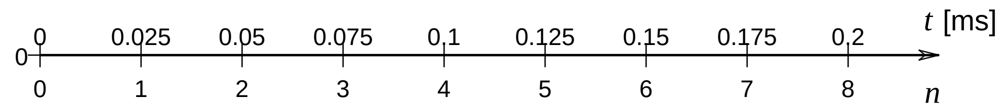
- it *may* come from sampling of analog signal
- but it may also be inherently discrete
- $n$ may correspond to: time, space, ...




However, the most popular interpretation is: periodic sampling in time.

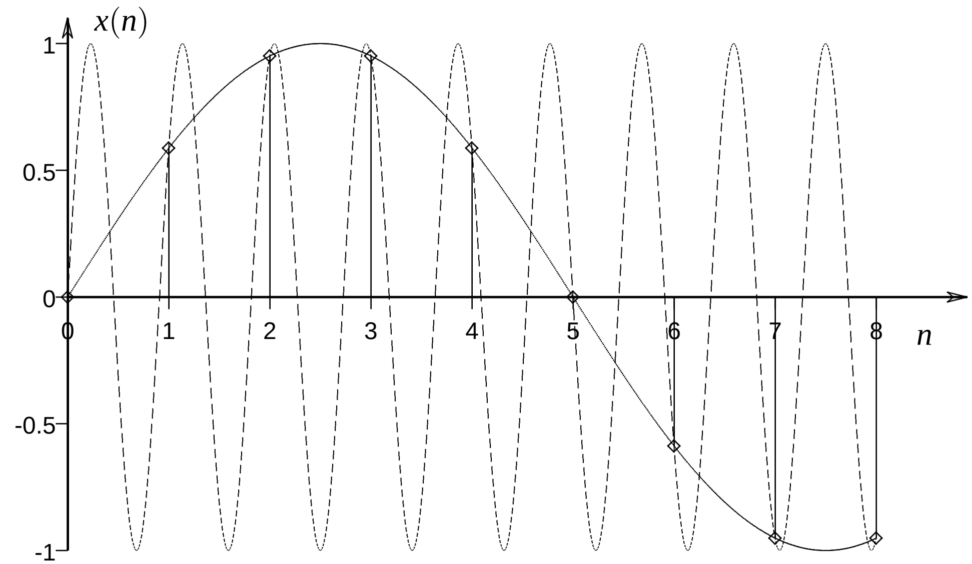
# Periodic sampling

$$n \longleftarrow\longrightarrow n \cdot T_s$$




$$x(n) = x_a(nT_s)$$

$$n = t/T_s, \quad T_s = 0.025\ [\mathrm{ms}]$$




Misinterpretations

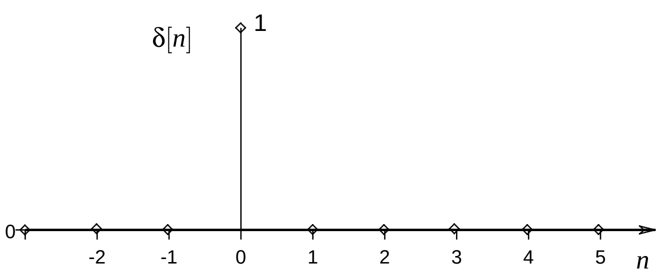
$\longrightarrow$ we do not know what is between points

a) $sin(n \cdot (1/5) \cdot \pi)$ or

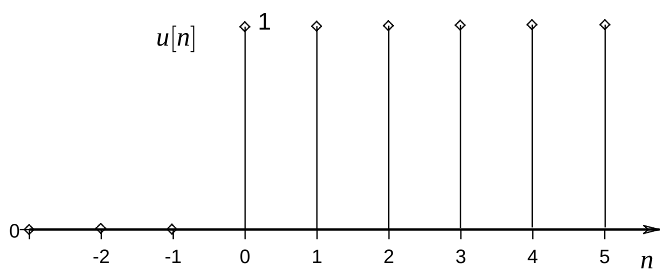
b) $sin(n \cdot (2 + 1/5) \cdot \pi)$ ?

# Number sequence (or DT signal) operations; basic sequences

| operation | notation | definition |
|---|---|---|
| sum | $z[n] = x[n] + y[n]$ | $\forall n \;\; z(n) = x(n) + y(n)$ |
| scale | $z[n] = \alpha \cdot y[n]$ | $\forall n \;\; z(n) = \alpha \cdot y(n)$ |
| shift | $z[n] = x[n - n_0]$ | $\forall n \;\; z(n) = x(n - n_0)$ |
| difference | $z[n] = x[n] - y[n]$ | $\forall n \;\; z(n) = x(n) - y(n)$ |
| product | $z[n] = x[n] \cdot y[n]$ | $\forall n \;\; z(n) = x(n) \cdot y(n)$ |




Unit sample sequence (DT impulse)




Unit step sequence

$$\delta[n] = u[n] - u[n-1]$$

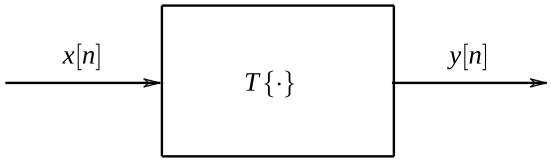
$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

# DT systems

A DT system: an operator mapping an input sequence $x[n]$ into an output sequence $y[n]$.

$$y[n] = T\{x[n]\}$$

$\longrightarrow$ A rule (formula) for computing output sequence values $y(n)$ from the input sequence values $x(n)$.




**Implementations:**

- PC program
- matlab m-file
- custom VLSI or FPGA
- programmable digital signal processor

Examples:

$$y(n) = 3 \cdot x(n)$$

$$y(n) = \frac{x(n) + x(n-1)}{2}$$

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

# Linear & time-invariant DT systems

## Linearity property

$$T\{\alpha_1 x_1[n] + \alpha_2 x_2[n]\} = \alpha_1 T\{x_1[n]\} + \alpha_2 T\{x_2[n]\}$$

in other words:

if

$x_1[n] \longrightarrow y_1[n]$

$x_2[n] \longrightarrow y_2[n]$

then

$\alpha x_1[n] \longrightarrow \alpha y_1[n]$ (scaling, homogeneity)

$x_1[n] + x_2[n] \longrightarrow y_1[n] + y_2[n]$ (additivity)

## Time invariance (shift invariance)

If

$$T\{x[n]\} = y[n]$$

then

$$\forall n_0, \quad T\{x[n-n_0]\} = y[n-n_0]$$

# Linear systems - examples

- $y(n) = 3 \cdot x(n)$ – is linear; it is also *memoryless*
- $y(n) = \frac{x(n)+x(n-1)}{2}$ (not memoryless):

$$T\{\alpha_1 x_1(n) + \alpha_2 x_2(n)\} = \frac{[\alpha_1 x_1(n) + \alpha_2 x_2(n)] + [\alpha_1 x_1(n-1) + \alpha_2 x_2(n-1)]}{2} =$$

$$= \frac{\alpha_1 x_1(n) + \alpha_1 x_1(n-1)}{2} + \frac{\alpha_2 x_2(n) + \alpha_2 x_2(n-1)}{2} =$$

$$= \alpha_1 \frac{x_1(n) + x_1(n-1)}{2} + \alpha_2 \frac{x_2(n) + x_2(n-1)}{2} = \alpha_1 y_1(n) + \alpha_2 y_2(n) \text{ cnd}$$

---

**(not L)** $y(n) = (x(n))^2$ because

$$T\{x_1(n) + x_2(n)\} = (x_1(n) + x_2(n))^2 = (x_1(n))^2 + (x_2(n))^2 + [2 \cdot x_1(n) x_2(n)]$$

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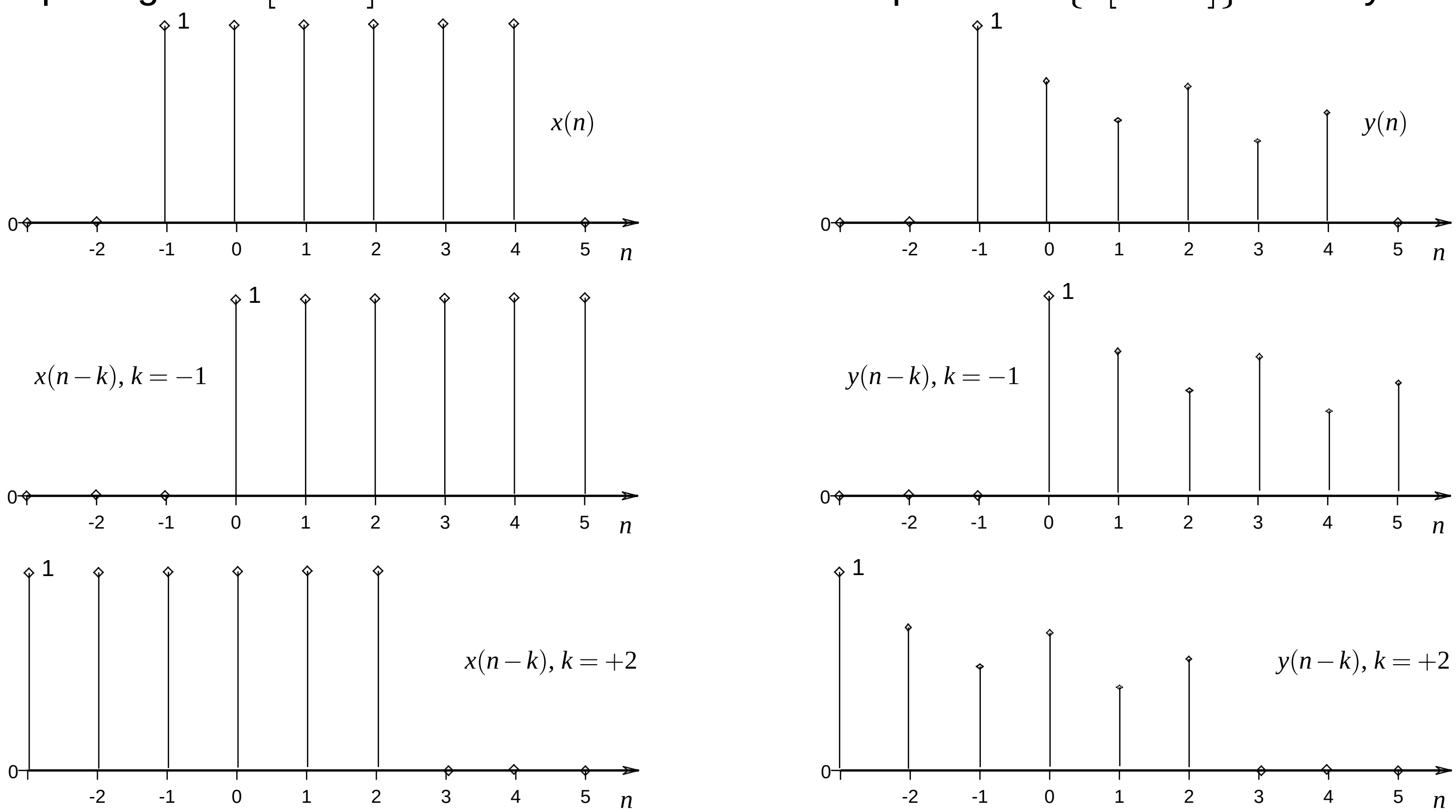
# shift example

Input signals $x[n-k]$. Responses $T\{x[n-k]\}$ of TI system $T\{.\}$



# Other properties: causality, stability

**causality**

$\longrightarrow y(n_0)$ depends only on $x(n),\ n \le n_0$ (*usually less important in DT implementations)*

**stability**

$\longrightarrow$ bounded input causes bounded output [BIBO]

bounded $\longrightarrow \exists B_x : \ \forall n \ |x(n)| \le B_x < \infty$

# Examples

**Decimator (compressor)**

$$y(n) = x(Mn)$$

$\longrightarrow$ L, but not TI (*prove it!*)

**1-st order difference**

forward: $y(n) = x(n+1) - x(n) \longrightarrow$ noncausal

backward: $y(n) = x(n) - x(n-1) \longrightarrow$ causal

**Accumulator**

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

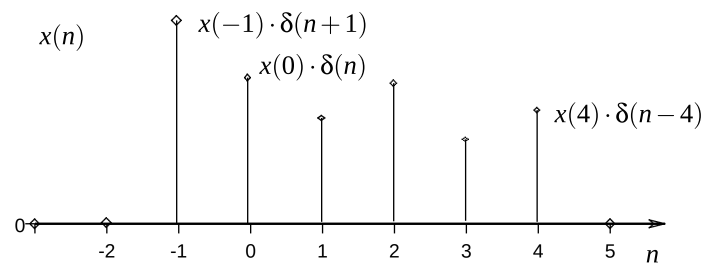
$\longrightarrow$ unstable; (*hint: feed it with* $u[n]$)

# LTI systems: impulse response

$h[n] = T\{\delta[n]\} \longrightarrow$ impulse response of $T\{.\}$
$h[n]$ characterizes completely system $T\{.\}$ – we may compute its response for any input $x[n]$.

- Decompose $x[n]$ into weighted sum of impulses $\delta[n-k]$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$




- Superpose responses (use LTI properties)

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$\longrightarrow$ this is a **convolution sum**

# Convolution example

(see scanned handcrafted version)

# Convolution properties

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

we denote as

$$y[n] = x[n] * h[n]$$

**Properties of "$*$"**

"$*$" is commutative: $x[n] * h[n] = h[n] * x[n]$

"$*$" distributes over addition $x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * (h_2[n])$

**System and $h[n]$**

- causality $\Leftrightarrow h[n] = 0, \;\; n < 0$. A hint: $y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$
- stability $\Leftrightarrow S = \sum_{k=-\infty}^{\infty} |h(k)| < \infty$

# Linear difference equations

... describe an important class of LTI systems.

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k), \quad a_0 = 1 \text{ (traditionally)}$$

or

$$\begin{aligned} y(n) \quad &= \quad -a_1 \cdot y(n-1) - a_2 \cdot y(n-2) - \ldots - a_n \cdot y(n-N) + \\ +b_0 \cdot x(n) \quad &+b_1 \cdot x(n-1) + b_2 \cdot x(n-2) + \ldots + b_n \cdot x(n-M) \end{aligned}$$

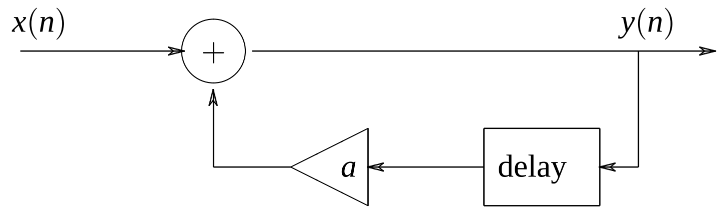
**Note:** if, for a given input $x_p[n]$, an output sequence $y_p[n]$ satisfies given difference equation,

$$y[n] = y_p[n] + y_h[n]$$

will also satisfy the equation, if $y_h[n]$ is a solution to $\sum_{k=0}^{N} a_k y(n-k) = 0$ (homogenous equation).

# Difference equation – example

An equation: $y(n) = a \cdot y(n-1) + x(n)$
with input
$x(n) = 0,\ n < 0$
$x(n) \neq 0,\ n > 0.$




$$
\begin{aligned}
y(0) &= a \cdot \mathbf{y}(-\mathbf{1}) + x(0) \\
y(1) &= a \cdot y(0) + x(1) \\
y(2) &= a \cdot y(1) + x(2) \\
&\dots
\end{aligned}
$$

Initial condition: $y(-1) = \alpha$
Let $x[n] = \delta[n]$

$$
\begin{aligned}
y(0) &= a \cdot \alpha + 1 \\
y(1) &= a(a \cdot \alpha + 1) = a^2\alpha + a \\
y(2) &= a^3\alpha + a^2 \\
&\dots \\
y(n) &= a^{n+1}\alpha + a^n
\end{aligned}
$$

# Difference equation – impulse response (example continued

$y(n) = a \cdot y(n-1) + x(n)$

Initial condition: $y(-1) = \alpha \; x[n] = \delta[n]$

Solution: $y(n) = a^{n+1}\alpha + a^n$

Find a homogenous part!

Stability:

$1 \; < \; a$**:** $\quad a^n \rightarrow \infty$

$0 < a < 1$**:** $\quad a^n \rightarrow 0$

$-1 < a < 0$**:** $\quad a^n \rightarrow 0$

$a \; < \; -1$**:** $\quad a^n \rightarrow$ ???