Test 2 (2007/8) **version B** – inst. spectrum, z-transform, filters

Please mark your name and test version on all your answer pages

1. (3 p.) The instantaneous spectrum $X(e^{j\theta}, n)$ of a signal

$$x(n) = \begin{cases} 1 & \text{for} \quad -M/2 \le n \le M/2 \quad \text{(Assume M is even)} \\ 0 & \text{otherwise} \end{cases}$$

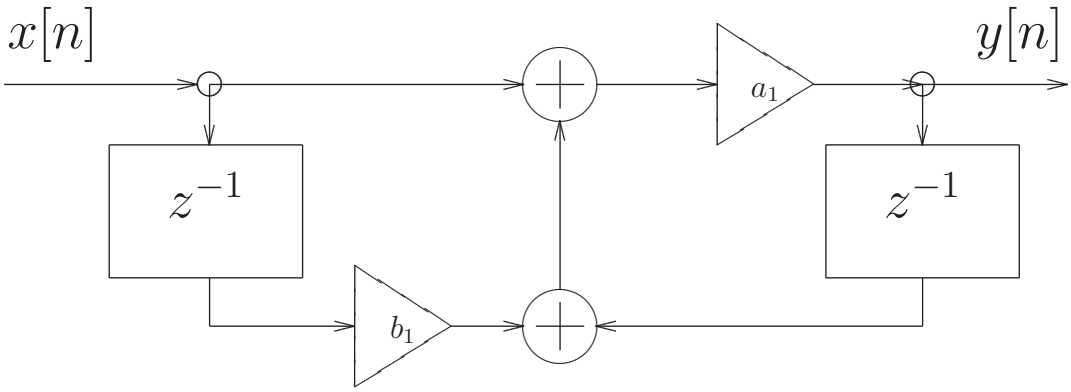
is computed using rectangular window $g(k)$ of length $K = 3M$.

   (a) For M=5 and given below values of n, calculate $X(e^{j\theta}, n)$ at $\theta = 0,\ \theta = \pi/2,\ \theta = \pi$ and sketch $|X(e^{j\theta}, n)|$ for all θ

   - $n = -M$
   - $n = 0$
   - $n = M$

   (b) *(optional - extra points)* Describe or sketch (without detailed calculations) $|X(e^{j\theta}, n)|$ for $\theta = \pi$ and $n = -3M,\ \ldots, 3M$.

2. (5 p.) Analyze a filter described with the following graph:



   (a) Find $H(z)$

   (b) Find conditions that assure filter stability.

   (c) Plot approximate $A(\theta)$ for $a_1 = -0.8$, $b_1 = 1$

   (d) With parameters as above, calculate response for $x(n) = \delta(n)$ and for $x(n) = -1 + cos(n\pi/2) + sin(n\pi)$

3. (2 p.) Calculate the z-transform and determine ROC (region of convergence) for the series:

   (a) -2$\delta[-n]$

   (b) $\delta[n]$+$\delta[n - n_0]$(n_0 is a constant)

   (c) $2u[-n]$

   (optional - extra points) Sketch the Fourier transforms for above signals, if they exist.

4. (2 p.) A causal lowpass FIR filter of the order 8 was designed from windowed Inverse Fourier Transform of the zero-phase ideal filter characteristics. A rectangular window was used. Ideal filter cutoff was at $\theta_b = \frac{2\pi}{4}$.

   (a) Plot the phase characteristics of the resulting filter. Find the group delay.

   (b) Sketch the amplitude characteristics of the resulting filter. Show and describe how the characteristics could change if other window type were used.

   (c) *(optional - extra points)* Calculate filter coefficients

$\Sigma = 12p\ T = 45\ min$