Lab 2 – Fourier transform, DFT, FFT

ver. March 21, 2016

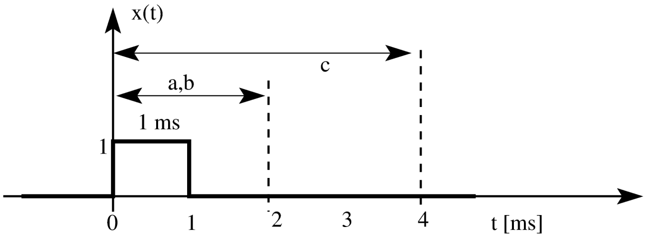
Entry test example questions

1. $x_a(t) = cos(2\pi f_a t)$ was sampled with sampling period T_s. Plot the { spectrum | N-point DFT } of $x[n]$ (f_a, T_s or f_s given, N given - whole number of periods or not)
2. A signal $x(n)$ with known Fourier spectrum $X(\theta)$ has been {inverted in time | decimated | modulated | ...}. Express mathematically what happened to the spectrum.
3. Calculate a DFT of a simple finite signal ($\delta(n-1)$, constant, $\{+1, -1, +1, \ldots\}$) - on paper

Lab exercises

Italics denote optional tasks.

1. Investigate a single square impulse of 1 ms length, sampled under different conditions (sampling frequency f_s and signal measurement duration T – see table).



case	f_s	T	N	N_1	A_{max}	k_{null}	f_n at null	f at null
$x_1[n]$	1 MHz	2 ms						
$x_2[n]$	10 kHz	2 ms						
$x_3[n]$	10 kHz	4 ms						

Copy the table to the report – you will later fill the empty table cells with answers to the following.
For each sampled signal ($x_1[n]$, $x_2[n]$, $x_3[n]$):

(a) Calculate total number of samples (fill in N column) and number of non-zero samples (fill in N_1 column)

(b) Create a simulated signal in Matlab (`[ones(1, N1), zeros(1, N-N1)]`)

(c) Calculate with Matlab and plot (on screen) magnitude of FFT (`plot(abs(fft(x...)))`)

(d) Find the maximum value in FFT (and fill in as A_{max})

(e) Find (and fill in) the index k_{null} of the first null in FFT values (Note: remember that Matlab numbering starts from 1, but we need the k for indexing frequency θ_k, so we start from 0).

Think of signal $x_1[n]$ as “sampled with such a high frequency that it is almost CT” and comment the spectrum differences between the three signals.

2. Simulate in Matlab 1024 points of following signals. Then, for each signal

- Plot the signal
- Plot magnitude of FFT (sketch it in the report)
- (for signals (a) and (b)) count number of zero crossings in FFT (note it in report)
- (for signal (c)) note the locations of the peak, compare peak width for the two frequencies

Signals:

(a) a 512 points square impulse (so you need to add 512 zeros to get 1024 samples)

(b) other (narrower) square impulses – fill them up to 1024

(c) sine wave (choose two frequencies to get integer and non-integer number of periods in window of 1024)

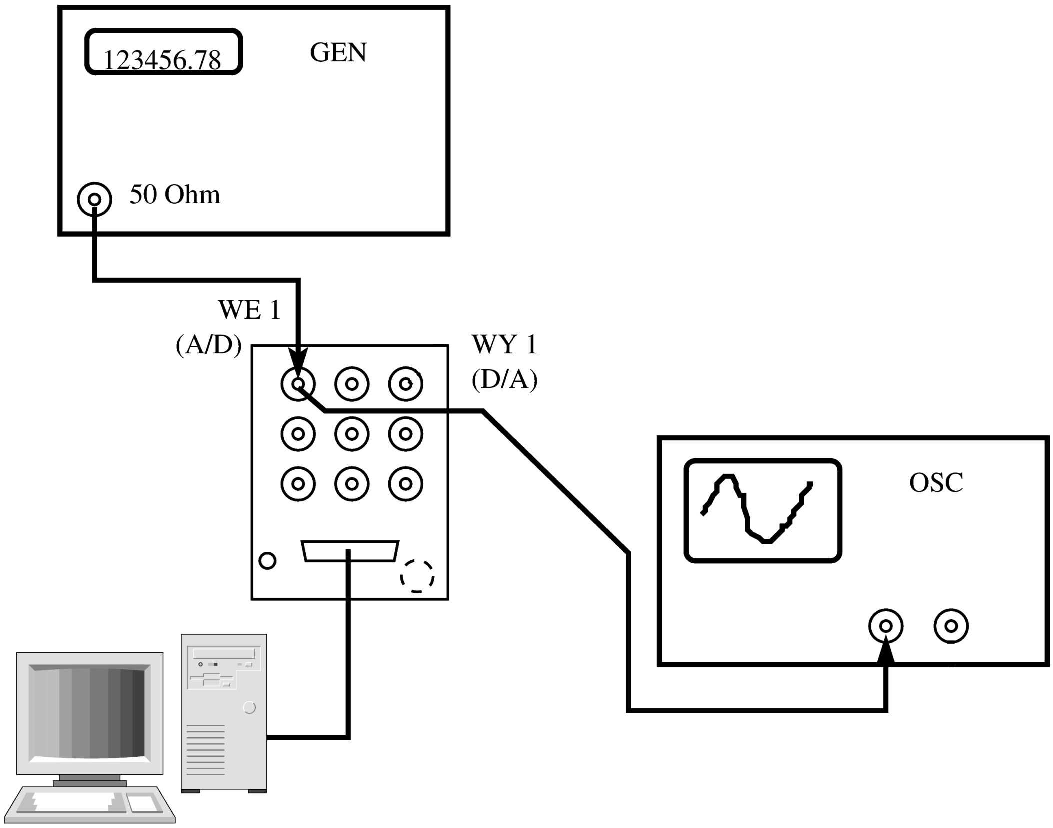
(d) $e^{jn\theta_c}$ - use `exp()` in Matlab; how many peaks do you see? why? Try different values of $0 < \theta_c \leq \pi$.

(e) a 32-point square impulse beginning at 0

(f) a 32-point square impulse beginning at $N_0 > 0$

3. Plot a spectrum of 512 samples of sine wave. Then, zero-pad them to 1024 and 2048 samples. Compare the resolution of FFT. Sketch abs(fft()) and note peak width. Compute IFFT. (plot real part of IFFT to cut off arithmetic errors). Hint: `fft(x,L)` automatically zero-pads signal x to length L.

4. Connect the equipment as presented below.



5. Capture 1024 samples of a live signal from a generator (use `LCPS_getdata(Nsamples,1,TsamplingInSeconds`
Choose some signal (sin, rectangular,...) and set the f and f_s using your own wisdom. Plot, labeling properly the horizontal axis:

   (a) the signal

   (b) its 1024-point FFT (magnitude, of course)

   (c) its 2^{12} *or even* ^{14}-point FFT (with zero-padding: `fft(x,N)` where N is the total length – with added zeros)

   Save the signal in some variable.

6. Compute spectra of different windows.

Fill in the table:

Window type	Mainlobe width	First sidelobe	highest sidelobe	Sidelobes change with f
	(normalized freq.)	(dB below mainl.)	(dB below mainl.)	(describe shortly)

In Matlab, window functions can be generated using: `rectwin, hamming, bartlett, blackman, hanning, kaiser`, with a scalar argument giving the length. For Kaiser – the second argument is β, use values between 1 and 8.

7. Do the following experiments to see the effect of windowing:
   (a) Plot a spectrum of 512 samples of sine wave. Choose the frequency to see the rectangular window effect clearly. If necessary, use zero-padding to see the spectrum better.
   (b) Use different window shapes, trying to obtain good, clear plot of the spectrum.
   (c) Demonstrate the signal separation properties of different windows - plot a spectrum of a sum of two sinusoids with similar frequencies and amplitudes, then with very different frequencies and amplitudes.
8. Repeat FFT plots from Ex. 5, using a window (e.g. Hamming) on the signal.

File: nwlab2 LaTeXed on March 21, 2016