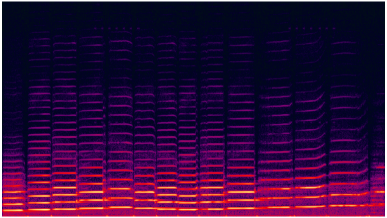
EDISP (Inst. Spectrum - STFT)
(English) Digital Signal Processing
Instantaneous spectrum
or
Short Time Fourier Transform lecture

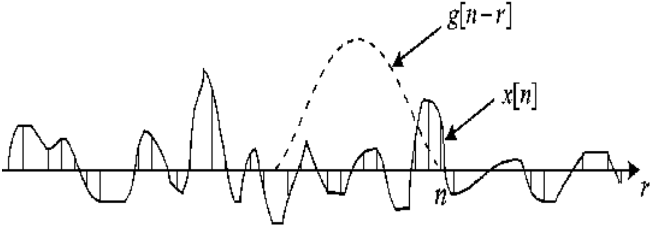
March 24, 2016

Signal properties changing in time

- FT/DFT etc: signal properties assumed constant in a whole analysis time
- True signals (e.g. speech, music, video): main information content in the **changes** of the signal properties
- (A simple idea) how to analyse such signals:
  - get a small section of a signal
  - assume properties stable inside section
  - analyze section (calculate spectrum)
  - move to next section (and repeat the procedure)
  - Finally draw a 2d-picture (abs() spectrum vs. time) $\longrightarrow$ *spectrogram*



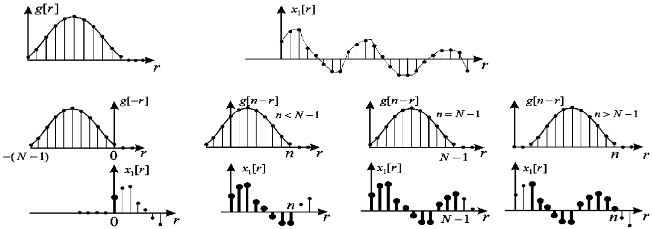
Formulation



$$X(n,\theta) = \sum_{r=-\infty}^{\infty} x[r]g[n-r]e^{-j\theta r}$$

- A window $g(n)$ of length L is non-zero if $n = 0,\ 1,\ \ldots,\ L-1$ (beware - others may define symmetrical windows)
- so n in $X(n,\theta)$ is the *end* of window
- The result depends on L and window type (recall windows lecture)

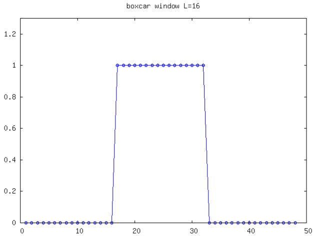
Sliding a window

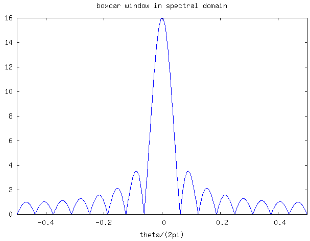


- edge effects
- resolution

Resolution in time or in frequency? (Gabor limit)

Can't have both :-)



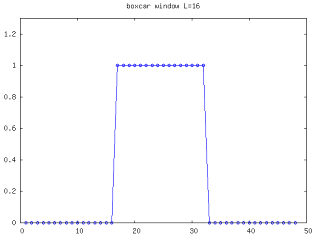


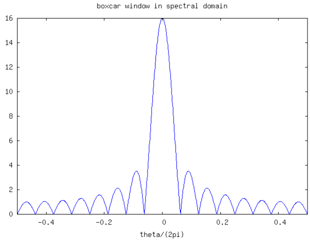
- Resolution in time $\approx L$ (window length)
- Resolution in frequency $\approx \frac{4\pi}{L}$
- And we also want low sidelobes (= "good" windows)
- $\longrightarrow$ good windows
  are bad windows

Hint: choose your window carefully to your application!
Gabor limit: *One cannot simultaneously sharply localize a signal in both the time domain and frequency domain.*

Resolution in time or in frequency? (Gabor limit)

Can't have both :-)



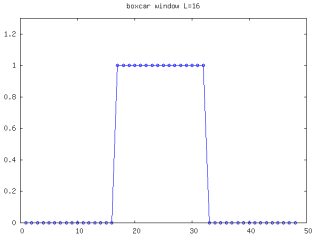


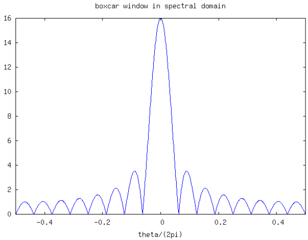
- Resolution in time $\approx L$ (window length)
- Resolution in frequency $\approx \frac{4\pi}{L}$
- And we also want low sidelobes (= "good" windows)
- $\longrightarrow$ good windows (with low sidelobes) are bad windows

Hint: choose your window carefully to your application!
Gabor limit: *One cannot simultaneously sharply localize a signal in both the time domain and frequency domain.*

Resolution in time or in frequency? (Gabor limit)

Can't have both :-)



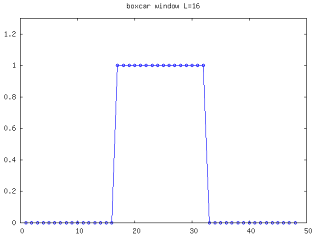


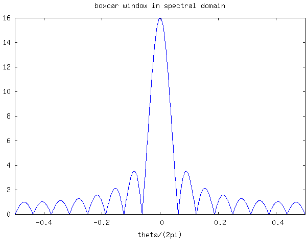
- Resolution in time $\approx L$ (window length)
- Resolution in frequency $\approx \frac{4\pi}{L}$
- And we also want low sidelobes (= "good" windows)
- $\longrightarrow$ good windows (with low sidelobes) are bad windows (have wide mainlobe

Hint: choose your window carefully to your application!
Gabor limit: *One cannot simultaneously sharply localize a signal in both the time domain and frequency domain.*

Resolution in time or in frequency? (Gabor limit)

Can't have both :-)



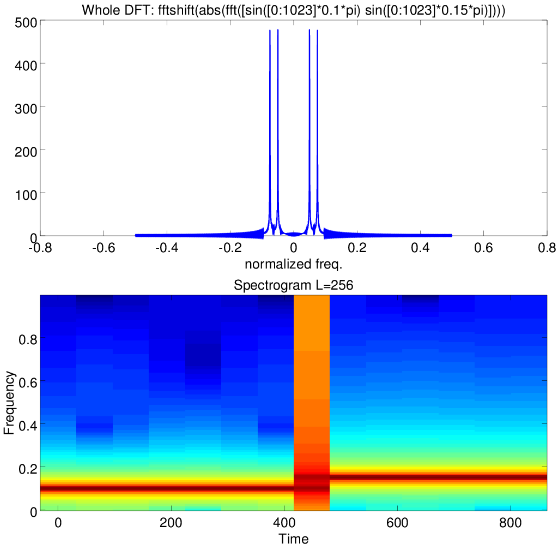


- Resolution in time $\approx L$ (window length)
- Resolution in frequency $\approx \frac{4\pi}{L}$
- And we also want low sidelobes (= "good" windows)
- $\longrightarrow$ good windows (with low sidelobes) are bad windows (have wide mainlobe and are effectively shorter in time)

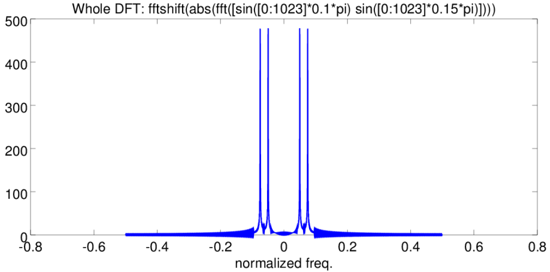
Hint: choose your window carefully to your application!

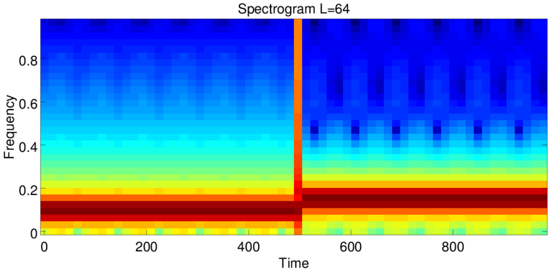
Gabor limit: *One cannot simultaneously sharply localize a signal in both the time domain and frequency domain.*

Resolution in time or in frequency? (Gabor limit)

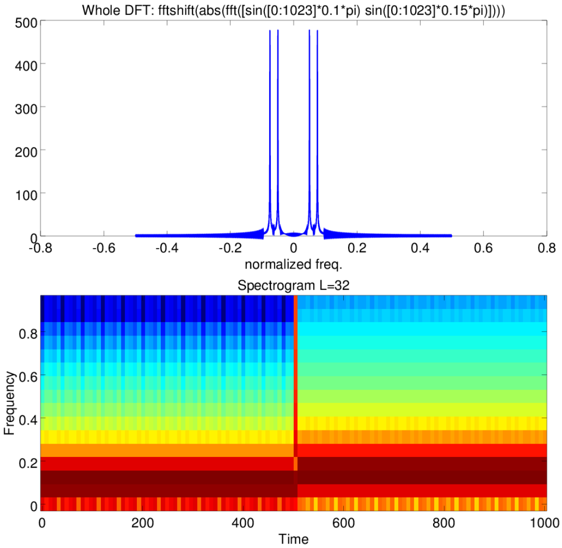


Resolution in time or in frequency? (Gabor limit)





Resolution in time or in frequency? (Gabor limit)



Wider view of the problem

Other names for the same:

- Short-Time Fourier Transform (STFT)
- Short-Term Fourier Transform (STFT)
- Time-Dependent Fourier Transform (TDFT)
- instantaneous spectrum

The result of STFT in display can be named:

- spectrogram
- waterfall plot (two different meanings: colormap moving in time or many linear plots overlaid)

Other approaches: Time-Frequency Transforms in general

- Wigner-Ville transform $W_x(n,\theta) = \sum_{r=-\infty}^{+\infty} x(n+r)x^*(n-r)e^{-j\theta 2r}$
- Wavelet transform (use time-concentrated basis functions)
- Chirplet transform
- ...