EDISP 2016l – Test 1, 28.04.2016 **version A**

Name: Jacek Mosurewin

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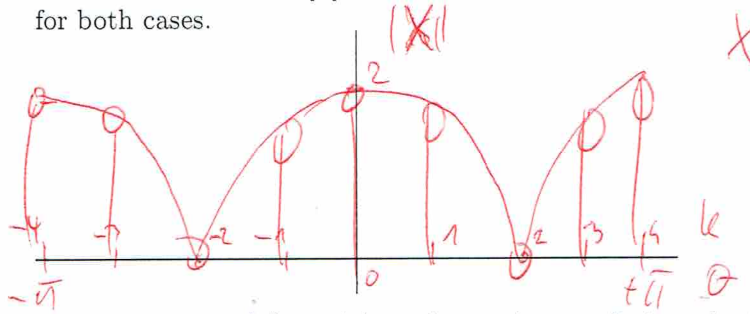
For short problems, try to write the answer in the provided space. Put your calculations and longer solutions on the reverse of this sheet or on an additional sheet marked with your name.

1. A DT system is described as follows: $y[n] = T(x[n]) = x[n] + \sum_{l=1}^{5} l(x[n-l]+x[n+l])$
   (a) (1 p.) Verify stability of the system calculating the B_y from B_x. Present calculations below. Is the system stable? yes/no: $\boxed{\text{yes}}$. Hint: you can always round B_y **up** if it is easier.
   Calculation: $|y(n)| \leq B_x + 2B_x + 4B_x + 6B_x + 8B_x + 10B_x = 31 B_x = B_y$
   (b) (1 p.) Can the system be analyzed with impulse response? yes/no: $\boxed{\text{yes}}$.
   Justify your answer: This is a linear combination of delayed samples, so it must be LTI

2. (2 p.) A periodic signal $x(t) = \cos(2\pi \cdot 7000t)$ was sampled with sampling frequency of 48 kHz.
   (a) Calculate normalized frequency f_n and normalized angular frequency θ of $x[n]$.
   Answer: $f_n = \boxed{7/48}$, $\theta = \boxed{2\pi \cdot 7/48}$. Calculation: $f_n = \frac{f}{f_s} = \frac{7\,kHz}{48\,kHz}$
   (b) What is the period of the resulting discrete-time signal $x[n]$? Answer: $\boxed{48}$. Calculation: $\cancel{48/7}$ is irreducible, so we will have 7 periods of $x(t)$ in 48 samples

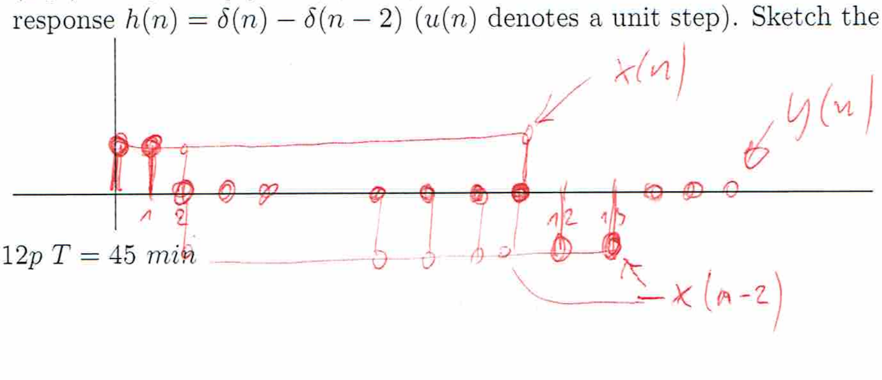
3. (3 p.) In a certain software library two functions to calculate FFT are provided – one does a 512-point FFT in 4 ms and other does a 256-point FFT in 1 ms. Both functions take a real-valued argument and produce a complex-valued DFT at output. Invent a possibly fast method to calculate a 1024-point FFT using multiple calls to the available functions plus some additional mathematics.
   How many calls of the functions will you need? Answer: $\boxed{4}$ calls of $\boxed{256}$-pt FFT.
   Describe the method on the reverse of this sheet(probably a sketch will be needed).
   Extra: try to guess the cost of multiplication and estimate the time needed for your transform.

4. (3 p.) Let $x[n]$ be defined as $x(n) = \delta(n)+\delta(n-2)$. Plot (on one picture) the magnitude of Fourier transform of $x[n]$ and of the 8 point DFT of $x[n]$; Mark the horizontal axis apropriately for both cases.



$X(e^{i\theta}) = 1 + e^{-i2\theta} = e^{-i\theta}(e^{+i\theta} + e^{-i\theta})$
$= e^{-i\theta} \cdot 2\cos\theta$

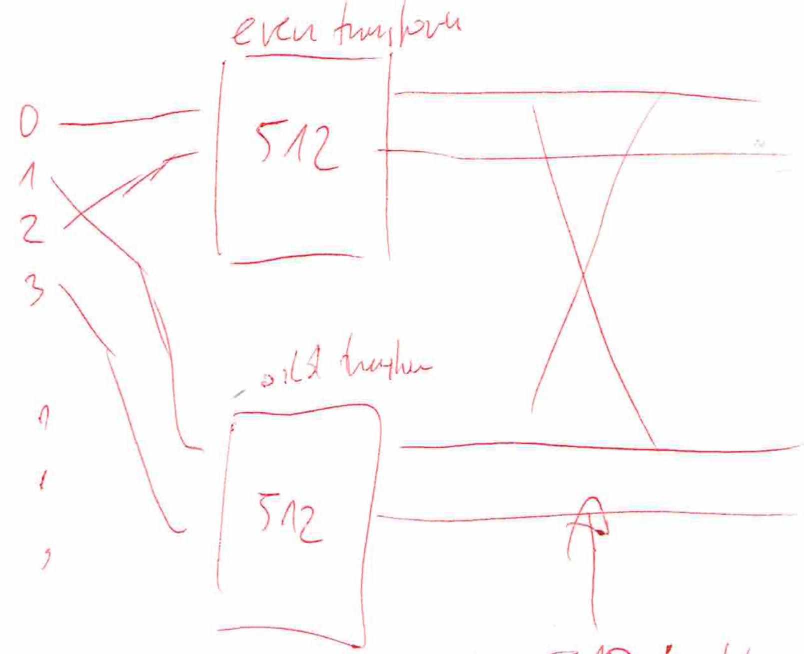
5. (2 p.) A signal $x[n] = u(n) - u(n-12)$ is applied to the input of an LTI system with impulse response $h(n) = \delta(n) - \delta(n-2)$ ($u(n)$ denotes a unit step). Sketch the output signal.



$\Sigma = 12p\ T = 45\ min$

3) Vers a

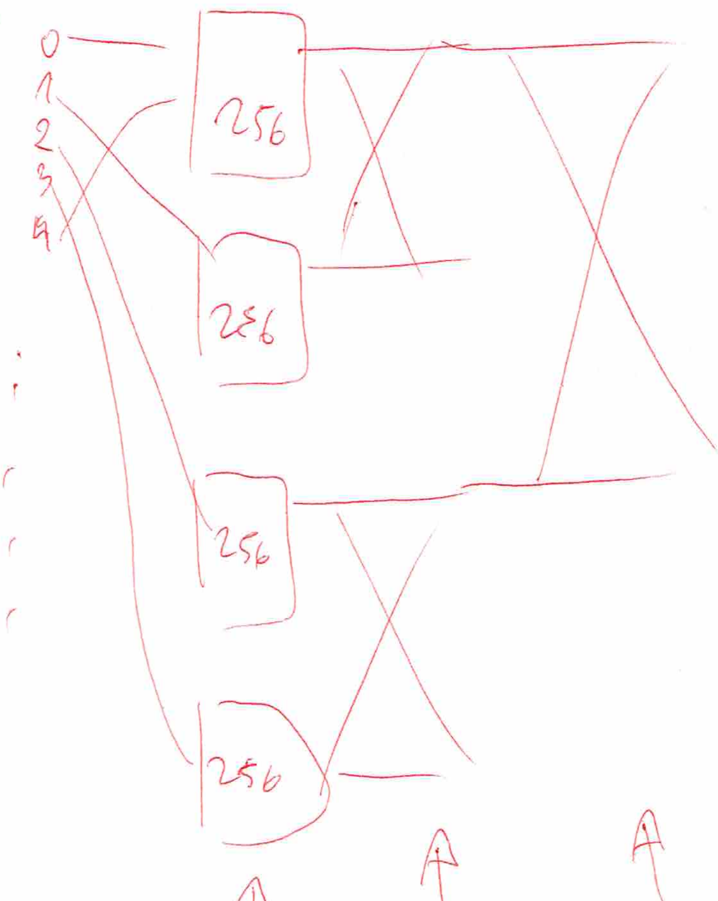
x(n)



512 butterflies

together: 2×4ms + 512 butterflies = about 8,5 ms

Version b:



4×1ms 2×256 butterflies 512 butterflies

I am guessing that b) can be faster, about 5 ms

exactly: FFT-256 is approx 8·128 butterflies, so 1024 butterflies can be approx 1ms

EDISP 2016l – Test 1, 28.04.2016 **version B**

Name: [illegible]

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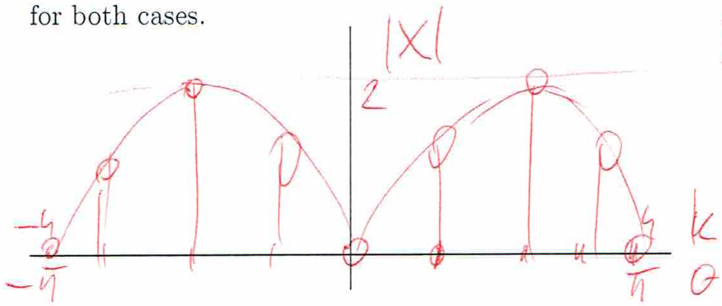
For short problems, try to write the answer in the provided space. Put your calculations and longer solutions on the reverse of this sheet or on an additional sheet marked with your name.

1. A DT system is described as follows: $y[n] = T(x[n]) = x[n] + \sum_{l=1}^{5} \frac{1}{l}(x[n-l] - x[n+l])$
   (a) (1 p.) Verify stability of the system calculating the B_y from B_x. Present calculations below.
   Is the system stable? yes/no: $\boxed{\text{yes}}$. Hint: you can always round B_y **up** if it is easier.
   Calculation:
   $|y(n)| \leqslant B_x\left(1 + 2\cdot\frac{1}{1} + 2\cdot\frac{1}{2} + 2\cdot\frac{1}{3} + 2\cdot\frac{1}{4} + 2\cdot\frac{1}{5}\right) < 5.4\cdot B_x$
   (b) (1 p.) Can the system be analyzed with impulse response? yes/no: $\boxed{\text{yes}}$.
   Justify your answer: This is a linear combination of delayed samples, so it must be LTI

2. (2 p.) A periodic signal $x(t) = \cos(2\pi \cdot 5000t)$ was sampled with sampling frequency of 48 kHz.
   (a) Calculate normalized frequency f_n and normalized angular frequency θ of $x[n]$.
   Answer: $f_n = \boxed{5/48}$, $\theta = \boxed{2\pi \cdot 5/48}$. Calculation: $f_n = f/f_s = 5\,kHz/48\,kHz$
   (b) What is the period of the resulting discrete-time signal $x[n]$? Answer: $\boxed{48}$. Calculation: 48/5 is irreducible, so we will have 5 periods of $x(t)$ in 48 samp

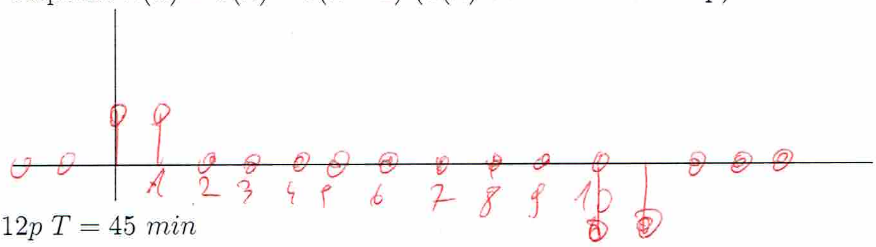
3. (3 p.) In a certain software library two functions to calculate FFT are provided – one does a 512-point FFT in 4 ms and other does a 256-point FFT in 2 ms. Both functions take a real-valued argument and produce a complex-valued DFT at output. Invent a possibly fast method to calculate a 1024-point FFT using multiple calls to the available functions plus some additional mathematics.
   How many calls of the functions will you need? Answer: $\boxed{2}$ calls of $\boxed{512}$-pt FFT.
   Describe the method on the reverse of this sheet(probably a sketch will be needed).
   Extra: try to guess the cost of multiplication and estimate the time needed for your transform.

4. (3 p.) Let $x[n]$ be defined as $x(n) = \delta(n) - \delta(n-2)$. Plot (on one picture) the magnitude of Fourier transform of $x[n]$ and of the 8 point DFT of $x[n]$; Mark the horizontal axis apropriately for both cases.



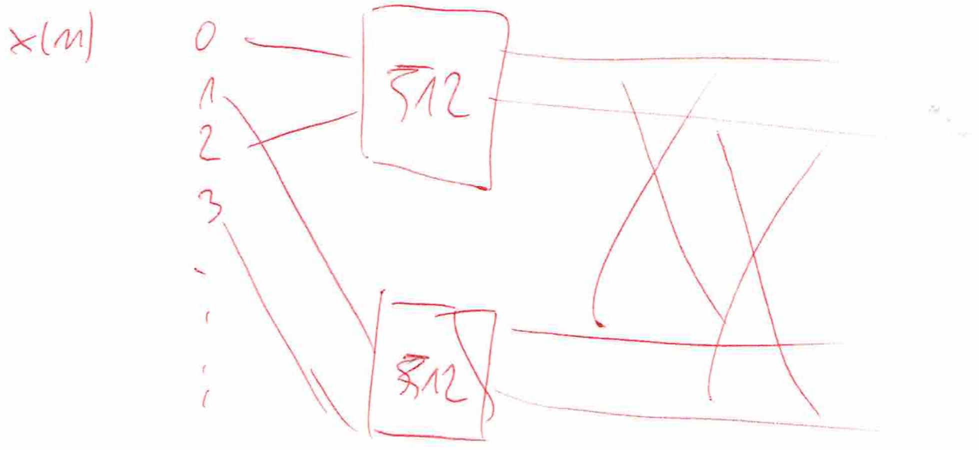
$X(e^{i\theta}) = 1 - e^{-i\theta} = e^{-i\theta}\cdot(e^{+i\theta} -$
$= j\,e^{-i\theta}\cdot 2\sin\theta$

5. (2 p.) A signal $x[n] = u(n) - u(n-10)$ is applied to the input of an LTI system with impulse response $h(n) = \delta(n) - \delta(n-2)$ ($u(n)$ denotes a unit step). Sketch the output signal.



$\Sigma = 12p\ T = 45\ min$

3) ver a



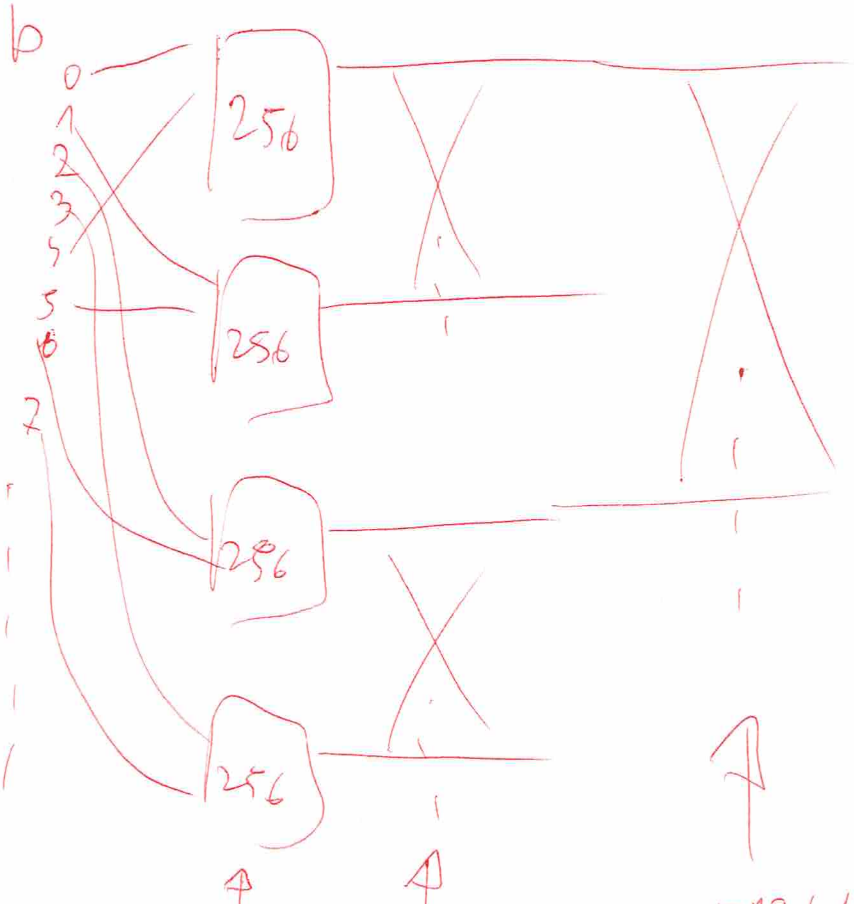
2x4ms

512 butterflies approx 1ms

together 9

a 512 pt FFT is approx 9 · 256 butterflies ≈ 2000 butterflies. So a butterfl
is about 2 μs.

ver b



4×2 ms

2×256 butterflies

512 butterflies

8 + 1 + 1 = 10 ms

Ver a is faster!!!