

EDISP (Filters 3)
(English) Digital Signal Processing
Two-dimensional signals & filters
lecture

January 3, 2011

Multidimensional signals

- ▶ Analogue K-D signal $x_a(t_1, t_2, t_3, \dots, t_K)$, t_k not necessarily time.
- ▶ Discretization (sampling) $\rightarrow x(n_1, n_2, n_3, \dots, n_K)$, *some signals are already discrete!*
 \rightarrow multidimensional discrete signal = multidimensional number series
- ▶ sampling periods $T_{s k}$ \rightarrow sampling frequencies $f_{s k} = \frac{1}{T_{s k}}$ not necessarily equal; (e.g some scanners have different h & v resolutions)
if t_k is spatial, $f_{s k}$ is *spatial frequency*
- ▶ Examples
 - ▶ 2-D picture
 - ▶ linear antenna array (t_1 – discrete or continuous space, t_2 – continuous time)
 - ▶ pulsed radar signal (“slow time” and “fast time”)

Image – a 2-D signal

- ▶ T_{s1}, T_{s2} – pixel dimensions; f_{s1}, f_{s2} – resolution (dpi, lines/mm)
- ▶ Fourier spectrum:

$$X(e^{j\theta_1}, e^{j\theta_2}) = \sum_{n_1=-\infty}^{+\infty} \sum_{n_2=-\infty}^{+\infty} x(n_1, n_2) e^{-jn\theta_1} e^{-jn\theta_2}$$

$\theta_k = \omega_k \cdot T_{sk} = \frac{2\pi f_k}{f_{sk}}$ – normalized angular frequency

- ▶ finite picture \rightarrow represented by a discrete spectrum

$$X(k, l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) e^{-j2\pi km/M} e^{-j2\pi ln/N}$$

- ▶ reconstruction by a discrete Fourier series

$$x(m, n) = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} X(k, l) e^{j2\pi km/M} e^{j2\pi ln/N}$$

2-D LTI systems

Linear and (*Time = shift in n_k*) Invariant (we extend the definition for 1-D systems)

- ▶ allows for analysis by impulse response
(unit impulse: $\delta(m, n) = 1$ if $m = n = 0$, $= 1$ otherwise)
impulse response is sometimes called Point Spread Function – PSF
- ▶ causality – meaningful if one of dimensions is time-related
- ▶ delay - in spatial dimension, zero-delay filter is the best
- ▶ 2-D convolution (linear filtering)

$$y(m, n) = \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} x(i, j) \cdot h(m-i, n-j)$$

- ▶ if $h(m, n) = h_1(m) \cdot h_2(n)$, we may decompose 2-D filtering into 2x(1-D)
(important for long impulse responses)

Images - practical remarks

We concentrate on monochrome images (B/W photos, print, raw images in medical, radar, sonar, satellite technology). Color images add some complexity, unimportant for the signal processing basics.

Understanding frequency concept:

- ▶ Horizontal frequency - changes in the horizontal direction (trunks in the forest)
- ▶ Vertical frequency - changes in the vertical direction (horizon)
- ▶ High frequencies - sharp edges, small features
- ▶ Low frequencies - soft edges, large areas of same intensity

Filtering

Filtering methods:

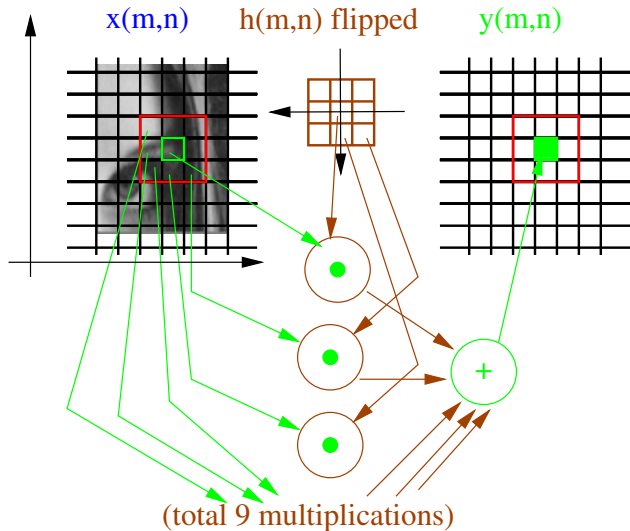
- ▶ Spatial (image) domain: convolution=weighted average of neighboring pixels
- ▶ Frequency domain: masking out (zeroing) parts of the 2D spectrum
- ▶ Nonlinear filters: some nonlinear manipulation on the set of neighboring pixels (e.g median)

How to behave at the image boundary with spatial filters?

- ▶ assume zeros outside (effect:dark areas at the boundaries)
- ▶ repeat last data row/column
- ▶ circular symmetry (effect: sky under the ground?)
- ▶ mirror symmetry (best for photo type images)

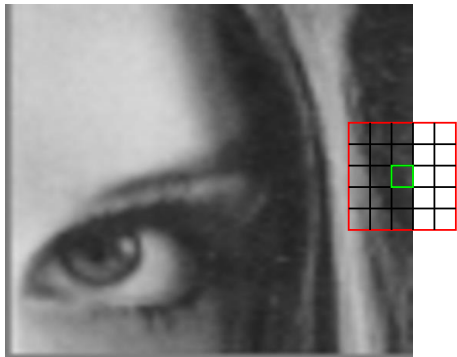
Spatial filter (2D convolution)

Operation on some vicinity of the current pixel.



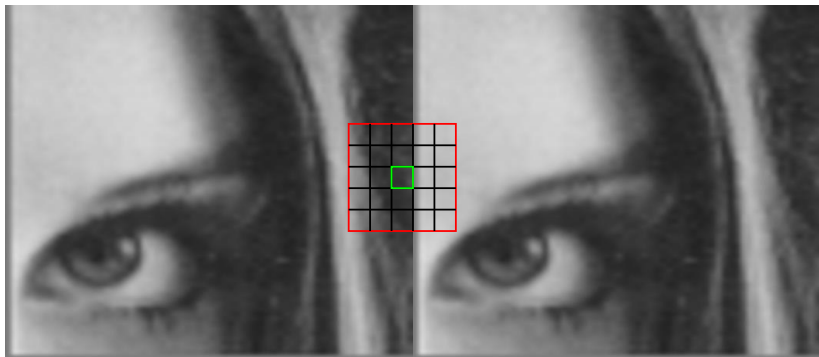
$$y(m,n) = \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} x(i,j) \cdot h(m-i, n-j)$$

Edge problem



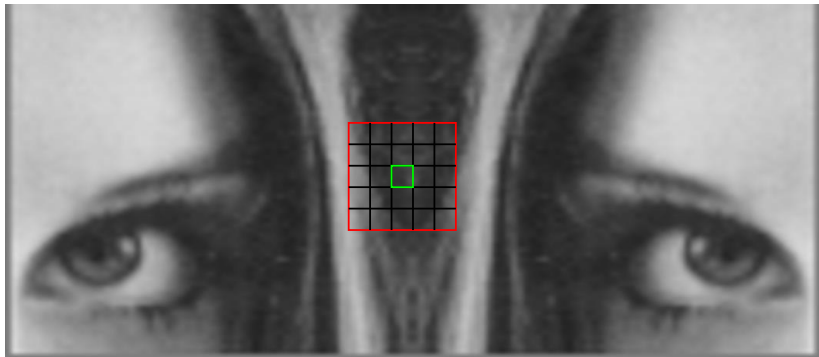
Assume zero outside

Edge problem



Assume circular copy

Edge problem



Assume flipped (mirror) copy

Spectral domain filtering (by FFT)

Operation on whole picture.

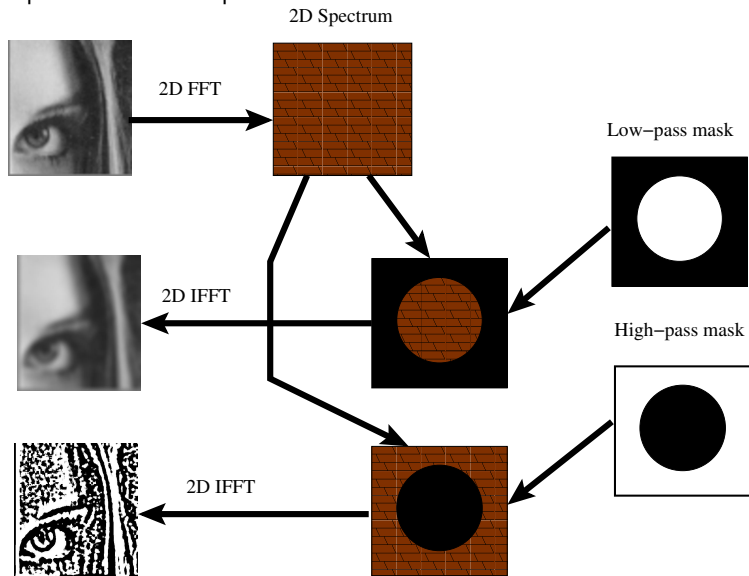


Image denoising

- ▶ AWGN - linear filtering
- ▶ impulsive noise (“salt and pepper”) - nonlinear methods

Median filter (nonlinear)

- ▶ removing impulses
- ▶ preserving edges
- ▶ method: replace a pixel with median from its vicinity (3x3, 5x5, ...)

Median filter

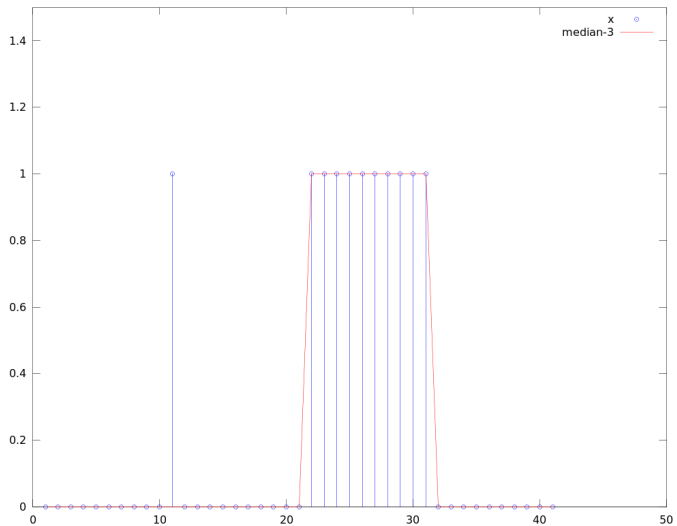
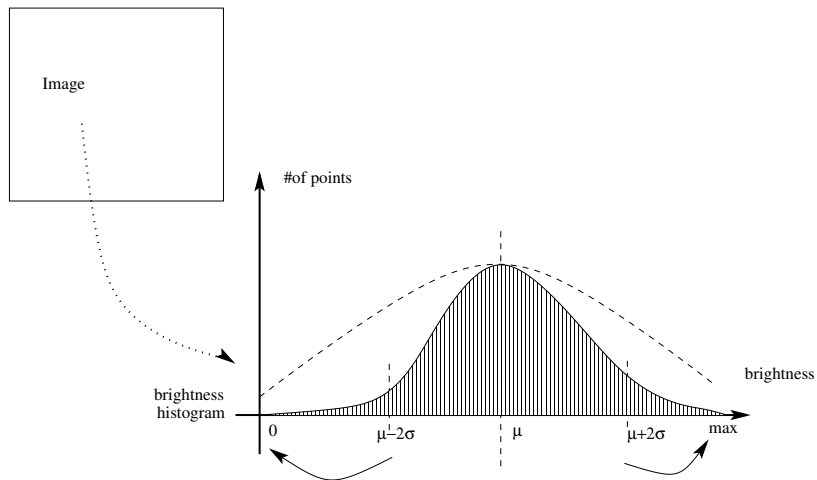


Image brightness normalization



Prepare for the lab!

- ▶ Understanding frequency in 2D
- ▶ Calculate a simple 2D Fourier transform: practice on 2x2 or 4x4

pictures:

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |

| | | | |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 |

| | | | |
|---|---|---|---|
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |

- ▶ Apply a linear filter to above pictures.
- ▶ Understand a median. Why it is a nonlinear operation?