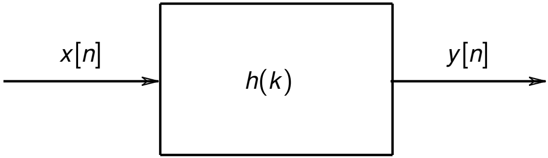
EDISP (Filters intro)
(English) Digital Signal Processing
Digital/Discrete Time/ filters introduction
lecture

January 10, 2011

DT system properties

An LTI system is described with its impulse response



$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

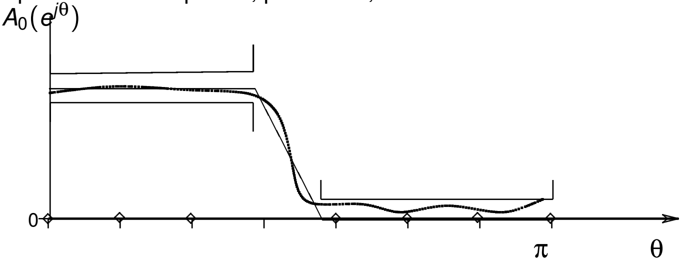
but we are interested in its characteristics in the frequency domain (frequency response)

$$H(e^{j\theta}) = H(z)|_{z=e^{j\theta}} = \sum_{n=-\infty}^{\infty} h(n) e^{-jn\theta}$$

Magnitude of fr. response	$A(\theta)$	$=$	$\lvert H(e^{j\theta})\rvert$
Phase of fr. response	$\varphi(\theta)$	$=$	$arg[H(e^{j\theta})]$
Group delay	$\tau(\theta)$	$=$	$-d\varphi(\theta)/d\theta$

Filter design

- Specification: stopband, passband, tolerances



- Approximation: find best rational function

$$\frac{b_0 + b_1 z^{-1} + \ldots + b_M z^{-M}}{1 + a_1 z^{-1} + \ldots + a_N z^{-N}} \quad \text{(IIR)}$$

  or

$$b_0 + b_1 z^{-1} + \ldots + b_M z^{-M} \quad \text{(FIR)}$$

  determine order and coefficients, check stability

- Implementation: structure, noise, hardware/software . . .

FIR filter design – window method

- Ideal filter: $A_0(\theta) = \begin{cases} 1 & \text{for} \quad |\theta| < \theta_p \\ 0 & \text{for} \quad \theta_p < |\theta| \leq \pi \end{cases}$ and zero phase
- Impulse response:

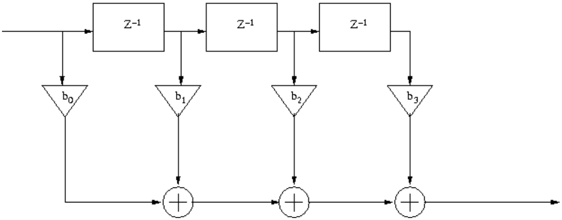
$$h_0(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_0\left(e^{j\theta}\right) e^{jn\theta}\, d\theta \;=\; \frac{\theta_p}{\pi} \frac{\sin n\theta_p}{n\theta_p}$$

  is non-causal and infinite!
- Make it finite: $h_P[n] \;=\; h_0[n]\, g[n]$ ($g[n] = 0$ for $|n| > P$)
- Shift it to be causal – delay by P samples: $h[n] = h_P[n-P]$

$\longrightarrow$ finally we obtain

$$H(z) \;=\; \sum_{n=0}^{2P} h(n)\, z^{-n} \;=\; z^{-P} H_P(z)$$

Implementations

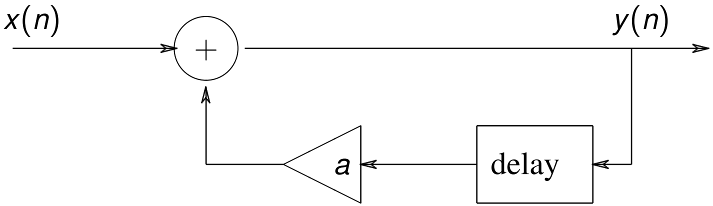


- Transversal (linear convolution)
- By $IFFT(FFT(x) \cdot FFT(h))$
- Lagrange structure: FIR with IIR inside

$$H(z) = \frac{(1 - z^{-N})}{N} \sum_{k=0}^{N-1} \frac{H(k)}{1 - e^{-j2\pi k/N} z^{-1}}$$

where $H(k)$ are samples of frequency response at $\theta_k = 2\pi K/N$; efficient if many $H(k)$ are almost zero.

Example



$$y(n) = x(n) + ay(n-1)$$
$$Y(z) = X(z) + aY(z)z^{-1}$$
$$Y(z) = X(z)\frac{1}{1-az^{-1}}$$
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-az^{-1}} = \frac{z}{z-a}$$

so $h(n) = a^n u(n)$ if we assume causal solution.

Example cont.

$$H(z) = \frac{z}{z-a}$$

$$H(e^{j\theta}) = \frac{e^{j\theta}}{e^{j\theta} - a}$$

$$|H(e^{j\theta})| = \frac{1}{|e^{j\theta} - a|}$$

$$|H(e^{j\theta})| = \frac{1}{|cos\theta - a + jsin\theta|} = \frac{1}{\sqrt{1 - 2a \cdot cos\theta + a^2}}$$