

Lab 2 – Fourier transform, DFT, FFT

ver. March 17, 2015

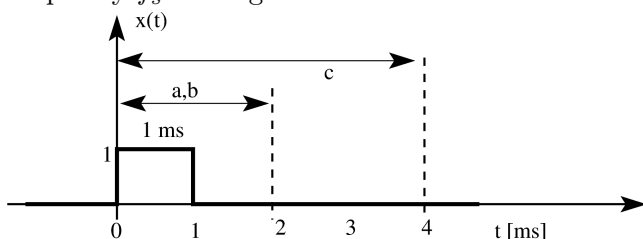
Entry test example questions

1. $x_a(t) = \cos(2\pi f_a t)$ was sampled with sampling period T_s . Plot the { spectrum | N-point DFT } of $x[n]$ (f_a , T_s or f_s given, N given - whole number of periods or not)
2. A signal $x(n)$ with known Fourier spectrum $X(\theta)$ has been {inverted in time | decimated | modulated | ...}. Express mathematically what happened to the spectrum.
3. Calculate a DFT of a simple finite signal ($\delta(n-1)$, constant, $\{+1, -1, +1, \dots\}$) - on paper

Lab exercises

Italics denote optional tasks.

1. Investigate a single square impulse of 1 ms length, sampled under different conditions (sampling frequency f_s and signal measurement duration T – see table).



case	f_s	T	N	N_1	A_{max}	k_{null}	f_n at null	f at null
$x_1[n]$	1 MHz	2 ms						
$x_2[n]$	10 kHz	2 ms						
$x_3[n]$	10 kHz	4 ms						

Copy the table to the report – you will later fill the empty table cells with answers to the following.

For each sampled signal ($x_1[n]$, $x_2[n]$, $x_3[n]$):

- (a) Calculate total number of samples (fill in N column) and number of non-zero samples (fill in N_1 column)
- (b) Create a simulated signal in Matlab (`[ones(1, N1), zeros(1, N-N1)]`)
- (c) Calculate with Matlab and plot (on screen) magnitude of FFT (`plot(abs(fft(x...)))`)
- (d) Find the maximum value in FFT (and fill in as A_{max})
- (e) Find (and fill in) the index k_{null} of the first null in FFT values (Note: remember that Matlab numbering starts from 1, but we need the k for indexing frequency θ_k , so we start from 0).

Think of signal $x_1[n]$ as “sampled with such a high frequency that it is almost CT” and comment the spectrum differences between the three signals.

2. Simulate in Matlab 1024 points of following signals. Then, for each signal

- Plot the signal
- Plot magnitude of FFT (sketch it in the report)
- (for signals (a) and (b)) count number of zero crossings in FFT (note it in report)
- (for signal (c)) note the locations of the peak, compare peak width for the two frequencies

Signals:

- (a) a 512 points square impulse (so you need to add 512 zeros to get 1024 samples)
 - (b) other (narrower) square impulses – fill them up to 1024
 - (c) sine wave (choose two frequencies to get integer and non-integer number of periods in window of 1024)
 - (d) $e^{jn\theta_c}$ - use `exp()` in Matlab; how many peaks do you see? why? Try different values of $0 < \theta_c \leq \pi$.
 - (e) a 32-point square impulse beginning at 0
 - (f) a 32-point square impulse beginning at $N_0 > 0$
3. Plot a spectrum of 512 samples of sine wave. Then, zero-pad them to 1024 and 2048 samples. Compare the resolution of FFT. Sketch `abs(fft())` and note peak width. Compute IFFT. (plot real part of IFFT to cut off arithmetic errors). Hint: `fft(x,L)` automatically zero-pads signal `x` to length `L`.
4. Capture 1024 samples of a live signal from a generator (use `getdata(Nsamples,1,TsamplingInSeconds)`). Choose some signal (sin, rectangular,...) and set the f and f_s using your own wisdom. Plot, labeling properly the horizontal axis:
- (a) the signal
 - (b) its 1024-point FFT (magnitude, of course)
 - (c) its 2^{12} or even 14 -point FFT (with zero-padding: `fft(x,N)` where N is the total length – with added zeros)

Save the signal in some variable.

5. Compute spectra of different windows.
- If you have time, do it from the Matlab commandline. Use window length of 64 and FFT with 4096 points (with zero-padding). Plot them using logarithmic (decibel) scale. In Matlab, window functions can be generated using: `rectwin`, `hamming`, `bartlett`, `blackman`, `hanning`, `kaiser`, with a scalar argument giving the length. For Kaiser – the second argument is β , use values between 1 and 8.
 - If you don't have enough time, use matlab tool `window`.

Fill in the table:

Window type	Mainlobe width (normalized freq.)	First sidelobe (dB below mainl.)	highest sidelobe (dB below mainl.)	Sidelobes change with f (describe shortly)

6. Do the following experiments to see the effect of windowing:
- (a) Plot a spectrum of 512 samples of sine wave. Choose the frequency to see the rectangular window effect clearly. If necessary, use zero-padding to see the spectrum better.
 - (b) Use different window shapes, trying to obtain good, clear plot of the spectrum.
 - (c) Demonstrate the signal separation properties of different windows - plot a spectrum of a sum of two sinusoids with similar frequencies and amplitudes, then with very different frequencies and amplitudes.
7. Repeat FFT plots from Ex. 4, using a window (e.g. Hamming) on the signal.