EDISP 2016z – Test 1, 15.12.2016 **version A**

Name: JACEK MISIUREWICZ

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For short problems, try to write the answer in the provided space. Put your calculations and longer solutions on the reverse of this sheet or on an additional sheet marked with your name.

1. A DT system is described as follows: $y[n] = T(x[n]) = x[n] + \sum_{l=1}^{+\infty}(0.9^l(x[n-l]+x[n+l]))$

   (a) (1 p.) Verify stability of the system calculating the B_y from B_x. Present calculations below.
   Is the system stable? yes/no: yes . Calculation:
   $y(n) \le B_x + \sum 0,9^l (B_x + B_x) = B_x \cdot \left(1 + 2 \cdot \frac{1}{1-0,9} - 2\right) = 19\, B_x$

   (b) (1 p.) Can the system be analyzed with impulse response? yes/no: yes .
   Justify your answer: It can be written as a linear difference equation, so it is LTI

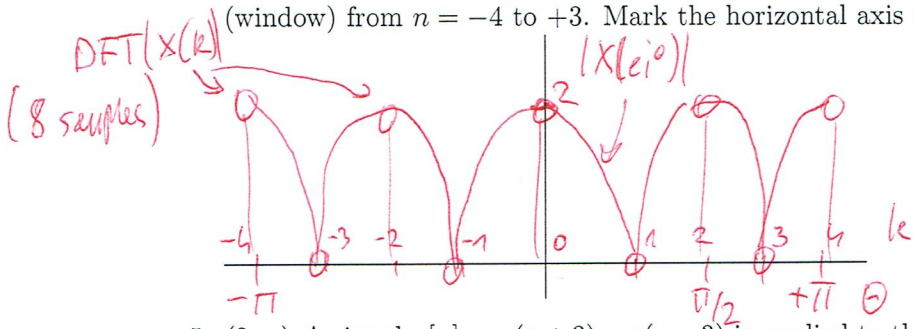
2. (2 p.) A periodic signal $x(t) = \cos(2\pi \cdot 6000t)$ was sampled with sampling frequency of 20 kHz.

   (a) Calculate normalized frequency f_n and normalized angular frequency θ of $x[n]$.
   Answer: $f_n =$ 0,3 , $\theta =$ $2\pi \cdot 0,3$. Calculation: $f_n = 6\,kHz / 20\,kHz = 0,3$

   (b) What is the period of the resulting discrete-time signal $x[n]$?
   Answer: $N_0 =$ 3 1/3 , $N =$ 10 . Calculation: $N_0 = 1/f_n = \frac{10}{3} \rightarrow N$

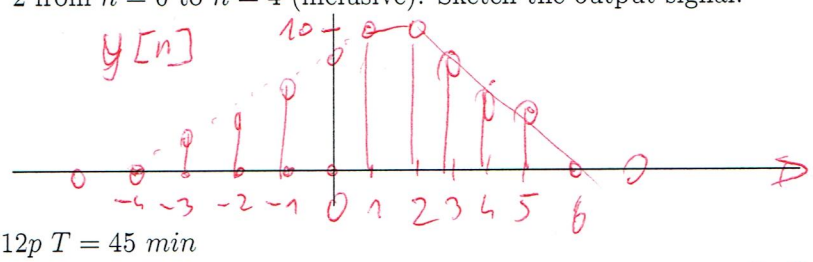
3. (3 p.) A software engineer has developed a piece of code which does a 512-point FFT on a single-core processor in 2 ms. Now he wants to reuse his code to calculate a 1024-point FFT. His idea is to run two 512-point FFTs on two separate cores, and then combine them into a 1024-point FFT on a single core. How many (real)×(real) multiplications will the combining part of code need? Answer: 2048 . Calculations: Combining N-order into 2N-order is one layer of butterflies, 512 butterflies = 512 complex (×) = 4·512 real (×)

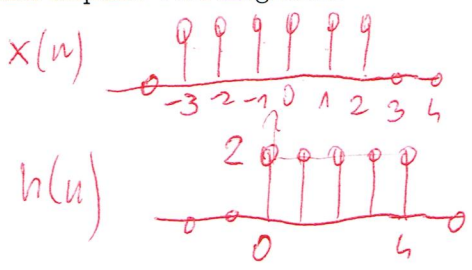
4. (3 p.) Let $x[n]$ be defined as $x(n) = \delta(n-2)+\delta(n-2)$. Plot (on one picture) the magnitude of Fourier transform of $x[n]$ and of the 8 point DFT of $x[n]$; assume the DFT summation interval (window) from $n = -4$ to $+3$. Mark the horizontal axis apropriately for both cases.



$X(e^{i\Theta}) = e^{i2\Theta} + e^{i2\Theta}$
$= 2\cos 2\Theta$
zero crossing at $\pi/4$ and $3\pi/4$

5. (2 p.) A signal $x[n] = u(n+3) - u(n-3)$ is applied to the input of an LTI system ($u(n)$ denotes a unit step). The impulse response $h(n)$ of the system is a rectangular impulse with magnitude 2 from $n = 0$ to $n = 4$ (inclusive). Sketch the output signal.





start at (-3), end at 2+5-1=5

$\Sigma = 12p\ T = 45\ min$

EDISP 2016z – Test 1, 15.12.2016 **version B**

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For short problems, try to write the answer in the provided space. Put your calculations and longer solutions on the reverse of this sheet or on an additional sheet marked with your name.

1. A DT system is described as follows: $y[n] = T(x[n]) = x[n] + \sum_{l=1}^{+\infty}(0.8^l(x[n-l]-x[n+l]))$

   (a) (1 p.) Verify stability of the system calculating the B_y from B_x. Present calculations below.
   Is the system stable? yes/no: yes . Calculation: ....
   $y(n) \le B_x + \sum 0{,}8^l (B_x + B_x) = B_x \cdot \left(1 + 2 \cdot \frac{1}{1-0{,}8} - 2\right) = 9 B_x$

   (b) (1 p.) Can the system be analyzed with impulse response? yes/no: yes .
   Justify your answer: ....
   It consists of summation and shifting and scaling, so it is LTI.

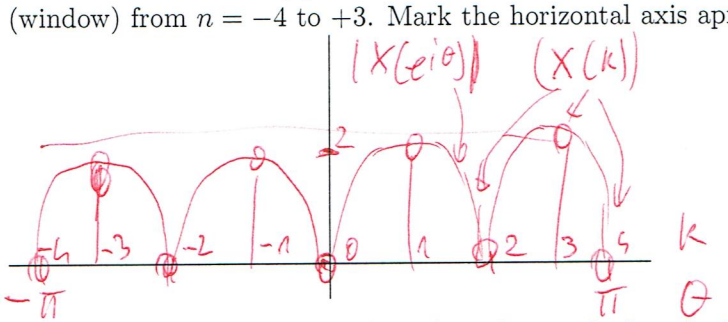
2. (2 p.) A periodic signal $x(t) = \cos(2\pi \cdot 600t)$ was sampled with sampling frequency of 20 kHz.

   (a) Calculate normalized frequency f_n and normalized angular frequency θ of $x[n]$.
   Answer: $f_n =$ 0,03 , $\theta =$ $2\pi \cdot 0{,}03$. Calculation: $f_n = 0{,}6\,kHz / 20\,kHz = 0{,}03$

   (b) What is the period of the resulting discrete-time signal $x[n]$?
   Answer: $N_0 =$ 33 1/3 , $N =$ 100 . Calculation: $N_0 = 1/f_n = \frac{100}{3} \rightarrow N$

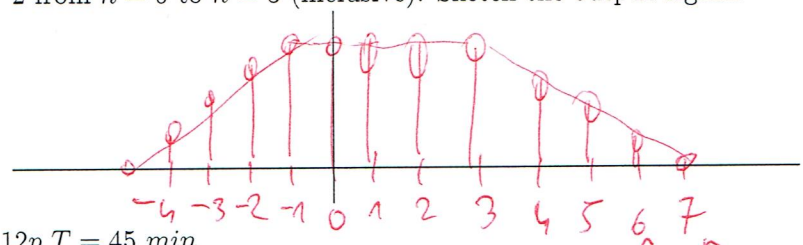
3. (3 p.) A software engineer has developed a piece of code which does a 256-point FFT on a single-core processor in 1 ms. Now he wants to reuse his code to calculate a 1024-point FFT. His idea is to run four 256-point FFTs on four separate cores, and then combine them into a 1024-point FFT on a single core. How many butterfly operations will the combining part of code need? Answer: 1024 . Calculations: Two layers of butterflies are needed. N/2 in each layer.

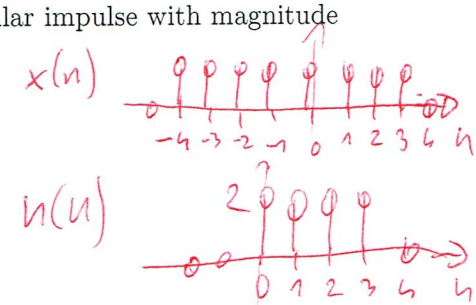
4. (3 p.) Let $x[n]$ be defined as $x(n) = \delta(n-2) - \delta(n-2)$. Plot (on one picture) the magnitude of Fourier transform of $x[n]$ and of the 8 point DFT of $x[n]$; assume the DFT summation interval (window) from $n = -4$ to $+3$. Mark the horizontal axis apropriately for both cases.



$X(e^{i\theta}) = e^{-i2\theta} - e^{+i2\theta} = 2j \sin 2\theta$

5. (2 p.) A signal $x[n] = u(n+4) - u(n-4)$ is applied to the input of an LTI system ($u(n)$ denotes a unit step). The impulse response $h(n)$ of the system is a rectangular impulse with magnitude 2 from $n = 0$ to $n = 3$ (inclusive). Sketch the output signal.





end at 3+4-1 = 6

$\Sigma = 12p\ T = 45\ min$