Name:_____

Solve long problems on an additional sheet,
marked with your name. For the short prob-
lems, try to write the answer in the provided
space. Put your calculations on the additional
sheet.

1	
2	
3	
4	
5	
Σ	

1. (6 p.) Let $x[n]$ be a signal obtained by sampling a continuous-time cosine wave of 0.125 ms
   period with sampling frequency of 48 kHz

   (a) Calculate the period N_0 and normalized angular frequency θ_0 of the $x[n]$.

   (b) Sketch the absolute value of DFT $X(k)$ for the transform size equal to $K = 4 \cdot N_0$.

   (c) Label the frequency axes carefully with index k and with θ values.

$N_0 = \ldots\ldots\ldots\ldots$
$\theta_0 = \ldots\ldots\ldots\ldots$
$\ldots\ldots\ldots\ldots\ldots$

2. (10 p.) A causal IIR filter is described by an equation

$$y(n)+0.8y(n-1) = -0.2x(n)+0.2x(n-1)$$

   (a) Find the transfer function $H(z)$ of the filter: $H(z)=\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

   (b) Find zeros and poles of the filter: $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

   (c) Sketch the graph of a filter implementation

   (d) Find the output for a discrete input signal defined as $x(n) = \delta(n-4)$

   (e) Find the output for a discrete input signal defined as $x(n) = 1 - (-1)^n$

3. A FIR filter is defined by its impulse response

$$h(n) = \begin{cases} (-1)^n & \text{for} \quad 0 \le n < 9 \\ 0 & \text{otherwise} \end{cases}$$

   (a) (2 p.) Plot the phase characteristics of the filter. Find the group delay.

   (b) (3 p.) Sketch the *approximate* amplitude characteristics of the filter (exact cal-
       culations aren't required). Try to be exact with frequency limits, pass- stop- and
       transition- band widths etc. Calculate the value of gain $(A(\theta))$ for DC and for $\theta = \pi$.

4. (6 p.) If we calculate the instantaneous spectrum of a signal using a window of length K,

   (a) the time resolution is proportional to $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

   (b) the frequency resolution is proportional to $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

(c) The rectangular window is good for ....
because ....

(d) If we use Blackmann window with the same length, the resolution in frequency will
be ....

5. *"Tricky questions": present your reasoning for each answer, otherwise scores will be lower.*

   (a) (3 p.) A DT system is described as: $T(x[n]) = 2x[n+2] - x[n-2] \cdot x[n-1]$; is T

      - stable yes:☐ or no:☐ explain why:...
      - causal yes:☐ or no:☐ explain why:...
      - linear yes:☐ or no:☐ explain why:...

   (b) (3 p.) Describe a filter that has to be used in the procedure of upsampling by 16x -
       filter type and passband limits ....
       ....

   (c) (4 p.) When calculating PSD (Power Spectral Density) of a random signal - is it
       better to take FFT of one long segment of data, or to split the given data into many
       shorter segments and do shorter FFT's? Explain why? ....
       ....
       ....

   (d) (2 p.) In a Kaiser window - what depends on the β parameter?
       Answer:...

   (e) (2 p.) The inverse Fourier transform of periodic signal is calculated by ☐ summation
       ☐ integration (choose answer, and explain why: ....)

   (f) (4 p.) Why does a digital signal processor usually have three memory buses?
       Answer:...
       Give an example of instruction that needs all the 3 memories.
       Answer:...

   (g) (2 p.) Calculate the $\mathcal{Z}$ transform of a signal $x[n] = \delta[n] + 2\delta[n-2] - 3\delta[n+2]$

       Answer:...

   (h) (4 p.) How many complex number multiplications do we need to compute a DFT of
       a finite-time signal of 2048 samples, using:

      - the definition formula? Answer: ☐ calculation:.............................
      - the FFT method? Answer: ☐ calculation:...................................

   (i) (3 p.) What length may be a filter impulse response if for many diferent input signals
       with 7 contiguous non-zero samples the output has 11 contiguous non-zero samples?
       Answer: ☐ calculation:..

   (j) (2 p.) Why cannot the previous answer be 100% sure? Show an example of bad
       guess. Answer: ....

$\Sigma = 56p$ $T = 90$ min