Lab 2 – Fourier transform, DFT, FFT

ver. March 22, 2017

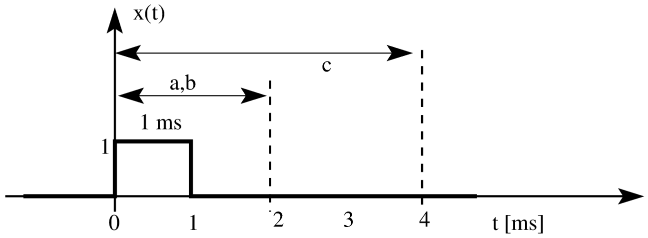
Entry test example questions

1. $x_a(t) = cos(2\pi f_a t)$ was sampled with sampling period T_s. Plot the { spectrum | N-point DFT } of $x[n]$ (f_a, T_s or f_s given, N given - whole number of periods or not)
2. A signal $x(n)$ with known Fourier spectrum $X(\theta)$ has been {inverted in time | decimated | modulated | ...}. Express mathematically what happened to the spectrum.
3. Calculate a DFT of a simple finite signal ($\delta(n-1)$, constant, $\{+1, -1, +1, \ldots\}$) - on paper

Lab exercises

Italics denote optional tasks. **Bold suggests what should be noted in the report.**

1. Investigate a single square impulse of 1 ms length, sampled under different conditions (sampling frequency f_s and signal measurement duration T – see table).



case	f_s	T	N	N_1	A_{max}	k_{null}	f_n at null	f at null
$x_1[n]$	1 MHz	2 ms						
$x_2[n]$	10 kHz	2 ms						
$x_3[n]$	10 kHz	4 ms						

Copy the table to the report. Then, fill the empty table cells with answers to the following. For each sampled signal ($x_1[n]$, $x_2[n]$, $x_3[n]$):

(a) Calculate total number of samples (**fill in N column**) and number of non-zero samples (**fill in N_1 column**)

(b) Create a simulated signal in Matlab (`[ones(1, N1), zeros(1, N-N1)]`)

(c) Calculate with Matlab and plot (on screen) magnitude of FFT (`plot(abs(fft(x...)))`)

(d) Find the maximum value in FFT (and **fill in as A_{max}**)

(e) Find (and **fill in**) the index k_{null} of the first null in FFT values (Note: Matlab numbering starts from 1, but we need the k for indexing frequency θ_k, so we must count from 0).

(f) **Fill in** normalized frequency f_n and physical frequency f calculated from k_{null}.

Think of signal $x_1[n]$ as "sampled with such a high frequency that it is almost CT" and **comment the spectrum differences between the three signals**.

2. Simulate in Matlab 1024 points of following signals. Then, for each signal

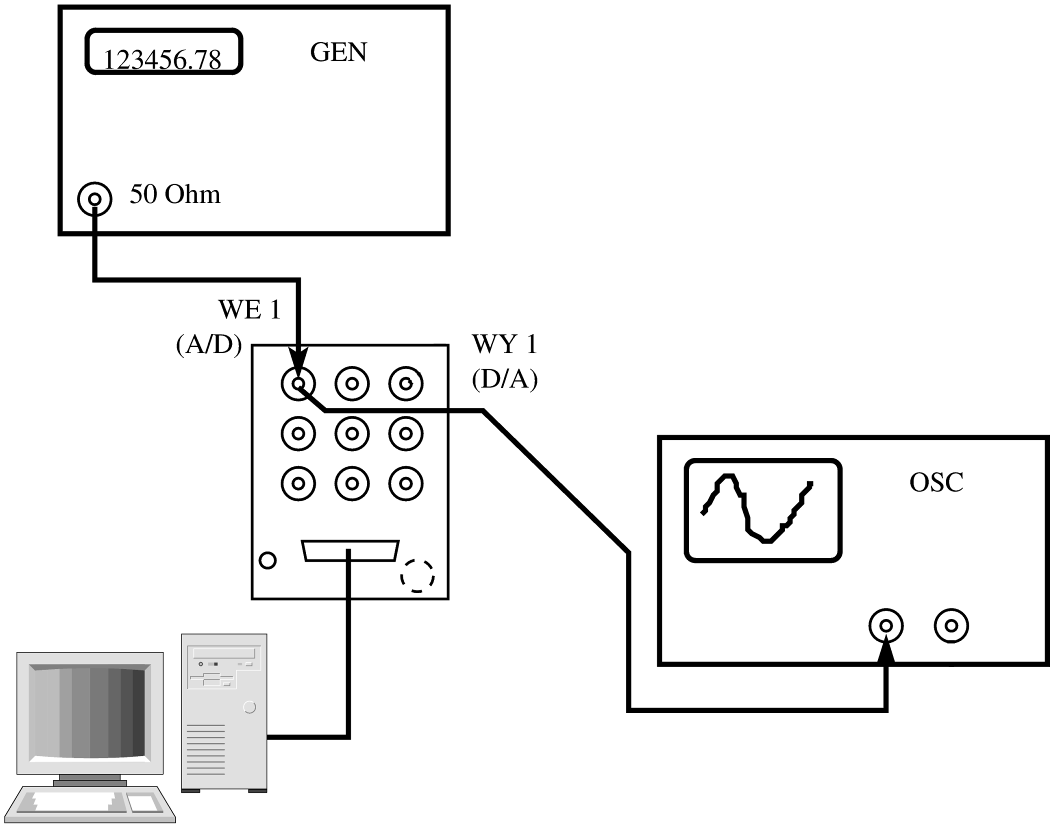
- Plot the signal (on the screen); for (c,d,e) signals **note** N_0 **(period length in samples)**
- Plot magnitude of FFT (**sketch it** in the report)
- (for signals (a) and (b)) count number of zero crossings in FFT (**note it in report)**
- (for signals (c & d)) **note** the locations of the peak, **compare peak width** for the two frequencies

Signals:

- (a) a 512 points square impulse (so you need to add 512 zeros to get 1024 samples)
- (b) other (narrower) square impulses – fill them up to 1024
- (c) sine wave with integer number of periods in window of 1024
- (d) sine wave with non-integer number of periods in window of 1024
- (e) $e^{jn\theta_c}$ - use `exp()` in Matlab; how many peaks do you see? why? Try different values of $0 < \theta_c \leq \pi$.
- *(f) a 32-point square impulse beginning at* $n = 0$
- *(g) a 32-point square impulse beginning at* $n = N_s > 0$

3. Plot a spectrum of 512 samples of sine wave. Then, zero-pad them to 1024 and 2048 samples. **Compare the resolution of FFT. Sketch abs(fft()) and note peak width**. Compute IFFT. (plot real part of IFFT to cut off arithmetic errors). Hint: `fft(x,L)` automatically zero-pads signal x to length L. **Comment the differences between original signal and one calculated by IFFT form FFT**.

4. Connect the equipment as presented below.



5. Capture 1024 samples of a live signal from a generator (use `xlive=LCPS_getdata(Nsamples,1,TsamplingInS`
Choose some signal (sin, rectangular,...) and set the f and f_s using your own wisdom.**Note your choice of signal type, f and f_s; note aproximate amplitude** (from the oscilloscope)
Plot, labeling properly the horizontal axis:
    - (a) the signal
    - (b) its 1024-point FFT (magnitude, of course)
    - (c) its 2^{12}- or even 2^{14}-point FFT (with zero-padding: `fft(x,N)` where N is the total length – with added zeros)

Make a sketch comparing (b) and (c) plots, describe differences with some words. Keep the variable `xlive` with the signal for future use.

6. Compute spectra of different windows. Ask teacher which/how many windows to choose.

   Copy the table and fill in (add rows for more windows):

Window type	Mainlobe width	First sidelobe	highest sidelobe	Sidelobes change with f
	(normalized freq.)	(dB below mainl.)	(dB below mainl.)	(describe shortly)

   In Matlab, window functions can be generated using: `rectwin, hamming, bartlett, blackman, hanning, kaiser`, with a scalar argument giving the length. For Kaiser – the second argument is β, use values between 1 and 8.

7. Do the following experiments to see the effect of windowing:

   (a) Plot a spectrum of 512 samples of sine wave. Choose the frequency to see the rectangular window effect clearly. If necessary, use zero-padding to see the spectrum better. **Sketch it in the report**.

   (b) Use different window shapes, trying to obtain good, clear plot of the spectrum.

   (c) Demonstrate the signal separation properties of different windows - plot a spectrum of a sum of two sinusoids with similar frequencies and amplitudes, then with very different frequencies and amplitudes. **Sketch these spectra in the report**.

8. Repeat FFT plots from Ex. 5, using a window (e.g. Hamming) on the signal. **Make a sketch and some comment to show differences** between spectra calculated with boxcar and with your chosen window.

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