November 4, 2011

Lab 2 – Fourier transform, DFT, FFT

Entry test example questions

1. $x_a(t) = cos(2\pi f_a t)$ was sampled with sampling period T_s. Plot the { spectrum | N-point DFT } of $x[n]$ (f_a, T_s or f_s given, N given - whole number of periods or not)
2. A signal $x(n)$ with known Fourier spectrum $X(\theta)$ has been {inverted in time | decimated | modulated | ...}. Express mathematically what happened to the spectrum.
3. Calculate a DFT of a simple finite signal (by pen and paper...)

Exercises

Italics denote optional tasks.

1. Simulate 2 ms of samples of a single square impulse of 1 ms length, sampled with:
   (a) 1 MHz
   (b) 10 kHz
   (c) 10 kHz, but use 4 ms of samples

   Remark: first, calculate by pencil and imagine (or even sketch) the signal, then produce it using ones(), zeros(), and [] operators in Matlab. Calculate (with Matlab) and plot amplitude of FFT's of all signals on one graph, keeping the real-world frequency axes the same and scaling the 1a signal 100 times down. *Find out from the FFT definition why the scaling is necessary (compare different length FFTs of a DC signal).*

   Think of 1a as "almost CT" signal and comment the spectrum differences.

2. Plot an FFT of 1024 points of following signals:
   (a) a 512 points square impulse
   (b) other (narrower) square impulses
   (c) sine wave (integer and non-integer number of periods in window)
   (d) $e^{jn\theta_c}$ (how many peaks do you see? why?) Try different values of $0 < \theta_c \leq \pi$.
   (e) a 32-point square impulse beginning at 0

(f) *a 32-point square impulse beginning at* $N_0 > 0$

(name the effects, note the number of zero places in spectrum etc.)

3. Plot a spectrum of 512 samples of sine wave. Then, zero-pad them to 1024 and 2048 samples. Compare the results. Compute IFFT. (plot real part of IFFT to cut off arithmetic errors). Hint: `fft(x,L)` automatically zero-pads signal x to length L.

4. Capture 1024 samples of a real signal from a generator. Choose some signal (sin, rectangular,...) and set the f and f_s using your own wisdom. Plot, labeling properly the horizontal axis:

   (a) the signal

   (b) its 1024-point FFT

   (c) its 2^{12} *or even* ^{14}-point FFT (with zero-padding)

   Save the signal in some variable.

5. Compute spectra of different windows. Note mainlobe width, sidelobe attenuation etc.

   (If you have enough time, use Matlab: `hamming, bartlett, blackman, hanning, kaiser`, otherwise use Windows program "anator").

6. Do the following experiments to see the effect of windowing:

   (a) Plot a spectrum of 512 samples of sine wave. Choose the frequency to see the rectangular window effect clearly. If necessary, use zero-padding to see the spectrum better.

   (b) Use different window shapes, trying to obtain good, clear plot of the spectrum.

   (c) Demonstrate the signal separation properties of different windows - plot a spectrum of a sum of two sinusoids with similar frequencies and amplitudes, then with very different frequencies and amplitudes.

7. Repeat FFT plots from Ex. 4, using a window (e.g. Hamming) on the signal.

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