Test 2 (2010/11) **version B** – inst. spectrum, z-transform, filters
Please mark your name and test version on all your answer pages

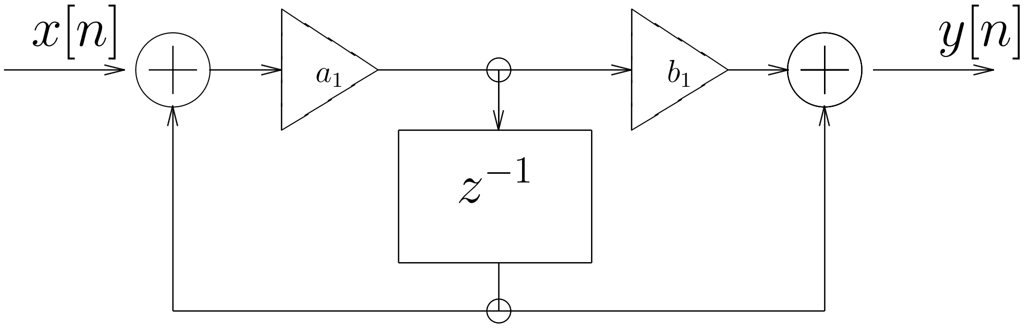
1. (3 p.) The instantaneous spectrum $X(e^{j\theta}, n)$ of a signal

$$x(n) = \delta(n+5) - \delta(n-5)$$

is computed using rectangular window $g(k)$ of length $K = 13$.

- sketch $|X(e^{j\theta}, n)|$ for all θ and calculate $X(e^{j\theta}, n)$ at $\theta = 0$ and $\theta = \pi$ for:
  (a) $n = -4$.
  (b) $n = 0$.
  (c) $n = +5$.

2. (5 p.) Analyze a filter described with the following graph:



Assume $a_1 = -0.9$ $b_1 = -1$. *Hint: introduce additional signal $v[n]$ at the output of the delay block; it will be cancelled when calculating Y/X.*

(a) Find $H(z)$, $h(n)$. Check if the filter is stable
(b) Sketch approximate $A(\theta)$
(c) Calculate response y(n) for $x(n) = \delta(n-1) + \delta(n+1)$
(d) Calculate response y(n) for $x(n) = 3 - sin(n\pi/2)$

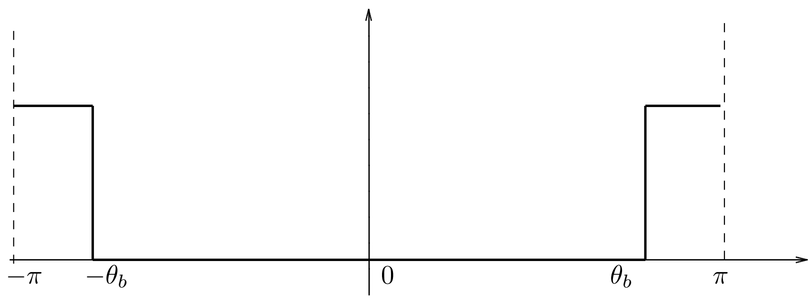
3. (2 p.) Calculate the z-transform and determine ROC (region of convergence) for the series:

(a) $\delta[n-1]$
(b) $\delta[n-3] - \delta[n+3]$
(c) $u[n+1] \cdot (-1)^n$

(optional - extra points) Sketch the Fourier transforms for above signals, if they exist.

4. (2 p.) A causal FIR filter of the order 10 was designed from windowed Inverse Fourier Transform of the zero-phase ideal filter frequency response (see figure). A rectangular window was used. Ideal filter cutoff was at $\theta_b = (7/8)\pi$.

(a) Sketch the phase characteristics $\phi(\theta)$ of the resulting filter.
(b) Find the approximate width of the transition band in the amplitude characteristics.
(c) Sketch the impulse response.



$\Sigma = 12p$ $T = 45$ min