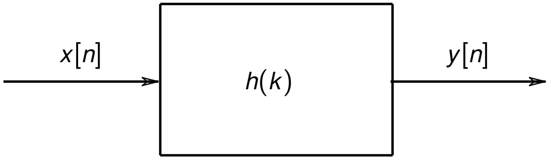
EDISP (Filters intro)
(English) Digital Signal Processing
Digital/Discrete Time/ filters introduction
lecture

November 26, 2007

DT system characteristics

An LTI system is described with its impulse response



$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

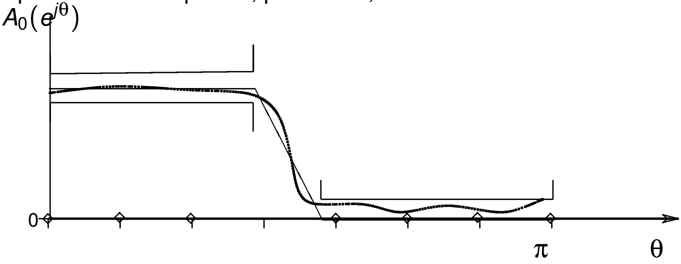
but we are interested in its characteristics in the frequency domain

$$H(e^{j\theta}) = H(z)|_{z=e^{j\theta}} = \sum_{n=-\infty}^{\infty} h(n) e^{-jn\theta}$$

Amplitude characteristics	$A(\theta)$	$=$	$\lvert H(e^{j\theta}) \rvert$
Phase characteristics	$\varphi(\theta)$	$=$	$arg[H(e^{j\theta})]$
Group delay	$\tau(\theta)$	$=$	$-d\varphi(\theta)/d\theta$

Filter design

- Specification: stopband, passband, tolerances



- Approximation: find best rational function

$$\frac{b_0 + b_1 z^{-1} + \ldots + b_M z^{-M}}{1 + a_1 z^{-1} + \ldots + a_N z^{-N}} \quad \text{(IIR)}$$

or

$$b_0 + b_1 z^{-1} + \ldots + b_M z^{-M} \quad \text{(FIR)}$$

determine order and coefficients, check stability

- Implementation: structure, noise, hardware/software ...

FIR filter design – window method

- Ideal filter: $A_0(\theta) = \begin{cases} 1 & \text{for} \quad |\theta| < \theta_p \\ 0 & \text{for} \quad \theta_p < |\theta| \leq \pi \end{cases}$ and zero phase
- Impulse response:

$$h_0(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_0\left(e^{j\theta}\right) e^{jn\theta} \, d\theta \; = \; \frac{\theta_p}{\pi} \, \frac{\sin n\theta_p}{n\theta_p}$$

  is non-causal and infinite!
- Make it finite: $h_P[n] \; = \; h_0[n] \, g[n]$ ($g[n] = 0$ for $|n| > P$)
- Shift it to be causal – delay by P samples: $h[n] = h_P[n-P]$

$\longrightarrow$ finally we obtain

$$H(z) \; = \; \sum_{n=0}^{2P} h(n) \, z^{-n} \; = \; z^{-P} H_P(z)$$

Implementations

- Transversal (linear convolution)
- By $IFFT(FFT(x) \cdot FFT(h))$
- Lagrange structure: FIR with IIR inside