EDISP 2003/2004 – Final exam 0, 26.01.2004

1. (12p) An ideal LP filter has the frequency characteristics

$$A(\theta) = \begin{cases} 1 & \text{for} \quad -\theta_g < \theta < \theta_g \\ 0 & \text{otherwise} \end{cases}$$

   (a) calculate impulse response of such a filter, assuming:
      i. $\phi(\theta) = const = 0$
      ii. $\phi(\theta) = -K\theta$ (K – an integer)
   (b) describe the procedure to design a causal FIR filter by approximating the ideal impulse response
   (c) how does the transition band width depend on the filter order when a rectangular window is used in the procedure?

2. (12p) A filter is described by an equation $y(n) = a \cdot y(n-1) + b \cdot x(n)$ (a, b are real, $x[n]$ - input signal, $y[n]$ - output signal)

   (a) Sketch the graph of this filter
   (b) Calculate the transfer function $H(z)$, find zeros and poles, find impulse response, find how the stability depends on a, b.
   (c) Let $a = 1/2$. Calculate the response for an input signal ($n \in -\infty \ldots + \infty$):
      i. $x(n) = const = 3$
      ii. $x(n) = 3 \cdot (-1)^n$

3. (12p) $x(t)$ is a continuous-time, periodic signal with limited mean power and with known period T. The signal is periodically sampled with period $t_s = T/K$ ($K > 0$ and is an integer) to obtain a DT signal $x[n]$.

   (a) Calculate the period of $x[n]$
   (b) Assuming $x(t)$ is harmonic, calculate the frequency of $x[n]$ (in radians per sample)
   (c) If N is the period of $x[n]$, make a sketch showing the difference between N and $2N$-point DFT of $x[n]$ (assume that the signal is periodic, but not necessarily harmonic)
   (d) Is it necessary to assume the $x(t)$ is band-limited for answering a) – c)? Present your reasoning.

4. (6x3p) *Tricky questions. Be careful and exact with answers, summation ranges etc.*

   (a) How many real number multiplications do we need to calculate 8-point DFT of a real signal, using FFT algorithm? Don't count sign-change as multiplication. How is the number reduced if we exploit the result symmetry?
   (b) How do we reconstruct a signal $x(n)$ from its DCT coefficients $X^c(k)$? Write the equation. (Don't try to use FFT...)
   (c) What is the period of $cos(\frac{3\pi}{8}n)$?
   (d) A filter $y(n) = x(n) + x(n-1)$ (x-input, y-output) filters a white noise signal $\xi(n)$ with zero mean and standard deviation $\sigma_\xi = 2$. Calculate the standard deviation of output signal σ_η.
   (e) A Bartlett window of length $2N-1$ is a convolution of two rectangular windows of length N. Calculate the mainlobe width of the Bartlett window.
   (f) Why does a digital signal processor need three separate memory banks? Explain clearly, present an example.

$\Sigma = 54p$ $T = 90$ min, $\lambda = 0.6\frac{p}{\min}$