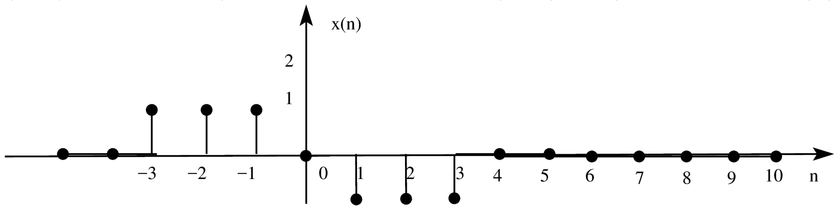
Test 2 2017l **version A** – inst. spectrum, z-transform, filters

Please mark your name <u>and test version</u> on all your answer pages

1. (3 p.) The STFT (instantaneous spectrum) $X(e^{j\theta}, n)$ of the signal $x(n)$ (see plot)

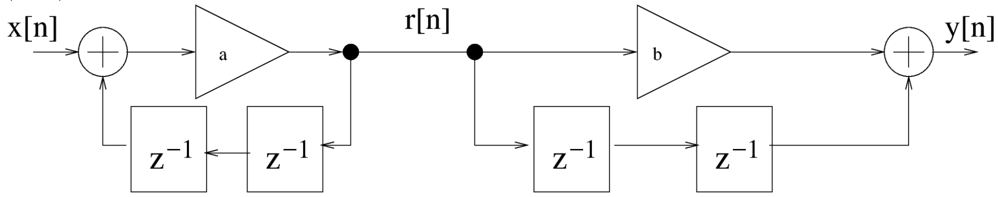


is computed using rectangular window $g(k)$ of length $K = 7$.

- For n given below, sketch $|X(e^{j\theta}, n)|$ for all θ;
then calculate numerical values of $X(e^{j\theta}, n)$ at $\theta = 0$, $\pi/2$ and π:
   (a) $n = -3$.
   (b) $n =\ 0$.
   (c) $n = +3$.

hint 1: Use the above plot to mark three positions of window.

2. (4 p.) Analyze a filter described with the following graph:



Assume $a = -0.81$, $b = +1$,

   (a) Find $H(z)$
   Hint: you may use $r(n)$ as a "helper" when writing the difference equation.
   (b) Find zeros/poles and plot their location on z-plane. Check if the filter is stable
   (c) Sketch approximate $A(\theta)$
   (d) Calculate response $y(n)$ for $x(n) = 3 + sin(n\pi/2)$
   (e) (extra points) Propose a modification of the filter graph, saving on delay blocks.

hint: $(1 - c)(1 + c) = 1 - c^2$, $(1 - \mathrm{j}c)(1 + \mathrm{j}c) = 1 + c^2$

3. (2 p.) Calculate the z-transform and determine ROC (region of convergence) for the series:
   (a) $\delta[n{+}2]$
   (b) $\delta[n{-}1] + \delta[n{+}1]$

4. (3 p.) Calculate a causal $x(n)$ when $X(z) = \frac{1}{1-e^{j3\pi/4}z^{-1}} + \frac{1}{1-e^{-j3\pi/4}z^{-1}}$
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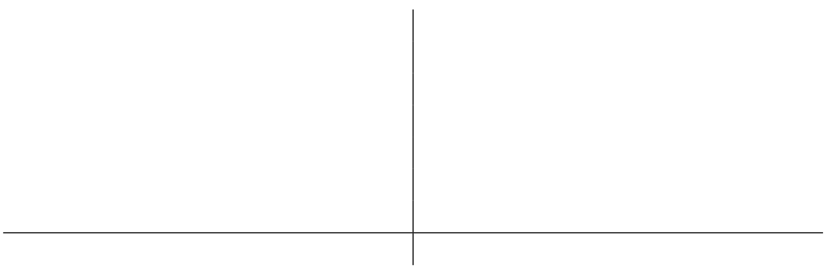
5. (3 p.) A <u>noncausal, zero-phase</u> lowpass FIR filter with impulse response length equal to 15 was designed from windowed Inverse Fourier Transform of ideal filter frequency response. A rectangular window was used. Ideal filter cutoff was at $\theta_b = (1/2)\pi$.
   (a) Calculate the group delay of the filter.
   (b) Find the approximate width of the transition band in the frequency response.
   (c) Sketch the impulse response.

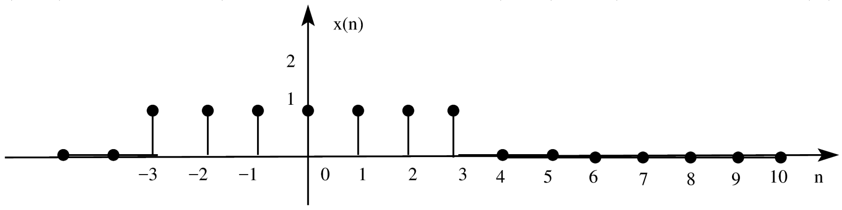


$\Sigma = 15p$ $T = 75$ min

Test 2 2017l **version B** – inst. spectrum, z-transform, filters

Please mark your name <u>and test version</u> on all your answer pages

1. (3 p.) The STFT (instantaneous spectrum) $X(e^{j\theta}, n)$ of the signal $x(n)$ (see plot)

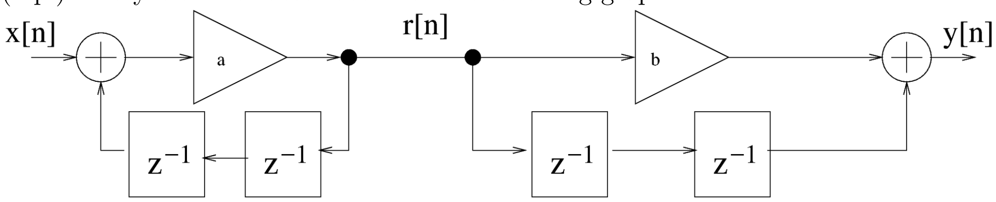


is computed using rectangular window $g(k)$ of length $K = 5$.

- For n given below, sketch $|X(e^{j\theta}, n)|$ for all θ;
  then calculate numerical values of $X(e^{j\theta}, n)$ at $\theta = 0$, $\pi/2$ and π:
  (a) $n = -3$.
  (b) $n = \ 0$.
  (c) $n = +2$.

hint 1: Use the above plot to mark three positions of window.

2. (4 p.) Analyze a filter described with the following graph:



Assume $a = +0.81$, $b = -1$,

(a) Find $H(z)$
Hint: you may use $r(n)$ as a "helper" when writing the difference equation.
(b) Find zeros/poles and plot their location on z-plane. Check if the filter is stable
(c) Sketch approximate $A(\theta)$
(d) Calculate response $y(n)$ for $x(n) = (-1)^n + cos(n\pi/2)$
(e) (extra points) Propose a modification of the filter graph, saving on delay blocks.

hint: $(1-c)(1+c) = 1-c^2$, $(1-\mathrm{j}c)(1+\mathrm{j}c) = 1+c^2$

3. (2 p.) Calculate the z-transform and determine ROC (region of convergence) for the series:

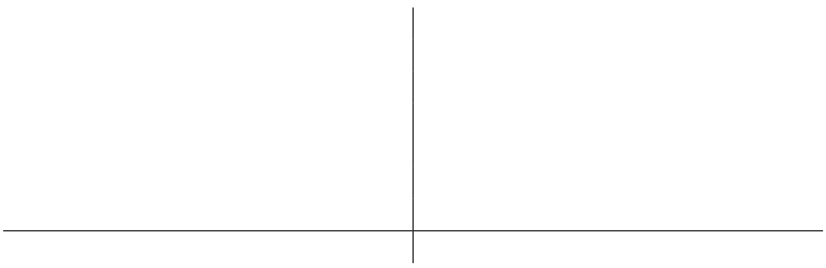
(a) $\delta[n-20]$
(b) $\delta[n-3] + \delta[n+3]$

4. (3 p.) Calculate a causal $x(n)$ when $X(z) = \frac{j}{1-e^{j\pi/4}z^{-1}} - \frac{j}{1-e^{-j\pi/4}z^{-1}}$
.

....

5. (3 p.) A <u>noncausal, zero-phase</u> lowpass FIR filter with impulse response length equal to 11 was designed from windowed Inverse Fourier Transform of ideal filter frequency response. A rectangular window was used. Ideal filter cutoff was at $\theta_b = (1/2)\pi$.

(a) Calculate the group delay of the filter.
(b) Find the approximate width of the transition band in the frequency response.
(c) Sketch the impulse response.



$\Sigma = 15p$ $T = 75$ min