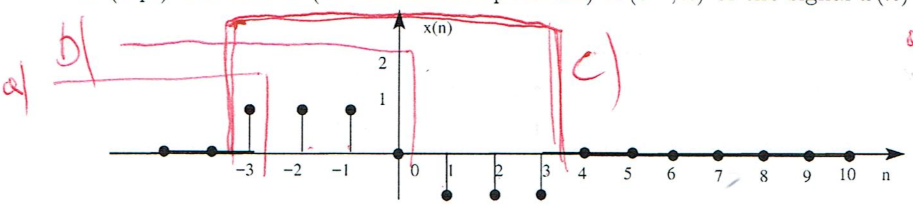
JACEK MISIUREWICZ

Test 2 2017l **version A** – inst. spectrum, z-transform, filters

Please mark your name and test version on all your answer pages

1. (3 p.) The STFT (instantaneous spectrum) $X(e^{j\theta}, n)$ of the signal $x(n)$ (see plot)



is computed using rectangular window $g(k)$ of length $K = 7$.

- For n given below, sketch $|X(e^{j\theta}, n)|$ for all θ; then calculate numerical values of $X(e^{j\theta}, n)$ at $\theta = 0$, $\pi/2$ and π:
  - (a) $n = -3$.
  - (b) $n =$ 0.
  - (c) $n = +3$.

hint 1: Use the above plot to mark three positions of window.

a) $n=-3$ $g \cdot x = \delta(n+3)$

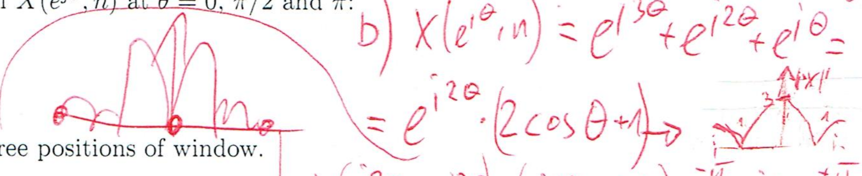
$X(e^{j\theta}, n) = e^{j3\theta}$

$|X(e^{j\theta}, n)| \Rightarrow$

$0, \pi/2, \pi$: $1, e^{j\frac{3\pi}{2}}, e^{j3\pi} = 1, j, -1$

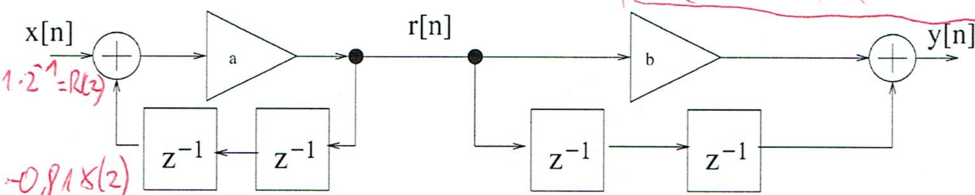
b) $X(e^{j\theta}, n) = e^{j3\theta} + e^{j2\theta} + e^{j\theta} =$

$= e^{j2\theta} \cdot (2\cos\theta + 1) \to$



c) $(e^{j3\theta} - e^{-j3\theta}) + (e^{j2\theta} - e^{-j2\theta}) + (e^{j\theta} - e^{-j\theta})$

2. (6 p.) Analyze a filter described with the following graph:

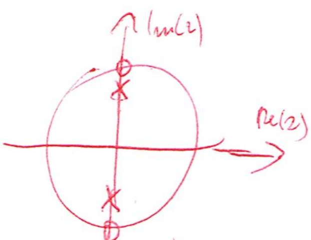


$-X(z) \cdot 0{,}81 - R(z) \cdot 0{,}81 \cdot z^{-1} = R(z)$

$R(z) \cdot (1 + 0{,}81 z^{-2}) = -0{,}81 X(z)$

Assume $a = -0.81$, $b = +1$,

$\frac{-0{,}81}{1 + 0{,}81 z^{-2}} \cdot \frac{1 + z^{-2}}{1} = \frac{1 + z^{-2}}{1 + 0{,}81 z^{-2}} \cdot (-0.81)$



(a) Find $H(z)$

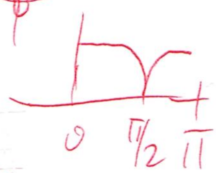
Hint: you may use $r(n)$ as a "helper" when writing the difference equation.

(b) Find zeros/poles and plot their location on z-plane. Check if the filter is stable

(c) Sketch approximate $A(\theta)$ $A(0) = A(\pi) = \frac{2 \cdot 0{,}81}{1{,}81} \approx 0.895$ $A(\pi/2) = \frac{0}{0{,}09} = 0$

(d) Calculate response $y(n)$ for $x(n) = 3 + sin(n\pi/2)$

(e) (extra points) Propose a modification of the filter graph, saving on delay blocks.



hint: $(1 - c)(1 + c) = 1 - c^2$, $(1 - jc)(1 + jc) = 1 + c^2$

$3 \cdot 0{,}895 = 2{,}68$

$\to 2{,}68 + 0 \cdot \sin(n\pi/2)$

3. (2 p.) Calculate the z-transform and determine ROC (region of convergence) for the series:

(a) $\delta[n+2]$ — z^2, all $|z| < \infty$

(b) $\delta[n-1] + \delta[n+1]$ — $z^{-1} + z^{1}$, $|z| < \infty$ $z \neq 0$

4. (3 p.) Calculate a causal $x(n)$ when

$$X(z) = \frac{1}{1 - e^{j3\pi/4} z^{-1}} + \frac{1}{1 - e^{-j3\pi/4} z^{-1}}$$

$u(n) \cdot (e^{j\frac{3\pi}{4}n} + e^{-j\frac{3\pi}{4}n}) =$

$= u(n) \cdot 2\cos(\frac{3\pi}{4} n)$

5. (3 p.) A noncausal, zero-phase lowpass FIR filter with impulse response length equal to ~~13~~ 15 was designed from windowed Inverse Fourier Transform of ideal filter frequency response. A rectangular window was used. Ideal filter cutoff was at $\theta_b = (1/2)\pi$.

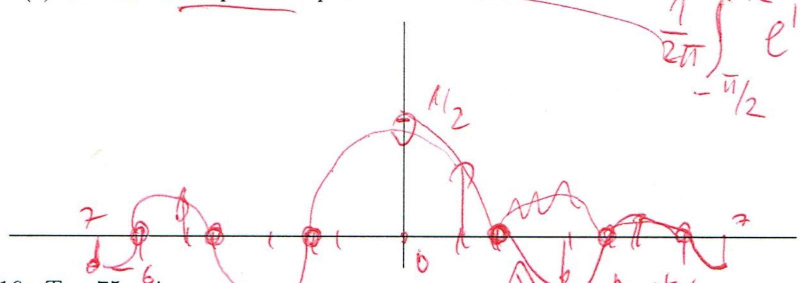
(a) Calculate the group delay of the filter. $\frac{d\,\theta \leftarrow \text{zero phase}}{d\varphi} = 0 \leftarrow$ zero order.

(b) Find the approximate width of the transition band in the frequency response. rect. window $\Rightarrow \frac{4\pi}{15}$

(c) Sketch the impulse response.

$\frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{jn\theta} d\theta = \frac{1}{jn2\pi} (e^{jn\pi/2} - e^{-jn\pi/2})$

$2\sin\frac{\pi}{2} n$



$\Sigma = 16p$ $T = 75$ min

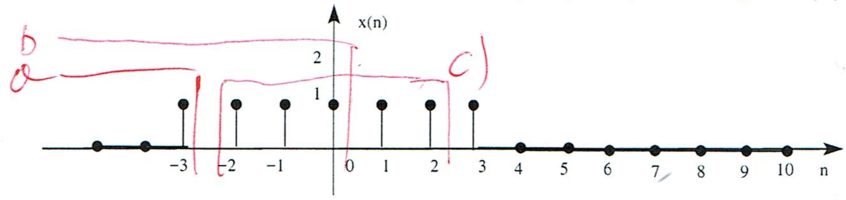
$\frac{1}{2} \leftarrow \frac{2\sin\frac{\pi}{2} \cdot 0}{2\pi \cdot 0}$

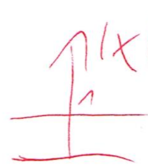
0

Jacek Misiurewicz

Test 2 20171 **version B** – inst. spectrum, z-transform, filters
Please mark your name <u>and test version</u> on all your answer pages

1. (3 p.) The STFT (instantaneous spectrum) $X(e^{j\theta}, n)$ of the signal $x(n)$ (see plot)

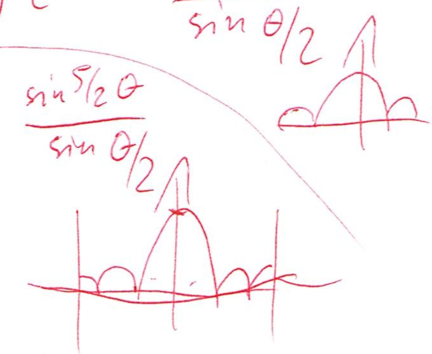




a) $e^{j3\theta}$ ...

b) $e^{j3/2\theta} \cdot \frac{\sin 4/2\theta}{\sin\theta/2}$

c) $\frac{\sin 5/2\theta}{\sin \theta/2}$

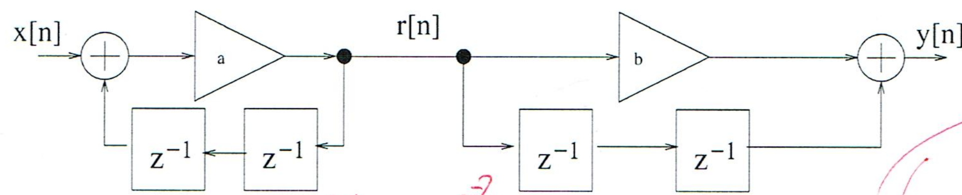


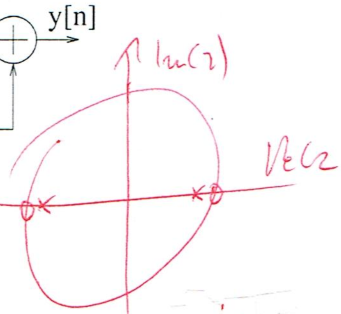
is computed using rectangular window $g(k)$ of length $K = 5$.

- For n given below, sketch $|X(e^{j\theta}, n)|$ for all θ; then calculate numerical values of $X(e^{j\theta}, n)$ at $\theta = 0$, $\pi/2$ and π:
  (a) $n = -3$.
  (b) $n = 0$.
  (c) $n = +\cancel{3}$. 2

hint 1: Use the above plot to mark three positions of window.

2. (~~5~~ 4 p.) Analyze a filter described with the following graph:





Assume $a = +0.81$, $b = \cancel{+}1$, $\frac{0,81}{1-0,81z^{-2}} \cdot \frac{z^{-2}-1}{1} = -\frac{1-z^{-2}}{1-0,81z^{-2}} 0,81 =$

(a) Find $H(z)$
Hint: you may use $r(n)$ as a "helper" when writing the difference equation.
(b) Find zeros/poles and plot their location on z-plane. Check if the filter is stable
(c) Sketch approximate $A(\theta)$ $A(0) = A(\pi) = 0$; $A(\pi/2) = \frac{2 \cdot 0,81}{1+0,81} \approx 0,895$, $\varphi(\pi/2) = 0$
(d) Calculate response $y(n)$ for $x(n) = -1^n + cos(n\pi/2)$
(e) (extra points) Propose a modification of the filter graph, saving on delay blocks.

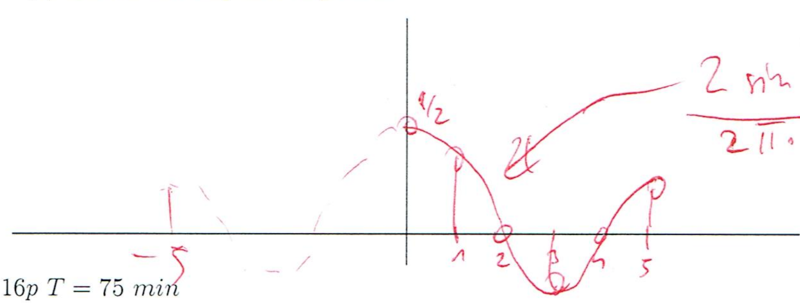
hint: $(1-c)(1+c) = 1 - c^2$, $(1 - jc)(1 + jc) = 1 + c^2$ → $0 \cdot (-1)^n + 0{,}895 \cdot \cos(n\pi/2)$

3. (2 p.) Calculate the z-transform and determine ROC (region of convergence) for the series:
(a) $\delta[n-20]$ z^{-20} all $z \neq 0$
(b) $\delta[n-3] + \delta[n+3]$ $z^{-3} + z^{+3}$, $|z| < \infty$ $z \neq 0$

4. (3 p.) Calculate a causal $x(n)$ when $\left(u(n) \cdot (j e^{j\pi/4 n} - j e^{-j\pi/4 n} \right) = u(n) \cdot -2 \sin\frac{\pi}{4} n$

$$X(z) = \frac{j}{1 - e^{j\pi/4} z^{-1}} - \frac{j}{1 - e^{-j\pi/4} z^{-1}}$$

5. (3 p.) A <u>noncausal, zero-phase</u> lowpass FIR filter with impulse response length equal to 11 was designed from windowed Inverse Fourier Transform of ideal filter frequency response. A rectangular window was used. Ideal filter cutoff was at $\theta_b = (1/2)\pi$.
(a) Calculate the group delay of the filter. → zero
(b) Find the approximate width of the transition band in the frequency response. $\frac{4\pi}{11}$
(c) Sketch the impulse response.



$\frac{2 \sin \pi/2 \cdot n}{2\pi \cdot n}$

$\Sigma = 16p$ $T = 75$ min