EDISP (Filters 3)
(English) Digital Signal Processing
Two-dimensional signals & filters
lecture

January 18, 2008

Multidimensional signals

- Analogue K-D signal $x_a(t_1,\ t_2,\ t_3,\ \dots,\ t_K)$, t_k not necessarily time.
- Discretization (sampling) $\longrightarrow x(n_1,\ n_2,\ n_3,\ \dots,\ n_K)$, *some signals are already discrete!*
  $\longrightarrow$ multidimensional discrete signal = multidimensional number series
- sampling periods $T_{s\ k} \longrightarrow$ sampling frequencies $f_{s\ k} = \frac{1}{T_{s\ k}}$ not necessarily equal; (e.g some scanners have different h & v resolutions) if t_k is spatial, $f_{s\ k}$ is *spatial frequency*
- Examples
  - 2-D picture
  - linear antenna array (t_1 – discrete or continuous space, t_2 – continuous time)
  - radar signal

Image – a 2-D signal

- $T_{s\,1},\ T_{s\,2}$ – pixel dimensions; $f_{s\,1},\ f_{s\,2}$ – resolution (dpi, lines/mm)
- Fourier spectrum:

$$X(e^{j\theta_1},\ e^{j\theta_2}) = \sum_{n_1=-\infty}^{+\infty} \sum_{n_2=-\infty}^{+\infty} x(n_1,\ n_2) e^{-jn\theta_1}\ e^{-jn\theta_2}$$

$\theta_k = \omega_k \cdot T_{sk} = \frac{2\pi f_k}{f_{sk}}$ – normalized angular frequency

- finite picture $\longrightarrow$ represented by a discrete spectrum

$$X(k,l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m,\ n) e^{-j2\pi km/M}\ e^{-j2\pi ln/N}$$

- reconstruction by a discrete Fourier series

$$x(m,n) = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} X(k\ ,l) e^{j2\pi km/M}\ e^{j2\pi ln/N}$$

2-D LTI systems

Linear and *(Time = shift in n_k)* Invariant (we extend the definition for 1-D systems)

- allows for analysis by impulse response
  (unit impulse: $\delta(m,n) = 1$ if $m = n = 0, = 1$ otherwise)
  impulse response is sometimes called Point Spread Function – PSF
- causality – meaningful if one of dimensions is time-related
- delay - in spatial dimension, zero-delay filter is the best
- 2-D convolution (linear filtering)

$$y(m,n) = \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} x(i,\ j) \cdot h(m-i,\ n-j)$$

- if $h(m,\ n) = h_1(m) \cdot h_2(n)$, we may decompose 2-D filtering into 2x(1-D) (important for long impulse responses)

Images - practical remarks

We concentrate on monochrome images (B/W photos, print, raw images in medical, radar, sonar, satellite technology). Color images add some complexity, unimportant for the signal processing basics.
Understanding frequency concept:

- Horizontal frequency - changes in the horizontal direction (trunks in the forest)
- Vertical frequency - changes in the vertical direction (horizon)
- High frequencies - sharp edges, small features
- Low frequencies - soft edges, large areas of same intensity

Filtering

Filtering methods:

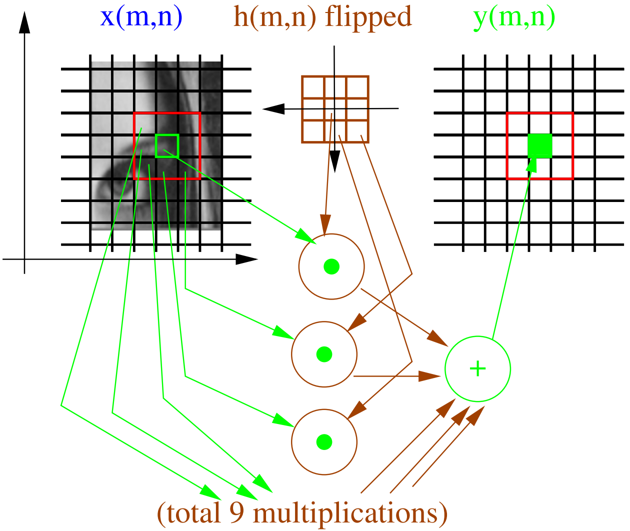
- Spatial (image) domain: convolution=weighted average of neighboring pixels
- Frequency domain: masking out (zeroing) parts of the 2D spectrum
- Nonlinear filters: some nonlinear manipulation on the set of neighboring pixels (e.g median)

How to behave at the image boundary with spatial filters?

- assume zeros outside (effect:dark areas at the boundaries)
- repeat last data row/column
- circular symmetry (effect: sky under the ground?)
- mirror symmetry (best for photo type images)

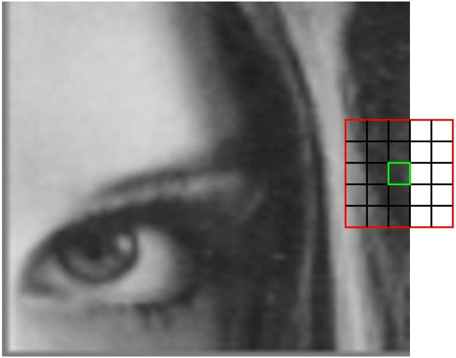
Spatial filter (2D convolution)

Operation on some vicinity of the current pixel.



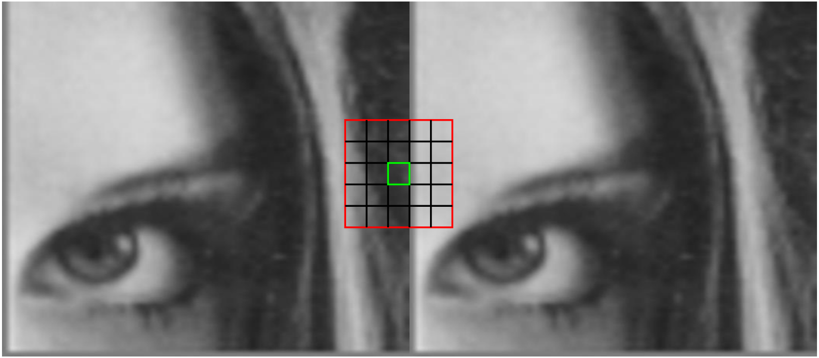
$$y(m,n)=\sum_{i=-\infty}^{+\infty}\sum_{j=-\infty}^{+\infty}x(i,\ j)\cdot h(m-i,\ n-j)$$

Edge problem



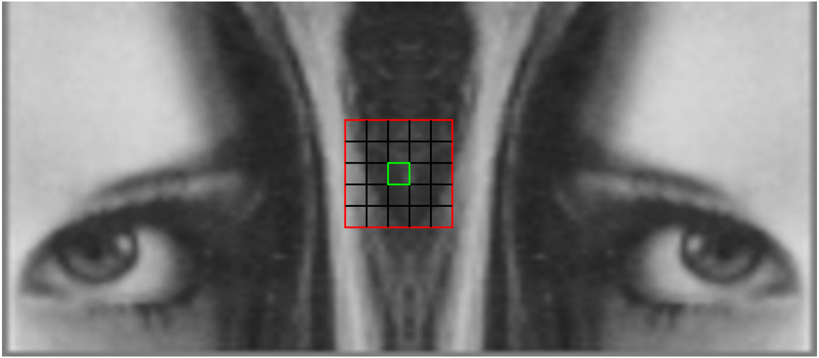
Assume zero outside

Edge problem



Assume circular copy

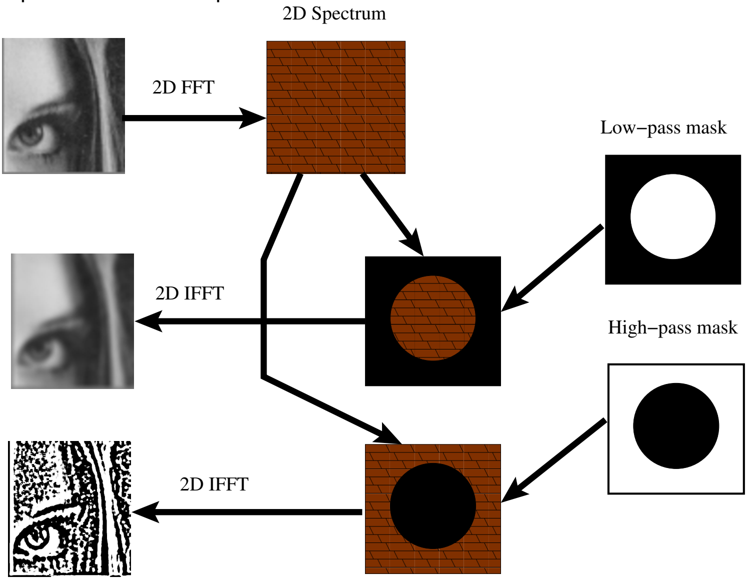
Edge problem



Assume flipped (mirror) copy

Spectral domain filering (by FFT)

Operation on whole picture.



Prepare for the lab!

- Understanding frequency in 2D
- Calculate a simple 2D Fourier transform: practice on 2x2 or 4x4 pictures:

0	0	0	0
1	1	1	1
0	0	0	0
1	1	1	1

0	1	0	1
0	1	0	1
0	1	0	1
0	1	0	1

1	0	1	0
0	1	0	1
1	0	1	0
0	1	0	1

- Apply a linear filter to above pictures.
- Understand a median. Why it is a nonlinear operation?