EDISP 2006/2007 – Final exam, 2nd approach, **version A** 5.02.2007

Name:_____

Solve long problems on an additional sheet, marked with your name. For the short problems, try to write the answer in the provided space. Put your calculations on the additional sheet.

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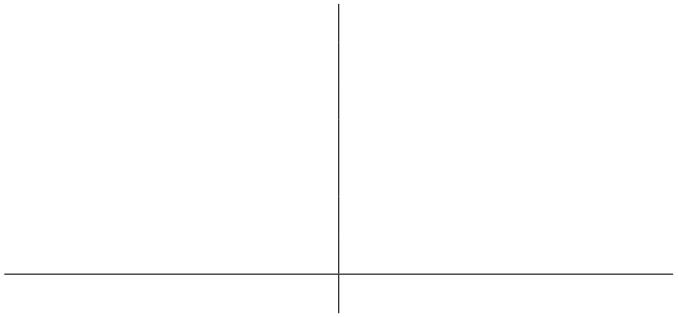
1. (6 p.) Let $x[n]$ be a signal obtained by sampling a continuous-time cosine wave of 5 Hz frequency with sampling period of 20 ms

   (a) Calculate the period N_0 and normalized angular frequency θ_0 of the $x[n]$.

   (b) Sketch the absolute value of DFT $X(k)$ for the transform size equal to $K = 2 \cdot N_0$.

   (c) Label the frequency axes carefully with index k and with θ values.



$N_0 =$ .....
$\theta_0 =$ .....
.....

2. (10 p.) A causal IIR filter is described by an equation

$$y(n) = 0.9y(n-1) + 0.1x(n) - 0.1x(n-1)$$

   (a) Find the transfer function $H(z)$ of the filter: $H(z)=$.....

   (b) Find zeros and poles of the filter: .....

   (c) Sketch the graph of a simple implementation of the filter

   (d) Find the output for a discrete input signal defined as $x(n) = 1 + (-1)^n$

   (e) Find the output for a discrete input signal defined as $x(n) = \delta(n-5)$

3. A causal bandpass FIR filter of the order 10 was designed from windowed Inverse Fourier Transform of the zero-phase ideal filter characteristics. A rectangular window was used. Ideal filter passband was from $\theta_l = \frac{\pi}{4}$ to $\theta_h = \frac{2\pi}{4}$.

   (a) (2 p.) Plot the phase characteristics of the resulting filter. Find the group delay.

   (b) (3 p.) Sketch the *approximate* amplitude characteristics of the resulting filter (exact calculations aren't required). Show and name the artefacts from the method nonideality.

4. (6 p.) If we calculate the instantaneous spectrum of a signal using a window of length K,

   (a) the frequency resolution is proportional to .....

   (b) the time resolution is proportional to .....

(c) the rectangular window is good for .....
because .....

(d) but we use other window shapes when the important thing is .....

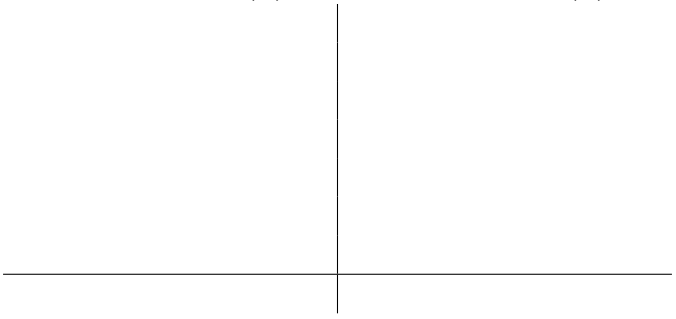
5. *"Tricky questions": present your reasoning for each answer, otherwise scores will be lower.*

(a) (3 p.) A DT system is described as: $T(x[n]) = 8x[n]+8x^2[n-4]$; is T

- stable yes:☐ or no:☐ explain why:.....
- causal yes:☐ or no:☐ explain why:.....
- linear yes:☐ or no:☐ explain why:.....

(b) (3 p.) Why is the DCT used for JPEG compression and DFT is not?.....
.....
.....

(c) (4 p.) A filter $y(n) = x(n) - 2x(n-1) + x(n-2)$ (x-input, y-output) filters a white noise signal $\xi(n)$ with zero mean and standard deviation $\sigma_\xi = 1$. Sketch the PSD of the input signal $\xi(n)$ and output signal $\eta(n)$.



(d) *(5 p.)DCT can be calculated using FFT as a tool. Invent and describe step-by-step how to do it for a variant of DCT, defined as $X(k) = \sum_{k=0}^{N-1} x(n)cos(nk\frac{\pi}{N-1})$.

(e) (2 p.) In a Kaiser window - what depends on the β parameter?
Answer:.....

(f) (2 p.) The inverse Fourier transform of limited energy signal is calculated by ☐ summation ☐ integration (choose answer, and explain why: .....)

(g) (4 p.) What does a digital signal processor MAC instruction do?
Answer:.....
Why is this operation implemented as one instruction?
Answer:.....

(h) (2 p.) Calculate the $\mathcal{Z}$ transform of a signal $x[n] = \delta[n]-\delta[n-1]+\delta[n+1]$

Answer:.....

(i) (4 p.) How many complex number multiplications do we need to compute a DFT of a finite-time signal of 1024 samples, using:

- the definition formula? Answer: ☐ calculation:.....
- the FFT method? Answer: ☐ calculation:.....

(j) (3 p.) What order may be a FIR filter if for many diferent input signals with 10 contiguous non-zero samples the output has 15 contiguous non-zero samples?
Answer: ☐ calculation:.....

(k) (2 p.) Why cannot the previous answer be 100% sure? Show an example of bad guess. Answer: .....

$\Sigma = 61p$ $T = 90$ min