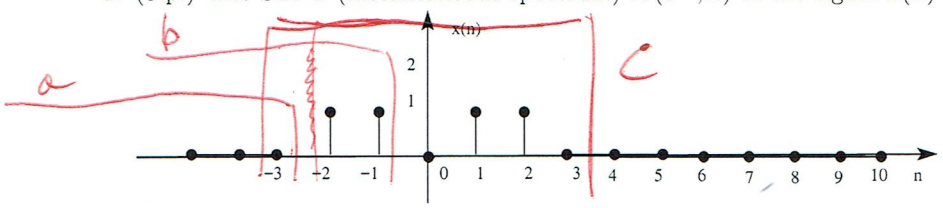
Test 2 2016l **version A** – inst. spectrum, z-transform, filters

Please mark your name and test version on all your answer pages

1. (3 p.) The STFT (instantaneous spectrum) $X(e^{j\theta}, n)$ of the signal $x(n)$ (see plot)

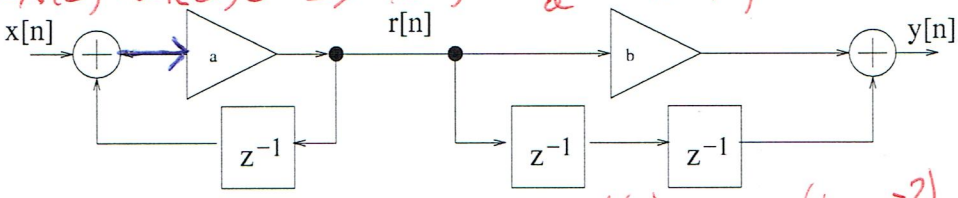


is computed using rectangular window $g(k)$ of length $K = 7$.

- For n given below, sketch $|X(e^{j\theta}, n)|$ for all θ;
  then calculate numerical values of $X(e^{j\theta}, n)$ at $\theta = 0$, $\pi/2$ and π:
  (a) $n = -3$.
  (b) $n = 0$.
  (c) $n = +3$.

hint 1: Use the above plot to mark three positions of window.
hint 2: In one case you will look for a Fourier transform of a sum of rectangle and delta.

2. (5 p.) Analyze a filter described with the following graph:



Assume $a = +9/10$, $b = -10/9$

(a) Find $H(z)$, $h(n)$.
Hint: you may use $r(n)$ as a "helper" when writing the difference equation.
(b) Find zeros/poles and plot their location on z-plane. Check if the filter is stable
(c) Sketch approximate $A(\theta)$
(d) Calculate response $y(n)$ for $x(n) = 3 + sin(n\pi/2)$

3. (2 p.) Calculate the z-transform and determine ROC (region of convergence) for the series:
(a) $\delta[n+20]$
(b) $\delta[n-1] - \delta[n+1]$
(c) $u[n] \cdot (-1)^{n-2}$

4. (3 p.) A certain filter has frequency response

$$H(\theta) = \begin{cases} 1 & \text{for} \quad -\pi/3 < \theta < \pi/3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Is the filter causal? If yes: find the group delay of the filter. If no: how to make it a causal FIR?
(b) Calculate $h(n)$ (find equation).
(c) Calculate values of $h(0)$, $h(1)$, $h(-1)$, $h(3)$

$\Sigma = 13p$ $T = 75$ *min*