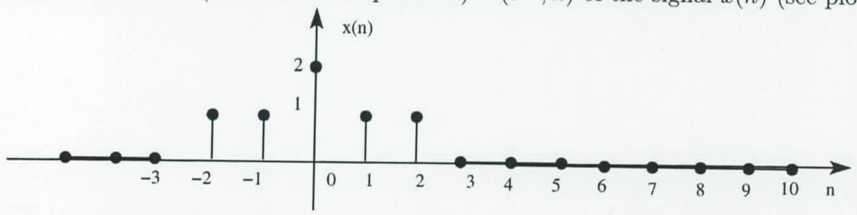
Test 2 (2013/14) **version A** – inst. spectrum, z-transform, filters
Please mark your name <u>and test version</u> on all your answer pages

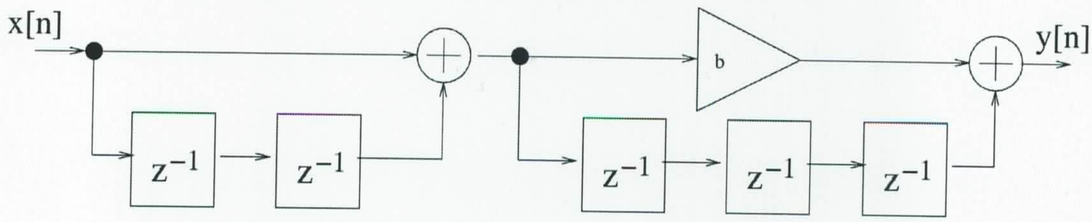
1. (3 p.) The STFT (instantaneous spectrum) $X(e^{j\theta}, n)$ of the signal $x(n)$ (see plot)



(hints: 1. Use the above plot to mark three positions of window. 2. Decompose signal into easy components if it seems complicated)
is computed using rectangular window $g(k)$ of length $K = 7$.

- For n given below, sketch $|X(e^{j\theta}, n)|$ for all θ and calculate $X(e^{j\theta}, n)$ at $\theta = 0,\ \pi/2$ and π
  (a) $n = -1$.
  (b) $n = +3$.
  (c) $n = +10$.

2. (4 p.) Analyze a filter described with the following graph:



Assume $b = 1$

(a) Find $H(z)$, $h(n)$.
(b) Find zeros/poles and plot their location on z-plane. Check if the filter is stable
(c) Sketch approximate $A(\theta)$
(d) Calculate response $y(n)$ for $x(n) = \delta(n-1) + \delta(n+1)$
(e) Calculate response $y(n)$ for $x(n) = cos(n\pi/4) + sin(n\pi/2)$

3. (3 p.) A filter is described with

$$H(z) = 1 + 2z^{-1} + \frac{-j}{1-(0.7+0.7j)z^{-1}} + \frac{+j}{1-(0.7-0.7j)z^{-1}}$$

(a) Is the filter stable?
(b) Find its impulse response.
(c) Find the $A(\pi/4)$ value.
(d) *(optional - extra points)* Sketch $A(\theta)$

4. (2 p.) Calculate the z-transform and determine ROC (region of convergence) for the series:
(a) $\delta[n{+}2]$
(b) $\delta[n{-}1] - \delta[n{+}1]$
(c) $u[n] \cdot (-1)^{n-2}$

5. (3 p.) A certain <u>ideal</u> FIR filter has impulse response

$$h_{ideal}(n) = \frac{1}{\pi}\frac{\sin n\pi/4}{n}$$

(a) What is the group delay of the ideal filter?
(b) Describe steps needed to make a practical (implementable in real time) filter from this ideal one, using no more than 11 multiplications per sample.
(c) Find group delay of the practical filter.
(d) *(optional - extra points)* Sketch the $A(\theta)$ of both filters (mark width of transition band).

$\Sigma = 15p\ T = 75\ min$

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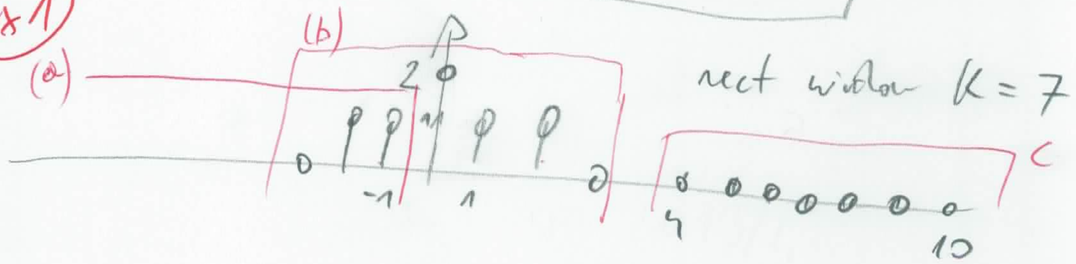
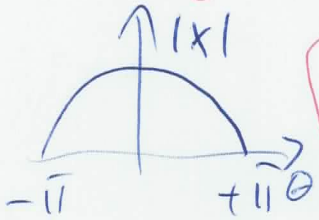
EDISP 2013/4

ex1

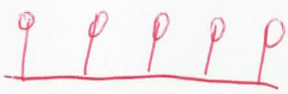


a) $\delta(n+2)+\delta(n+1) \xrightarrow{\mathcal{F}} e^{j2\theta}+e^{j\theta} = \begin{cases} \theta=0 \to 2 \\ \theta=\pi/2 \to 1+j \\ \theta=\pi \to 0 \end{cases}$

rect with ~~ln~~ = 2 →

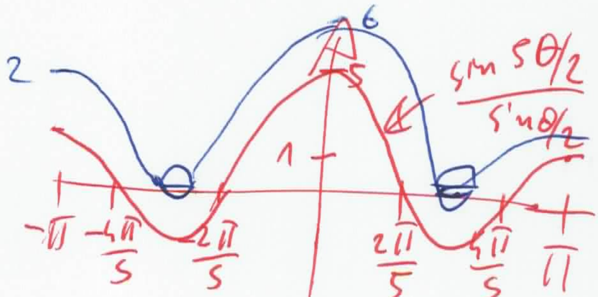


b)



$L=5$

$+\ \delta[n] \to \dfrac{\sin 5\frac{\theta}{2}}{\sin\frac{\theta}{2}} + 1$



$= \begin{cases} \theta=0 \to 5+1=6 \\ \theta=\pi/2 \quad \dfrac{\sin 5\pi/4}{\sin\pi/4}+1 \\ \theta=\pi \end{cases}$

dai numeri simula

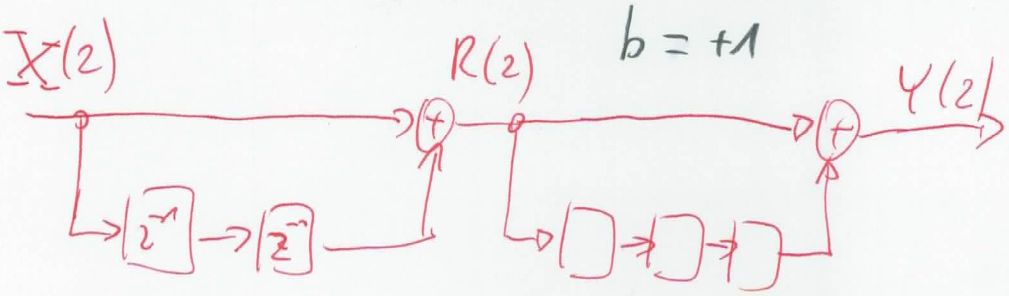
$\dfrac{-0.7}{0.7}+1=0$

$\dfrac{\sin 5\pi/2 \to 1}{\sin \pi/2 \to 1} + 1 = 2$

c) ZERO

T2A

ex2



a)

$$R(z) = X(z) + X(z)z^{-2} \rightarrow \frac{R(z)}{X(z)} = 1 + z^{-2}$$

$$Y(z) = R(z) + R(z)z^{-3} \Rightarrow \frac{Y(z)}{R(z)} = 1 + z^{-3}$$

$$\frac{Y(z)}{X(z)} = H(z) = (1+z^{-2})(1+z^{-3}) = 1 + z^{-2} + z^{-3} + z^{-5}$$

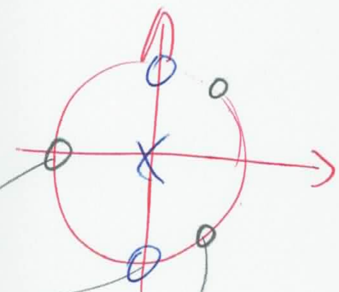
$$h(n) = [1, 0, 1, 1, 0, 1] \text{ or } (\delta(n) + \delta(n+2) + \delta(n+3 + \delta(n+5))$$

b) no poles, except at $z = 0$

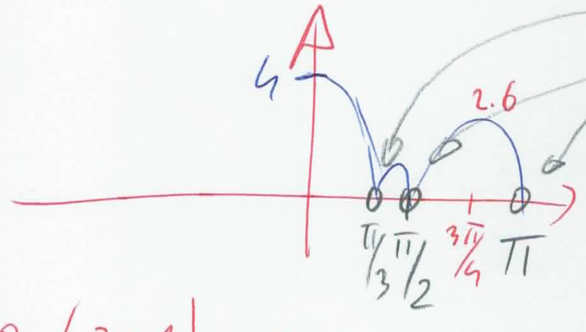
zeros: at $z^{-2} = -1$: $+i$ and $-i$

at $z^{-3} = -1$: $-1, e^{i\pi/3}, e^{-i\pi/3}$

FIR → stable



c) $A(\theta)$



at $\theta = 0$ $(z = 1) \rightarrow +4$

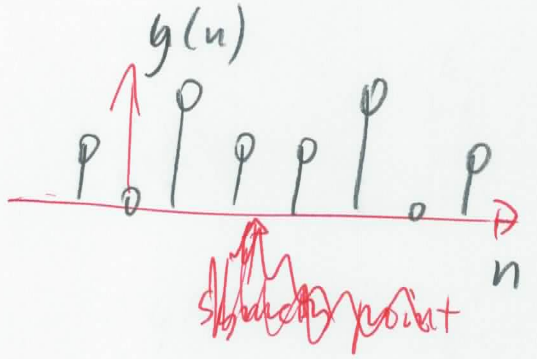
at $\theta = 3\pi/4$ $(z = e^{i3\pi/4}) \rightarrow 1 - i + \frac{\sqrt{2}}{2}(1+i) + \frac{\sqrt{2}}{2}(1-i) =$

$= 1 + \sqrt{2} + i$, abs() ≈ 2.6

ex2 cont.

d) $x(n) = \delta(n-1) + \delta(n+1)$

$$y(n) = h(n-1) + h(n+1) = \begin{array}{l} \,[1\ \underset{n=0}{0}\ 1\ 1\ 0\ 1] \quad h(n+1) \\ +\quad [1\ 0\ 1\ 1\ 0\ 1] \quad h(n-1) \\ = [1\ 0\ 2\ 1\ 1\ 2\ 0\ 1] \quad y(n) \end{array}$$



e) $x(n) = \cos n\frac{\pi}{4} + \cos n\pi$

$A(\pi) = 0$

$H\left(\frac{\pi}{4}\right) = 1 - j + \frac{\sqrt{2}}{2}(-1-j) + \frac{\sqrt{2}}{2}(-1+j) =$

$= 1 - \sqrt{2} - j \,, \quad A\left(\frac{\pi}{4}\right) \approx \sqrt{1.16} \approx 1.08$

$\varphi\left(\frac{\pi}{4}\right) = \operatorname{arctg} \frac{-1}{1-\sqrt{2}} = ? = -\frac{5\pi}{8}$

This is a FIR with $h(n)$ symmetrical around $n=2.5$ so it is a constant group delay filter with group delay of 2.5

so $\varphi(\theta) = -2.5 \cdot \theta \,, \quad \varphi\left(\frac{\pi}{4}\right) = -\frac{5\pi}{8}$

This works only if there are no zeros between $\theta = 0$ and $\theta = \pi/4$

answer $y \approx 1.08 \cdot \cos\left(n\frac{\pi}{4} - \frac{5\pi}{8}\right)$

T2A

ex 3

a) Poles are at $(0.7 \pm 0.7i)$, $|d_k| = \sqrt{0.98} < 1$

so STABLE

b) $h(n) = \delta(n) + 2\delta(n-1) + u(n)\cdot\left(-ie^{i\pi/4 n} + ie^{-i\pi/4 n}\right)\cdot\sqrt{0.98}^n$

$d_1 = d_2^* = \sqrt{0.98}\cdot e^{i\pi/4}$

$e^{-i\pi/4 n} - e^{i\pi/4 n} = -2i \sin \pi/4 n$

so $h(n) = \delta(n) + 2\delta(n-1) + u(n)\cdot\sqrt{0.98}^n\cdot 2\sin(\pi/4\, n)$

c) $A\left(\frac{\pi}{4}\right) = \left| 1 + 2\frac{\sqrt{2}}{2}(1-j) + \frac{-j}{1 - 0.7(1+j)\cdot\frac{\sqrt{2}}{2}(1-i)} + \frac{+j}{1 - 0.7(1-j)\cdot\frac{\sqrt{2}}{2}(1-i)} \right|$

$e^{i\pi/4} = \frac{\sqrt{2}}{2}\cdot(1+i)$

denominator very small

$\approx 1 - 2 * 2i$, not so small

$\frac{\sqrt{2}}{2} \approx 0.707$

$\frac{0.7\cdot\sqrt{2}}{2}\cdot 2 \approx 0.99!$

so $A\left(\frac{\pi}{4}\right) \approx \underline{100}$

T2A

(ex4)

$\delta[n+2] \to z^{2}$

$\delta[n-1] - \delta[n+1] \to z^{-1} - z$

$(-1)^{n-2} = (-1)^{n} \quad u[n]\cdot(-1)^{n-2} \to \frac{1}{1+z}$

(ex5)

a) Ideal filter has $h(n)$ symmetrical around 0, so group delay is 0

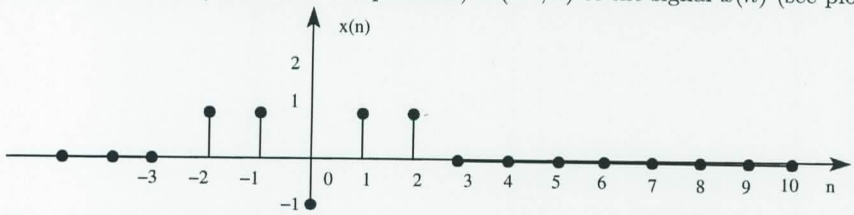
b) - Make finite $h(n)$ (cut with a window with $L = 11$)

- Then shift by $L-1/2$ to make causal

c) $\frac{L-1}{2} = 5$ and this is the group delay.

Test 2 (2013/14) **version B** – inst. spectrum, z-transform, filters
Please mark your name and test version on all your answer pages

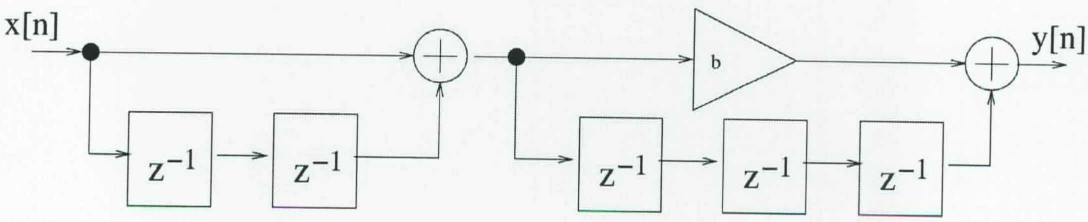
1. (3 p.) The STFT (instantaneous spectrum) $X(e^{j\theta}, n)$ of the signal $x(n)$ (see plot)



(hints: 1. Use the above plot to mark three positions of window. 2. Decompose signal into easy components if it seems complicated)
is computed using rectangular window $g(k)$ of length $K = 5$.

- For n given below, sketch $|X(e^{j\theta}, n)|$ for all θ and calculate $X(e^{j\theta}, n)$ at $\theta = 0$, $\pi/2$ and π
  (a) $n = -1$.
  (b) $n = +2$.
  (c) $n = +9$.

2. (4 p.) Analyze a filter described with the following graph:



Assume $b = -1$

(a) Find $H(z)$, $h(n)$.
(b) Find zeros/poles and plot their location on z-plane. Check if the filter is stable
(c) Sketch approximate $A(\theta)$
(d) Calculate response $y(n)$ for $x(n) = \delta(n-1) + \delta(n+1)$
(e) Calculate response $y(n)$ for $x(n) = cos(n\pi/4) + cos(n\pi)$

3. (3 p.) A filter is described with

$$H(z) = 1 + 2z^{-1} + \frac{1}{1-(0.7+0.7j)z^{-1}} + \frac{1}{1-(0.7-0.7j)z^{-1}}$$

(a) Is the filter stable?
(b) Find its impulse response.
(c) Find the $A(\pi/4)$ value.
(d) *(optional - extra points)* Sketch $A(\theta)$

4. (2 p.) Calculate the z-transform and determine ROC (region of convergence) for the series:

(a) $\delta[n-2]$
(b) $\delta[n-3] - \delta[n+3]$
(c) $u[n] \cdot (-1)^{n-3}$

5. (3 p.) A certain ideal FIR filter has impulse response

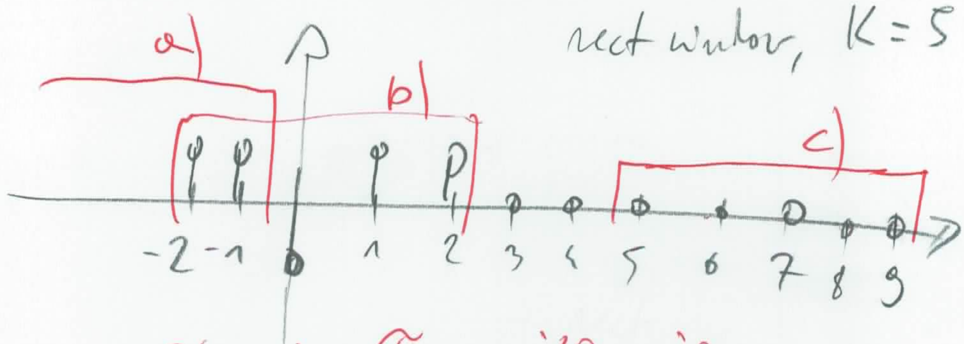
$$h_{ideal}(n) = \frac{1}{\pi}\frac{\sin \overset{n}{\pi}/3}{n}$$

(a) What is the group delay of the ideal filter?
(b) Describe steps needed to make a practical (implementable in real time) filter from this ideal one, using no more than 21 multiplications per sample.
(c) Find group delay of the practical filter.
(d) *(optional - extra points)* Sketch the $A(\theta)$ of both filters (mark width of transition band).

$\Sigma = 15p\ T = 75\ min$

T2B

ex 1



a) $\delta(n+2)+\delta(n+1) \xrightarrow{\mathcal{F}} e^{j2\theta}+e^{j\theta}$

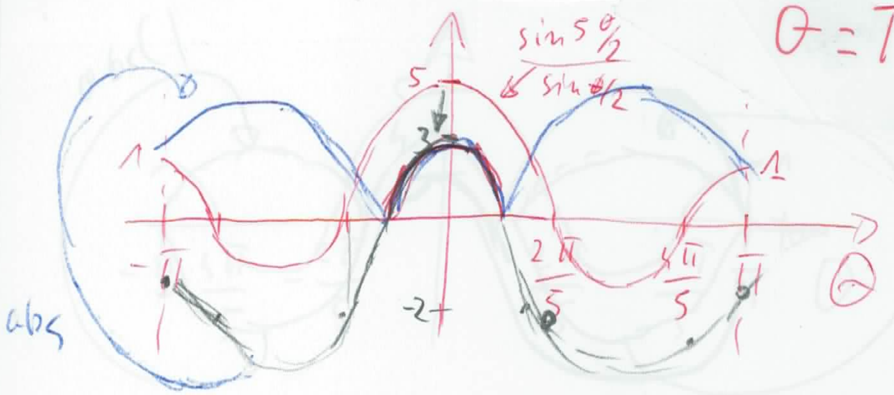
or rect with $L=2 \rightarrow |X(\theta)| = \dfrac{\sin\theta}{\sin\theta/2}$

$\theta=0 \rightarrow 2$; $\theta=\pi/2 \rightarrow 1+j$; $\theta=\pi \rightarrow 0$

b) $\dfrac{\text{φφφφφ}}{L=5} - 2\delta[n] \rightarrow \dfrac{\sin 5\frac{\theta}{2}}{\sin\frac{\theta}{2}} - 2$

$\theta=0 \rightarrow 5-2=3$; $\theta=\pi/2 \rightarrow \dfrac{\sin 5\frac{\pi}{4}}{\sin\pi/4} - 2 = -2$

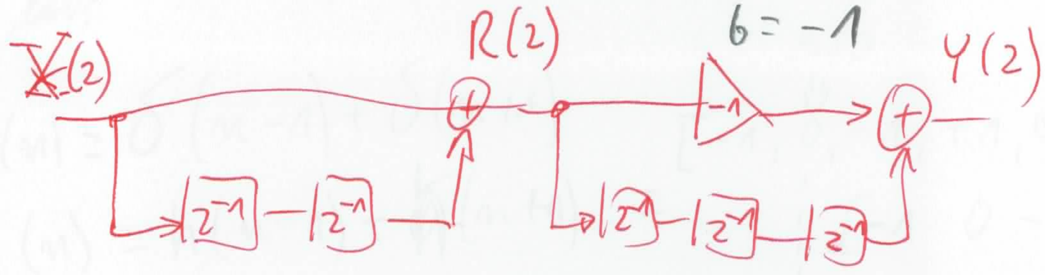
$\theta=\pi \rightarrow 1-2=-1$



T2B

T2B

ex2



a) $R(z) = X(z) + X(z)z^{-2} \rightarrow \dfrac{R(z)}{X(z)} = 1 + z^{-2}$

$Y(z) = -R(z) + R(z)z^{-3} \rightarrow \dfrac{Y(z)}{R(z)} = z^{-3} - 1$

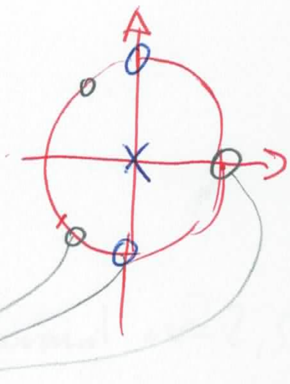
$$\frac{Y(z)}{X(z)} = H(z) = (1+z^{-2})(z^{-3}-1) = -1 - z^{-2} + z^{-3} + z^{-5}$$

$h(n) = [-1, 0, -1, +1, 0, +1]$ or $(-\delta(n) - \delta(n-2) + \delta(n-3) + \delta(n$

b) No poles (except a multiple pole at $z=0$)

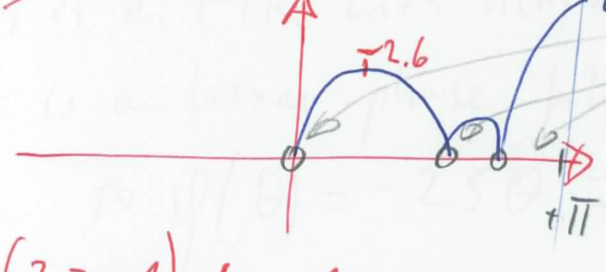
Zeros: at $z^{-2} = -1$: $+j$ and $-j$

at $z^{-3} = 1$: $1, e^{j\frac{2\pi}{3}}, e^{-j\frac{2\pi}{3}}$



FIR $\Rightarrow$ stable

c) $A(\theta)$



at $\theta = \pi$ $(z = -1)$: $-1 - 1 - 1 - 1 = -4$, abs() = 4

at $\theta = \pi/4$ $\left(z = \frac{\sqrt{2}}{2}(1+j)\right.$ or $\left.z = e^{j\pi/4}\right)$: $-1 + j + \frac{\sqrt{2}}{2}(-1-j) + \frac{\sqrt{2}}{2}(-1+j) =$

$= -1 - \sqrt{2} + j$, abs() $\approx \sqrt{7} \approx 2.6$

T2B

ex3

a) Poles at $(0.7 + 0.7i)$ $|\alpha_k| = \sqrt{0.98} < 1$

so stable

b) $h(n) = \delta(n) + 2\delta(n-1) + u(n) \cdot \sqrt{0{,}98} \cdot \left(e^{i\pi/4} + e^{-i\pi/4}\right)$

$\alpha_1 = \alpha_2^* = \sqrt{0{,}98} \cdot e^{i\pi/4}$

so $h(n) = \delta(n) + 2\delta(n-1) + u(n) \cdot \sqrt{0.98}^{\,n} \cdot 2\cos\left(\pi/4 \cdot n\right)$

c) $$A\left(\frac{\pi}{4}\right) = \left| 1 + 2\frac{\sqrt{2}}{2}(1-i) + \frac{1}{1 - \underbrace{0.7(1+i) \cdot \frac{\sqrt{2}}{2}(1-i)}_{\approx 0.99}} \right.$$

≈ 0.01

$$+ \frac{1}{1 - 0.7(1-i) \cdot \frac{\sqrt{2}}{2}(1-i)}$$

so $A\left(\frac{\pi}{4}\right) \approx 100$

T2B

ex4

$\delta[n-2] \to \delta[n]$ shifted $\to 1 \cdot 2^{-2} = 2^{-2}$

$\delta[n-3] - \delta[n+3] \xrightarrow{\text{linearity}} 2^{-3} - 2^{3}$

$u[n] \cdot (-1)^{n-3} \longrightarrow -\dfrac{1}{1+2}$

$(-1)^{(n-3)} = (-1)\cdot(-1)\cdot(-1)\cdot(-1)^n$

ex5

a) The ideal filter has symmetrical $h(n)$ around 0, so
group delay $= 0$.

b) Make $h(n)$ finite -> cut with a window
(e.g. rectangular) 21 multiplications $\Rightarrow L = 21$
Then shift by $(L-1)/2$ to make causal.

c) $\dfrac{(L-1)}{2} = 10$ and this is group delay.