EDISP
(English) Digital Signal Processing

Jacek Misiurewicz
e-mail: jmisiure@elka.pw.edu.pl
Institute of Electronic Systems
Warsaw University of Technology
Warsaw, Poland

February 22, 2017

General information

Lectures 2h/week, Thu, 08:15-10

Labs ≈4h/2weeks: Monday 8:15-12, room CS203. See the schedule. (Fri grp?)
First meeting for all students – 9:15, see webpage or the blackboard
Labs start with an "entry test"!!!

Contact J. Misiurewicz, (jmisiure@elka.pw.edu.pl) room 454.

Web page *http://staff.elka.pw.edu.pl/˜jmisiure/*
⟶ Slides – evening before lecture (usually ;-))

Homeworks Announced as a preparation for the tests.

Exams Two short tests within lecture hours (see the lab schedule) and a final exam during the exam session (TBA).

Scoring:

2x10%	=	20%	tests
6x5%	=	30%	lab + entry test (lab 0 – not scored)
		50%	final exam
2x2%	=	4%	extra for homeworks (maybe even more)

Short path `if [(score`$\geq$`41)&&(tests`$\geq$`15)&&(test2`$\geq$`5)]; then score`$*=2$`; fi`
"if" conditions are evaluated once, before re-doing tests etc.

Books

base book The course is based on selected chapters of the book:
A. V. Oppenheim, R. W. Schafer, *Discrete-Time Signal Processing*, Prentice-Hall 1989 (or II ed, 1999; also previous editions: *Digital Signal Processing*).

free book A free textbook covering some of the subjects can be found here: `http://www.dspguide.com/pdfbook.htm` *The book is slightly superficial, but nice*

good book Edmund Lai, *Practical Digital Signal Processing for Engineers and Technicians*, Newnes (Elsevier), 2003

exercise book Vinay K. Ingle, John G. Proakis, *Digital Signal Processing using MATLAB*, Thomson 2007;*Helps understand Matlab usage in the lab (but is NOT a lab base for us)*

Additional books available in Poland:
R.G. Lyons, Wprowadzenie do cyfrowego przetwarzania sygnałów (WKiŁ 1999)
Craig Marven, Gilian Ewers, Zarys cyfrowego przetwarzania sygnałów, WKiŁ 1999 [en: A simple approach to digital signal processing, Wiley & Sons, 1996]
Tomasz P. Zieliński, Od teorii do cyfrowego przetwarzania sygnałów, WKiŁ 2002

A schedule was here - see the webpage for an updated version!

Course aims *or what I will check when it comes to grading*

- knowing the mathematical fundamentals of discrete-time (DT) signal processing: DT signals, normalized frequency notion, DT systems, LTI assumption, impulse response, stability of a system
- understanding the DT Fourier transforms and know how to apply them to simple DT signal analysis
- knowing basic window types and their usage for FT and STFT
- understanding the description of a DT system with a graph, difference equation, transfer function, impulse response, frequency response
- being able to apply Z-transform in analysis of a simple DT system
- understanding filtering operation and the process of DT filter design; being able to use computer tools for this task
- knowing the basic ways of implementing DSP algorithms (with PC, signal processor, hardware FPGA)
- understanding 2D signal processing basics: 2D convolution/filtering, 2D Fourier analysis
- being able to use a numerical computer tool (Matlab, Octave or similar) for simulating, analyzing and processing of DT signals

What Is EDISP All About ;-)

Theory Discrete-time signal processing

Practice Digital signal processing

Application examples:

Filters Guitar effects, radar, software radio, medical devices...
A digital filter does not lose tuning with aging, temperature, humidity...

Adaptive filters Echo canceller, noise cancellation (e.g. hands-free microphone in a car),...

Discrete Fourier Transform/FFT Signal analyzer, OFDM modulation, Doppler USG, ...

Random signals Voice compression, voice recognition....

2D signals Image processing, USG/CT/MRI image reconstruction, directional receivers, ...

Upsampling/Interpolation CD audio output, ...

Oversampling CD audio D/A conversion (example)

Please have a look at the black/green-board.

Notice & remember some things:

- Upsampling
- Filtering (and what happens to the signal spectrum)
- Frequency response (frequency characteristics) of a filter
- Trade-off: we simplify analog part by doing a tough job on the digital side

Some notes:

- First order LP filter: $A(f) = \frac{1}{\sqrt{1+(2\pi fRC)^2}}$ $\longrightarrow$ 6 dB per octave (20 dB per decade)
- To obtain 80 dB of attenuation we need 4 decades (10^4 times cutoff frequecy)
- N-th order filter: $A(f) = \frac{1}{\sqrt{1+(2\pi fRC)^{2N}}}$

Signal classes

Continuous or Discrete **amplitude** and **time**.

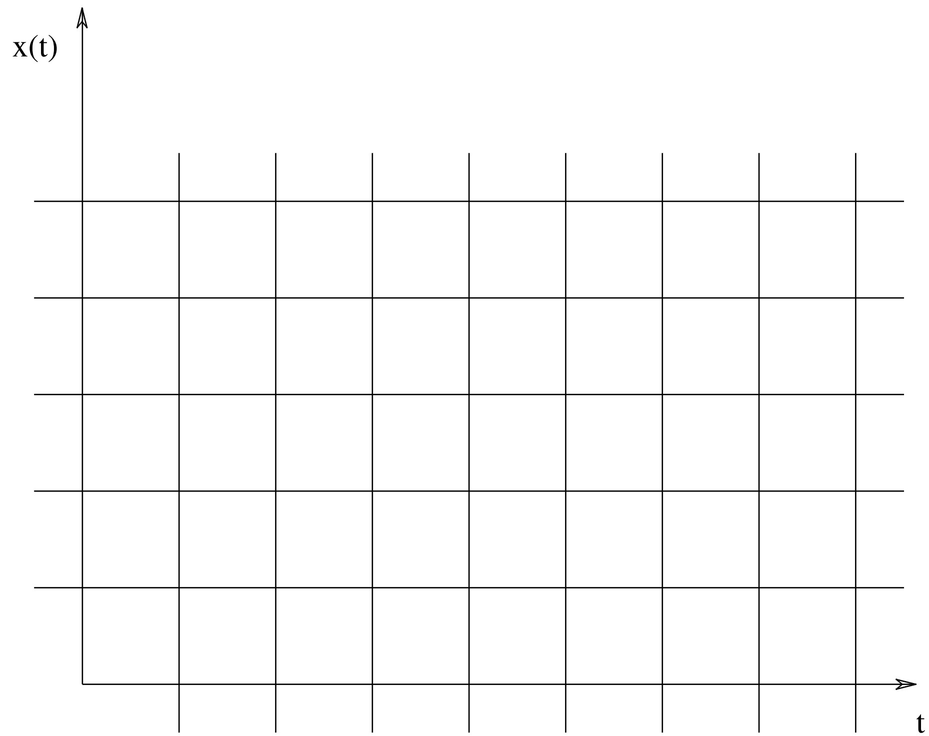
CA-CT $\longrightarrow$ "analog" signals

DA-CT $\longrightarrow$

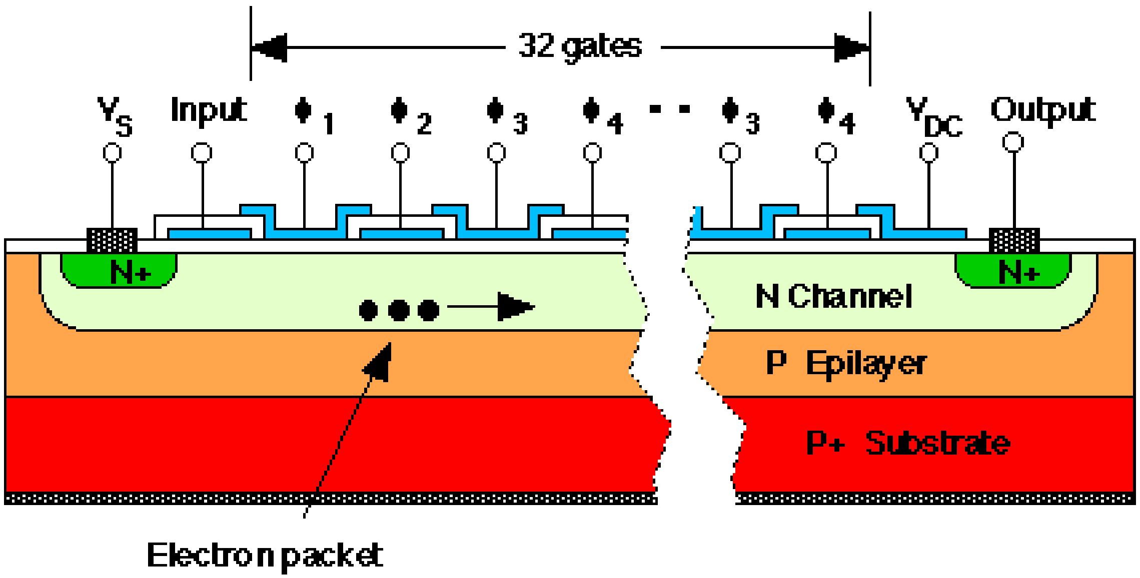
CA-DT $\longrightarrow$ CCD, SC, SAW devices

DA-DT $\longrightarrow$ digital devices

We'll speak mainly about DT properties; only in some subject DA will be of importance.



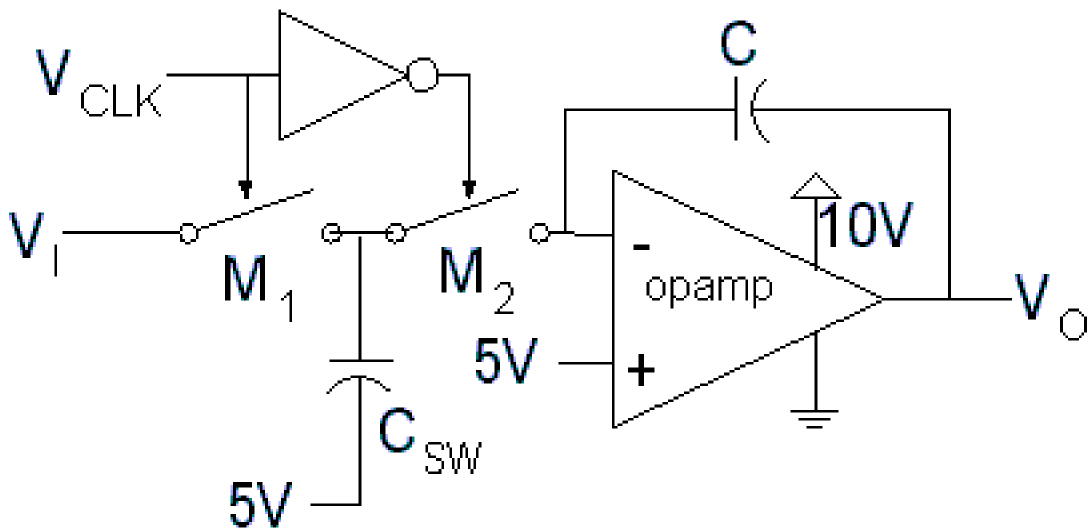
(side remark:) CCD device – continuous amplitude, discrete time



Charge is transferred on the clock edge (discrete time!).

Clock is usually polyphase (2-4 phases).

SC device (another CA-DT example)



DT signal representations

DT signal $\longleftarrow\longrightarrow$ a number sequence

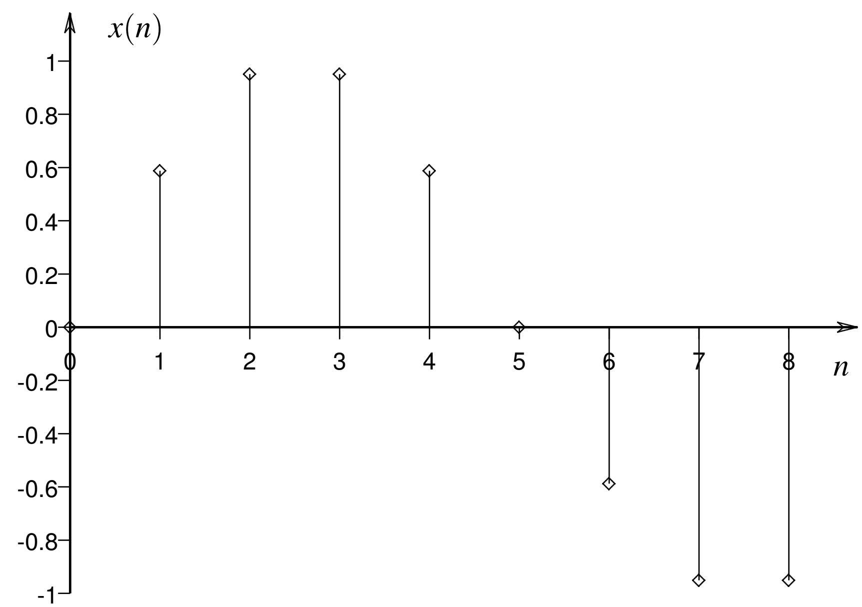
$$x[n] = \{x(n)\}$$

$x[n]$ is a number sequence (or ...)

$x(n)$ is a $n-$th sample

$\longrightarrow$ $x(n)$ is *undefined* for $n \notin \mathbb{Z}$

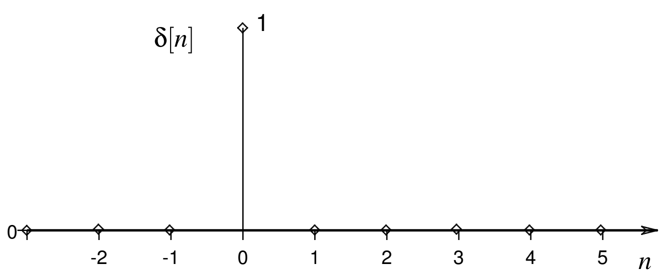
- it *may* come from sampling of analog signal
- but it may also be inherently discrete
- n may correspond to: time, space, ...



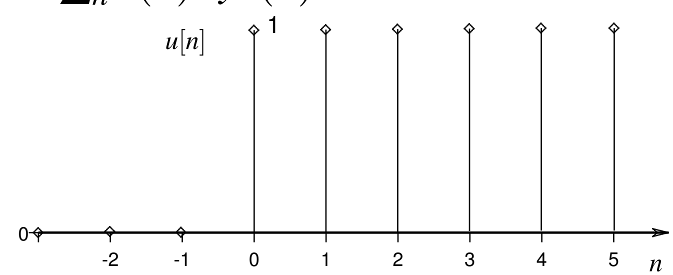
However, the most popular interpretation is: periodic sampling in time.

Number sequence (or DT signal) operations; basic sequences

operation	notation	definition	
sum	$z[n] = x[n] + y[n]$	$\forall n \;\; z(n) = x(n) + y(n)$	$\longrightarrow \overset{\downarrow}{\bigoplus} \longrightarrow$
scale	$z[n] = \alpha \cdot y[n]$	$\forall n \;\; z(n) = \alpha \cdot y(n)$	$\longrightarrow \triangleright \longrightarrow$
shift	$z[n] = x[n - n_0]$	$\forall n \;\; z(n) = x(n - n_0)$	$\longrightarrow \boxed{z^{-n}} \longrightarrow$
difference	$z[n] = x[n] - y[n]$	$\forall n \;\; z(n) = x(n) - y(n)$	
product	$z[n] = x[n] \cdot y[n]$	$\forall n \;\; z(n) = x(n) \cdot y(n)$	
scalar product	$c = \langle x[n],\; y[n] \rangle$	$c = \sum_n x(n) \cdot y^*(n)$	



Unit sample sequence (DT impulse)



Unit step sequence

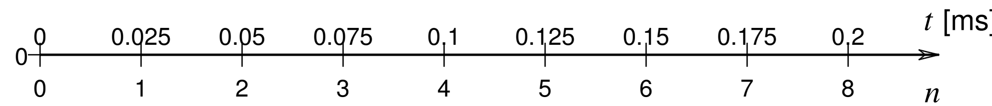
$$\delta[n] = u[n] - u[n-1]$$

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

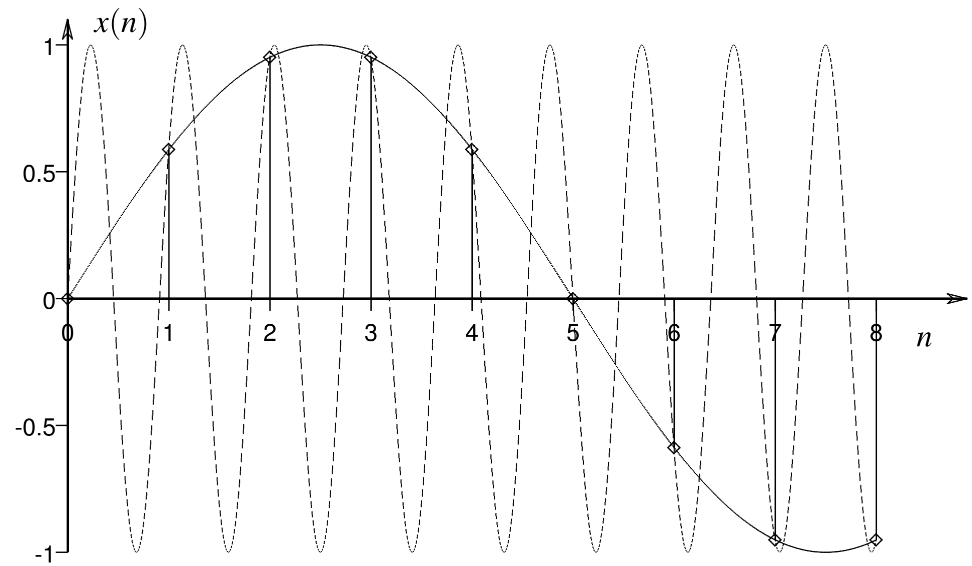
Periodic sampling

$$n \longleftarrow\longrightarrow n \cdot T_s$$

$$x(n) = x_a(nT_s)$$



$$n = t/T_s, \;\; T_s = 0.025 \text{ [ms]}$$



Misinterpretations

$\longrightarrow$ we do not know what is between points

a) $sin(n \cdot (1/5) \cdot \pi)$ or

b) $sin(n \cdot (2 + 1/5) \cdot \pi)$?

We have to **know** which one to choose $\longrightarrow$ sampling theorem

The Sampling Theorem

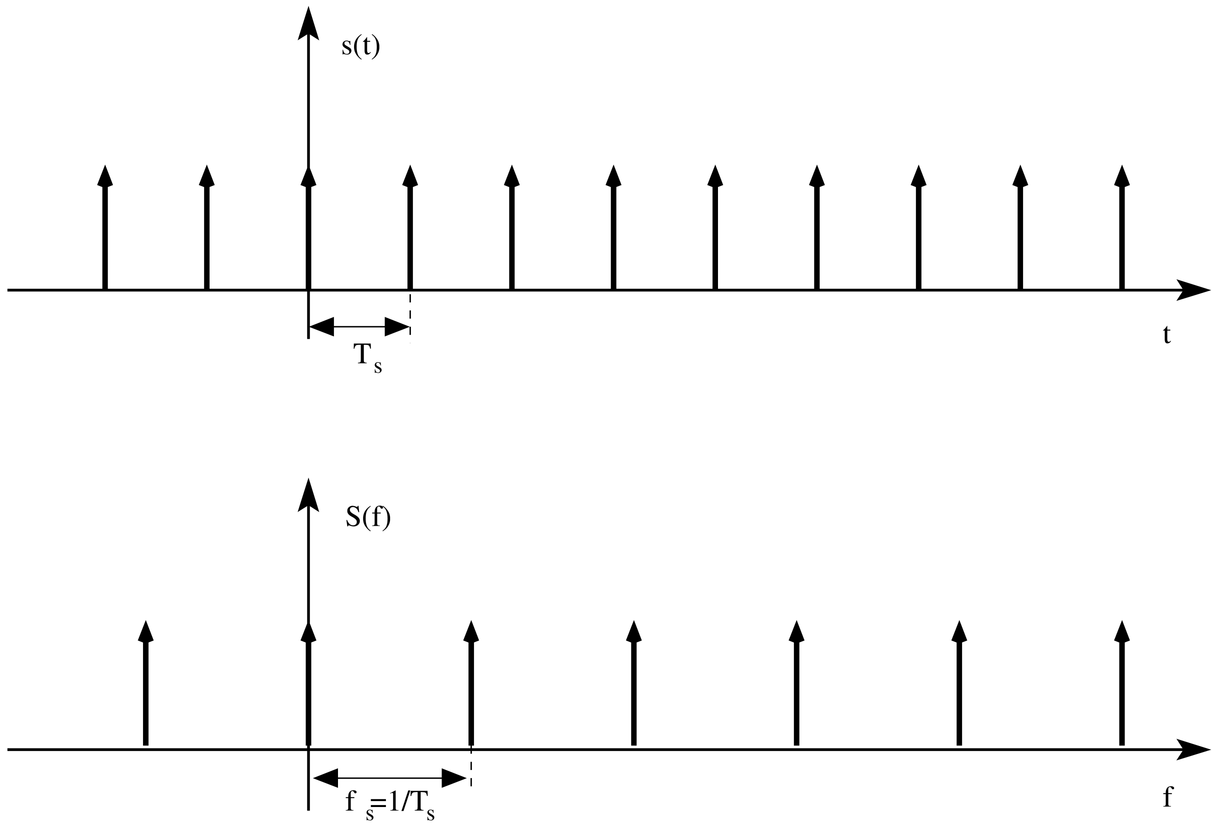
Named also after:

- 1915 Edmund T. Whittaker (UK)
- 1928 Harry Nyquist [ny:kvist] (SE) $\longrightarrow$ (US)
- 1928 Karl Küpfmüller (DE)
- 1933 Vladimir A. Kotelnikov (USSR)
- 1946 Gábor Dénes (HU) $\longrightarrow$ Dennis Gabor (UK)
- 1949 Claude E. Shannon (US)
- Cardinal Theorem of Interpolation Theory

If a signal is bandlimited with f_b, the reconstruction is possible from samples taken with $f_s > 2f_b$

Nyquist frequency: $f_s/2$, Nyquist rate: $2f_b$

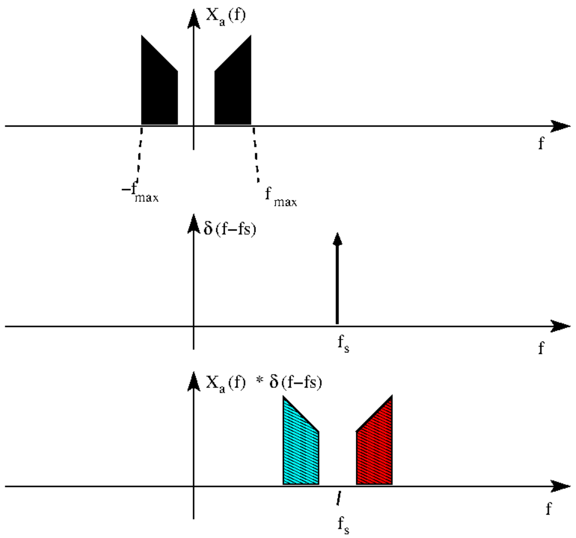
“Sha” function and its spectrum

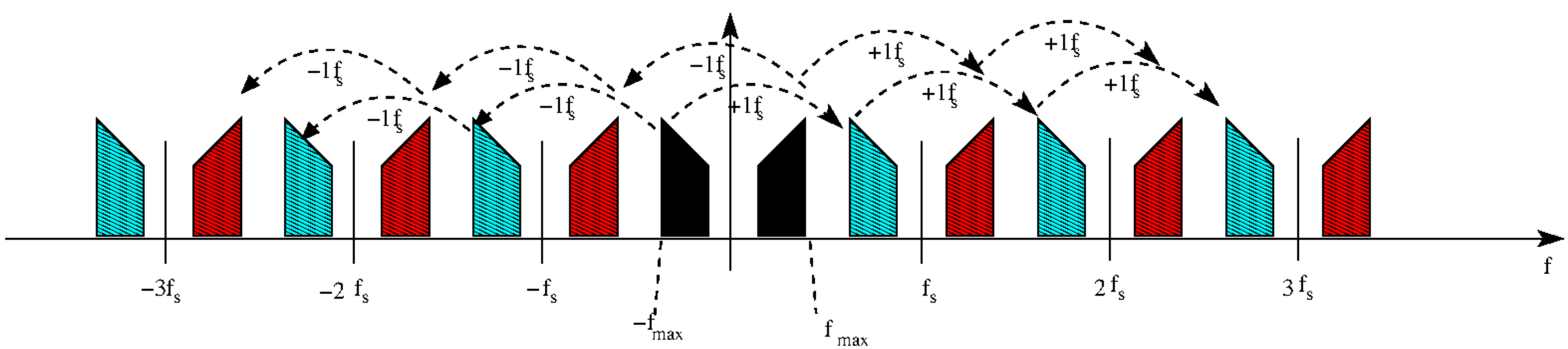


Шш

山
mountain

Sampling = convolution





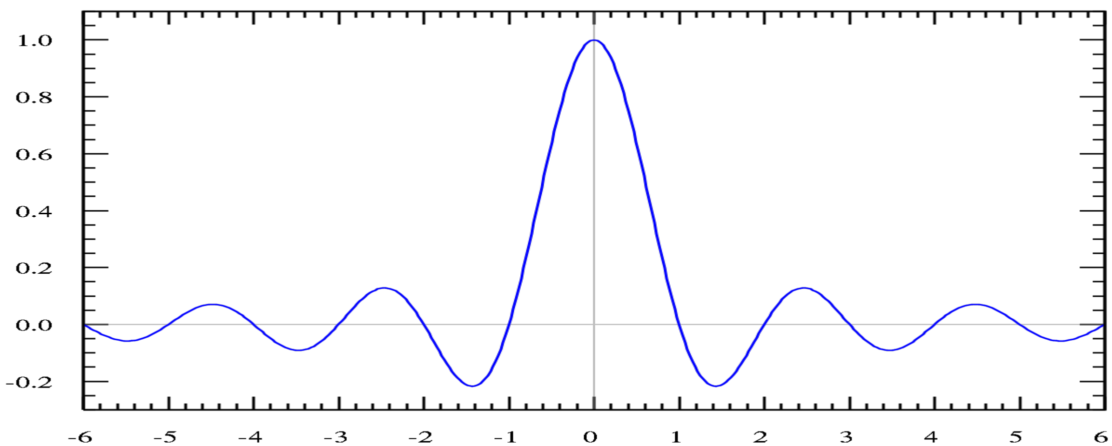
Reconstruction

Reconstruction: interpolation, (*sinus cardinalis* $\mathrm{sinc} = \mathrm{Sa} = \frac{\sin(\pi x)}{\pi x} = \mathrm{j}_0(\pi x)$)

$$x(t) = \sum_{n=-\infty}^{\infty} x[n] \cdot \mathrm{sinc}\left(\frac{t - nT}{T}\right)$$

lowpass filtering (Küpfmüller filter) (DE)

$$x(t) = \left(\sum_{n=-\infty}^{\infty} x[n] \cdot \delta(t - nT)\right) * \mathrm{sinc}\left(\frac{t}{T}\right)$$



Sampling rates in audio processing

`http://en.wikipedia.org/wiki/Sampling_rate`

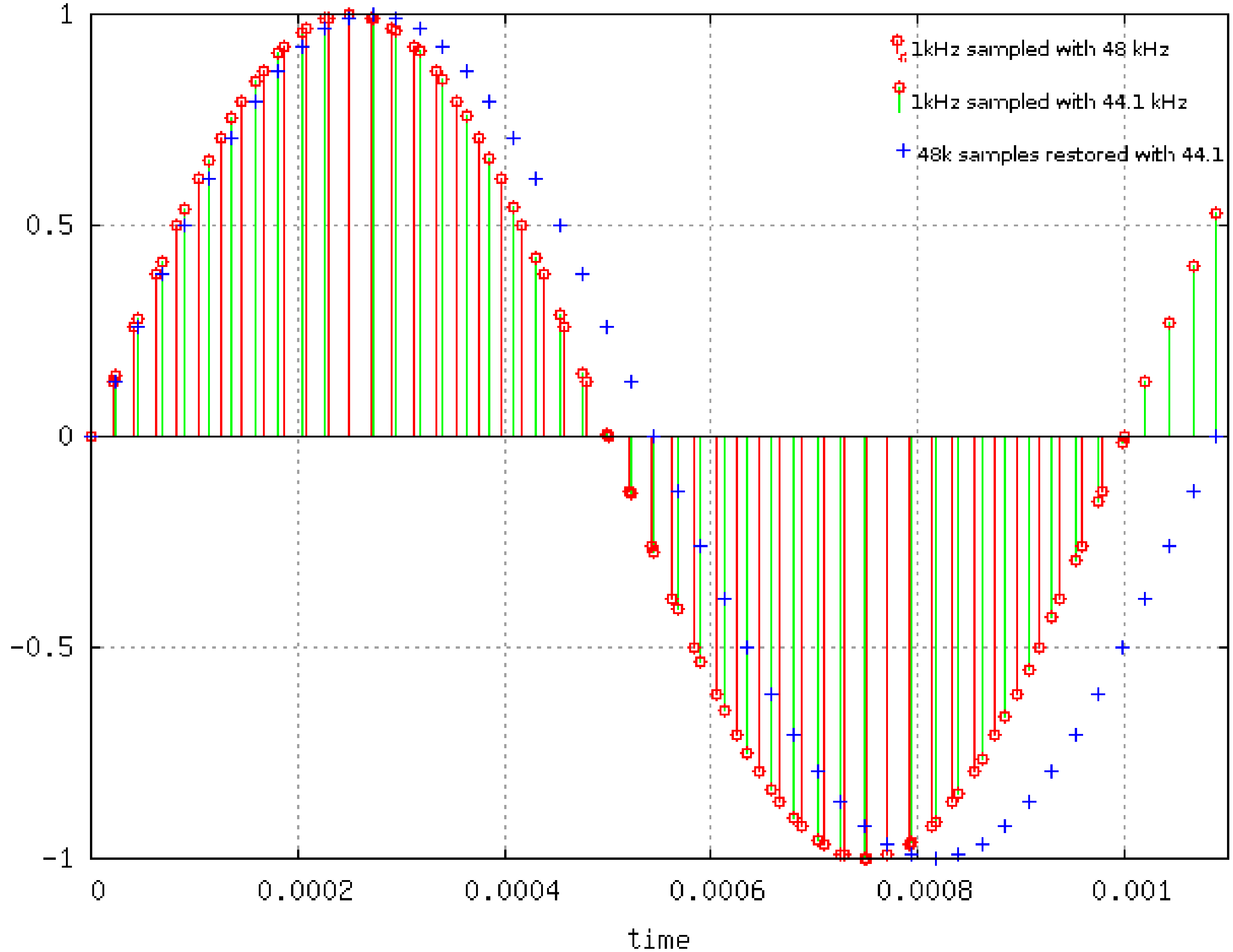
In digital audio, common sampling rates are:

- 8,000 Hz - telephone, adequate for human speech
- 22,050 Hz - radio
- 32,000 Hz - miniDV digital video camcorder, DAT (LP mode)
- 44,100 Hz - audio CD, also most commonly used with MPEG-1 audio (VCD, SVCD, MP3) *compatible with PAL (625 line) and NTSC (528 line) dot frequency*
- 48,000 Hz - digital sound used for miniDV, digital TV, DVD, DAT, films and professional audio
- 96,000 or 192,000 Hz - DVD-Audio, some LPCM DVD tracks, BD-ROM (Blu-ray Disc) audio tracks, and HD-DVD (High-Definition DVD) audio tracks
- 2.8224 MHz - SACD, 1-bit sigma-delta modulation process known as Direct Stream Digital, (Sony and Philips)

Frequency in a DT signal

	CD audio system	DAT audio system
Sampling:	44100 Hz	48000 Hz
Nyquist:	22050 Hz	24000 Hz
t_s	22.676μs	20.833μs
1kHz: samples per period	44.1	48
1kHz: moved from CD to DAT	1kHz	48/44.1=1.0884 kHz

We need a good definition of frequency!


1
0.5
0
-0.5
-1
0
0.0002
0.0004
0.0006
0.0008
0.001
time
1kHz sampled with 48 kHz
1kHz sampled with 44.1 kHz
48k samples restored with 44.1

DT signal frequency concept

Continuous time cosine:		Discrete time cosine:	Normalized...
$x_a(t) = \cos\omega t$ $\omega = 2\pi f$	$\omega \in \mathbb{R}$	$x(n) = \cos\omega n T_s$ $x(n) = \cos 2\pi f\, n\frac{1}{f_s}$ $x(n) = \cos\theta n$	...time: $n = t/T_s$ ...frequency: $f_n = \frac{f}{f_s}$ ...ang. freq.: $\theta = 2\pi\frac{f}{f_s}$
$T = \frac{1}{f} = \frac{2\pi}{\omega}$ $x(t) = x(t+kT)$	$\leftarrow$ period ? $\rightarrow$	$N_0 = \frac{1}{f_n} = \frac{2\pi}{\theta}$ $x(n) = x(n+kN)$ $x(n+N)$ defined only if $N \in \mathbb{Z}$	
Always	$\leftarrow$ periodic $\rightarrow$	only if $N_0 = N/M$ (!!)	

Normalized angular frequency θ: interval of 2π may be assumed as $[0, 2\pi)$ or $[-\pi, \pi)$.

$$\cos n(\theta + k\cdot 2\pi) = \cos(n\theta + n\cdot k\cdot 2\pi) = \cos n\theta$$

Normalized frequency example

$x_a(t) = \cos \omega t$ with $\omega = 1000 \cdot 2 \cdot \pi$ (1kHz)
Let us sample it with $f_s = 48\ kHz$
$x(n) = x_a(nT_s) = x_a(n/f_s) = \cos(1000 \cdot 2\pi \cdot n/48000) = \cos(\frac{2\pi}{48}n)$
or
$x_a(t) = \cos \omega t$ with $\omega = 2000 \cdot 2 \cdot \pi$ (2kHz)
Sampled with $f_s = 96\ kHz$
$x(n) = x_a(nT_s) = x_a(n/f_s) = \cos(2000 \cdot 2\pi \cdot n/96000) = \cos(\frac{2\pi}{48}n)$

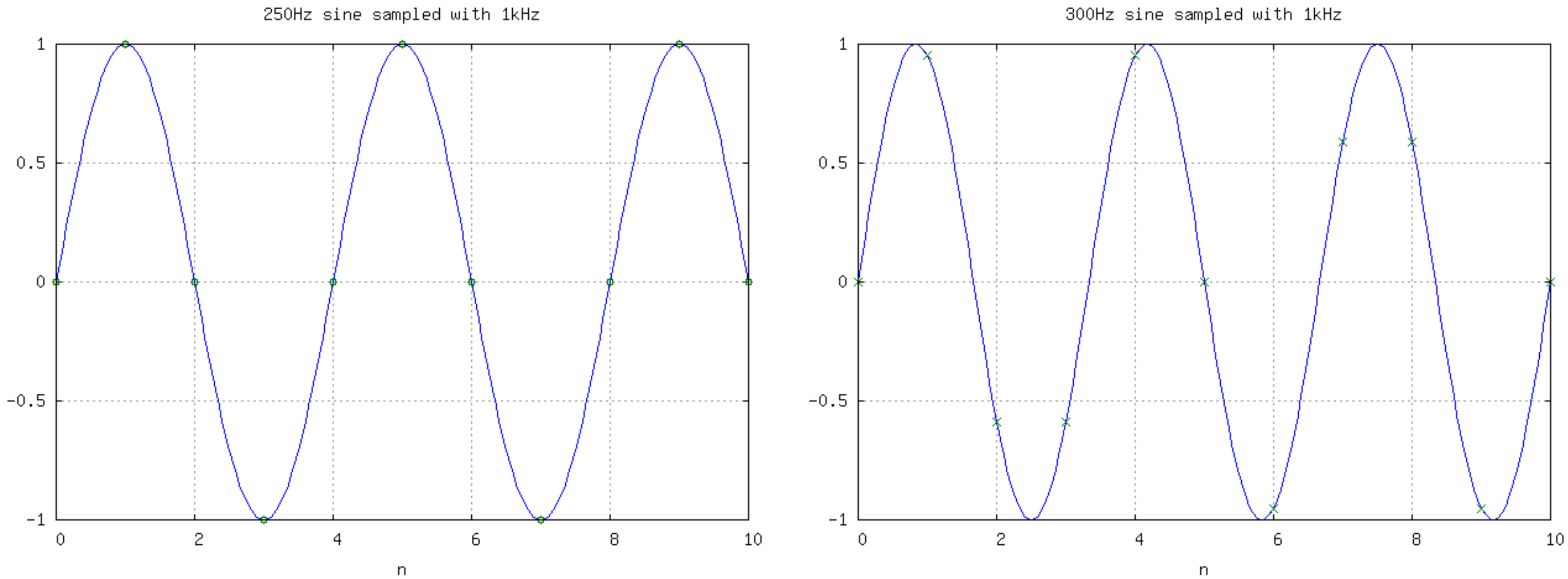
$\longrightarrow$ signals identical after sampling

- Extract important parameter: $\theta = \frac{2\pi}{48}$
- ...and we may write it down as $x(n) = \cos(\theta n)$

$\longrightarrow$ Normalized (angular) frequency $(2\pi) \cdot \frac{f}{f_s}$ determines the properties of the sampled signal, and now it is not important what was the frequency of x_a (only how it was related to f_s).

Periodicity example

Periodicity of a *number series* is not the same as the periodicity of a CT signal



Period of a sine wave is a real number: $x(t)$ exists for $t \in \mathbb{R}$.

With a number series the period must be an integer, because $x(n)$ exists only for $n \in \mathbb{Z}$.