EDISP 2006/2007 – Final exam, 1st approach, **version A** 26.01.2007

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Solve long problems on an additional sheet, marked with your name. For the short problems, try to write the answer in the provided space. Put your calculations on the additional sheet.

1. A DT system is described as follows:

$$T(x[n]) = 8x[n]+8x[n-4]$$

   (a) (2 p.)is T
   - stable yes:☐ or no:☐ explain why:.......................................
   - causal yes:☐ or no:☐ explain why:.......................................
   - linear yes:☐ or no:☐ explain why:.......................................

   (b) (2 p.) find the impulse respone of the system T ....

   (c) (2 p.) find the step response ....

2. (6 p.) Let $x[n]$ be a signal obtained by sampling a continuous-time sine wave of 10 kHz frequency with sampling period of 25 μs

   (a) Calculate the period K and normalized angular frequency θ of the $x[n]$.

   (b) Sketch the absolute value of DFT $X(k)$ for the transform size equal to $2K$.

   (c) Label the frequency axes carefully with index k and with θ values.

3. (10 p.) A causal IIR filter is described by its poles: $p_1 = +0.5j$, $p_2 = -0.5j$. We know that the DC gain is equal to 1, and the location of poles determines the frequency characteristics of the filter.

   (a) Find the transfer function $H(z)$ of the filter

   (b) Write the difference equation for the filter

   (c) Sketch the graph of a simple implementation of the filter

   (d) Find the output for a discrete input signal defined as $x(n) = 1 + (-1)^n$

   (e) Find the output for a discrete input signal defined as $x(n) = \delta(n-5)$

4. A causal lowpass FIR filter of the order 10 was designed from windowed Inverse Fourier Transform of the zero-phase ideal filter characteristics. A rectangular window was used. Ideal filter cutoff was at $\theta_b = \frac{\pi}{3}$.

(a) (2 p.) Plot the phase characteristics of the resulting filter. Find the group delay.

(b) (3 p.) Sketch the *approximate* amplitude characteristics of the resulting filter (exact calculations aren't required). Show and name the artefacts from the method nonideality.

5. *"Tricky questions": remember to present your reasoning for each answer, otherwise scores will be lower.*

   (a) (3 p.) What is the difference between DCT and DFT? ....
   ....
   ....

   (b) (3 p.) A filter $y(n) = x(n) + x(n-1)$ (x-input, y-output) filters a white noise signal $\xi(n)$ with zero mean and standard deviation $\sigma_\xi = 1$. Sketch the PSD of the input signal $\xi(n)$ and output signal $\eta(n)$.

   (c) *(5 p.)DCT can be calculated using FFT as a tool. Invent and describe step-by-step how to do it for a variant of DCT, defined as $X(k) = \sum_{k=0}^{N-1} x(n)cos(nk\frac{\pi}{N-1})$.

   (d) (3 p.) Why does a digital signal processor need three separate memory banks?
   Answer:..

   (e) (3 p.) What is the minimum width of a transition band in an LP FIR filter of order $K = 8$ designed with window method?
   Answer: $\boxed{}$ calculation:...

   (f) (3 p.) How do we reconstruct a signal from its DFT coefficients $X(k)$? Describe the difference between the periodic and finite-time signal assumption.................
   ....
   ....

   (g) (4 p.) How many complex number multiplications do we need to compute a 2-dimensional DFT of a 2D finite-time signal of NxN samples, using the definition formula (**not** the FFT method).
   Answer: $\boxed{}$ calculation:...

   (h) (6 p.) A CT signal $x(t)$ has been sampled with the sampling frequency of 64 kHz. The 32-point FFT $X(k)$ of the resulting signal $x(n)$ is almost all zeros, except two points, one of them is $X(4) = 1$. What is the other non-zero point? (Answer: $\boxed{}$).What are the possible values of the frequency of $x(t)$ if we know that:

      i. a **good** antialiasing filter was used (Answer: ....)
      ii. an antialiasing filter was **not** used (Answer:....................................)

   (i) (3 p.) How many nonzero samples may be (at max) in the output of a 4-th order FIR filter when the input signal has 10 non-zero samples?
   Answer: $\boxed{}$ calculation:...

$\Sigma = 60p\ T = 90\ min$