

## Lab 2 – DT signal frequency, Fourier transform

### Entry test example questions

1.  $x_a(t) = \cos(2\pi f_a t)$  was sampled with sampling period  $T_s$ . Find normalized frequency, normalized angular frequency  $\theta$  or period of the sampled signal. ( $f_a$ ,  $T_s$  or  $f_s$  given, their proportion rational or irrational...)
2. A signal  $x(n)$  with known Fourier spectrum  $X(\theta)$  has been {inverted in time — decimated — modulated — ...}. Express mathematically what happened to the spectrum.
3. Calculate a DFT of a simple finite signal (by pen and paper...)

### Exercises

*Italics denote optional tasks.*

1. Implement  $y(n) = a \cdot y(n-1) + x(n)$ , accepting  $a$  and initial  $y$  as parameters. Test impulse response with zero initial condition, initial cond. response, then the combination of both for  $0 < a < 1$ .
2. Experiment with different values of  $a$ . ( $1$ ,  $-1$ ,  $> 1$ ,  $< 0$  etc.).
3. Plot a sinusoid with normalized frequency  $f_n = f/f_s$  equal to  $0.1$ ,  $0.3$ ,  $0.5$ ,  $0.9$ ,  $1.1$ ,  $2.1$  ( $\theta = 2\pi f_n$ ,  $x(n) = \sin(n\theta)$ ). Note number of samples in period. Explain the plots - try to draw the underlying CT signal.
4. Use program `anator` to display real-time signal and its spectrum – measure a sinusoid with different relations of  $f$  and  $f_s$  ( $f < f_s/2$ ,  $f \approx f_s/2$ ,  $f > f_s/2$ , etc.). Comment the plots. ( $f_s \approx 38\text{kHz}$ )
5. Simulate 2 ms of samples of a single square impulse of 1 ms length, sampled with:
  - (a) 1 MHz
  - (b) 10 kHz
  - (c) 10 kHz, but use 4 ms of samples

Plot amplitude of FFT's of all signals on one graph, keeping the real-world frequency axes the same and scaling the 5a signal 100 times down. *Find out from the FFT definition why the scaling is necessary (compare different length FFTs of a DC signal).*

Think of 5a as “almost CT” signal and comment the spectrum differences.

6. Plot an FFT of 1024 points of following signals:

- (a) a 512 points square impulse
- (b) other (narrower) square impulses
- (c) sine wave (integer and non-integer number of periods in window)
- (d)  $e^{jn\theta_c}$  (how many peaks do you see? why?) Try different values of  $0 < \theta_c \leq \pi$ .
- (e) *a 32-point square impulse beginning at 0*
- (f) *a 32-point square impulse beginning at  $N_0 > 0$*

(name the effects, note the number of zero places in spectrum etc.)

7. Plot a spectrum of 512 samples of sine wave. Then, zero-pad them to 1024 and 2048 samples. Compare the results. Compute IFFT. (plot real part of IFFT to cut off arithmetic errors). Hint: `fft(x,L)` automatically zero-pads signal x to length L.

8. Compute spectra of different windows. Note mainlobe width, sidelobe attenuation etc.

(If you have enough time, use Matlab: `hamming`, `bartlett`, `blackman`, `hanning`, `kaiser`, otherwise use Windows program “anator”).

9. Do the following experiments to see the effect of windowing:

- (a) Plot a spectrum of 512 samples of sine wave. Choose the frequency to see the rectangular window effect clearly. If necessary, use zero-padding to see the spectrum better.
- (b) Use different window shapes, trying to obtain good, clear plot of the spectrum.
- (c) Demonstrate the signal separation properties of different windows - plot a spectrum of a sum of two sinusoids with similar frequencies and amplitudes, then with very different frequencies and amplitudes.