Homework2/0809+ – z-transform, inst. spectrum, filters

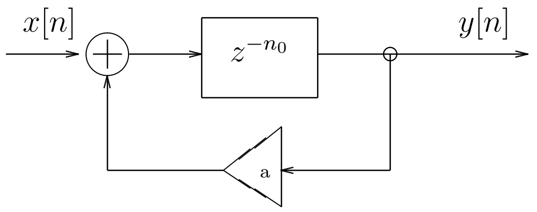
1. The instantaneous spectrum $X(e^{j\theta}, n)$ of a signal

$$x(n) = \begin{cases} z(n) & \text{for} \quad n = 10, \ \ldots, 49 \\ 0 & \text{otherwise} \end{cases}$$

is computed using rectangular window $g(k)$ of length $K = 10$.
Assume i) $z(n) = 1$ ii) $z(n) = e^{j\theta_0 \cdot n}$ iii) $z(n) = sin(\theta_0 \cdot n)$

   (a) How does the mainlobe width of $X(e^{j\theta}, n)$ change with n (make approximate sketch).
   (b) What is the minimum width of the mainlobe (write an expression)
   (c) If we change K, or the window type – how will the mainlobe change?
   (d) Sketch the value of $X(e^{j\theta}, n)$ versus n with $\theta = \theta_0$.

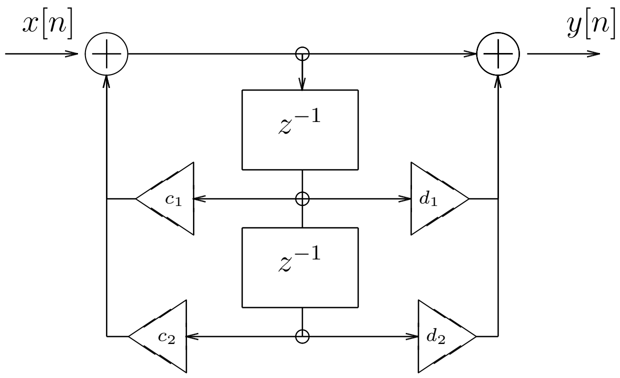
2. Analyze an IIR filter with graph as follows



For plots, assume $a = 0.5$, $n_0 = 2$

   (a) find $h[n]$, $H(z)$
   (b) Plot zeros and poles of the filter
   (c) How the filter stability depends on n_0 and a?
   (d) Plot filter amplitude characteristics
   (e) Find filter response for $x(n) = 1 + cos(n\pi)$

3. Analyze a filter described by graph:



   (a) Find $H(z)$
   (b) Find the conditions on c_1, c_2, d_1, d_2 to assure filter stability.
   (c) find $h(n)$ for $c_1 = 0$, $c_2 = 0$, $d_1 = -2$, $d_2 = 1$
   (d) Sketch $A(\theta)$ for $c_1 = 1.8 \cdot \cos(\pi/4)$, $c_2 = -.81$, $d_1 = -2$, $d_2 = 1$.

Hint: Mark a signal after the first adder with $r(n)$, then eliminate it from the equations.

4. Calculate the z-transform and its region of convergence for the following series:

   (a) $\delta[n]$
   (b) $u[n] - u[n - n_0]$
   (c) $u[n] \cdot 0.4^n$
   (d) $u[-n] \cdot -0.4^n$

5. Assuming phase equal to zero, calculate the impulse response of:

   (a) an ideal lowpass filter with cutoff frequency of θ_b
   (b) an ideal highpass filter with cutoff frequency of $\pi - \theta_b$
   (c) an ideal bandpass filter with passband of $\theta_c \pm \theta_b$

   Then, assume $\theta_b = pi/4$ and try to design a filter with rectangular window method, with order of 7. Try to calculate the filter characteristics by hand, then check with a computer.

Additional problems. . .

. . . may be found in *Oppenheim and Schafer with Buck*:

- z-transform: basic problems from Chapter 3 ("The z-transform")
- filters: basic problems from Chapter 5 ("Transform analysis of LTI systems")
- filter design: first few basic problems from Chapter 7 ("Filter design techniques")
- instantaneous spectrum: problems 10.9, 10.13 – 10.20 (problems from Chapter 10 "Fourier analysis of signals using the DFT" that have "TDFT" or "time-dependent FT" inside)

note: if yout have other editions of O. and S., the chapter numbering will be different, but contents are similar.

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