EDISP (2D sig)
(English) Digital Signal Processing
Two-dimensional signals & filters
lecture

May 31, 2017

Multidimensional signals

- Analogue K-D signal $x_a(t_1,\ t_2,\ t_3,\ \ldots,\ t_K)$, t_k not necessarily time.
- Discretization (sampling) $\longrightarrow x(n_1,\ n_2,\ n_3,\ \ldots,\ n_K)$, *some signals are already discrete!*
  $\longrightarrow$ multidimensional discrete signal = multidimensional number series
- sampling periods $T_{s\ k} \longrightarrow$ sampling frequencies $f_{s\ k} = \frac{1}{T_{s\ k}}$ not necessarily equal; (e.g some scanners have different h & v resolutions)
  if t_k is spatial, $f_{s\ k}$ is *spatial frequency*
- Examples
  - 2-D picture
  - linear antenna array (t_1 – discrete or continuous space, t_2 – continuous time)
  - pulsed radar signal (“slow time” and “fast time”)

ULA - Uniform Linear Array (of antennas)

SDMA (a.k.a. BDMA) - Space (Beam) Division Multiple Access

Smart LTE antenna

Two dimensions in classsical radar

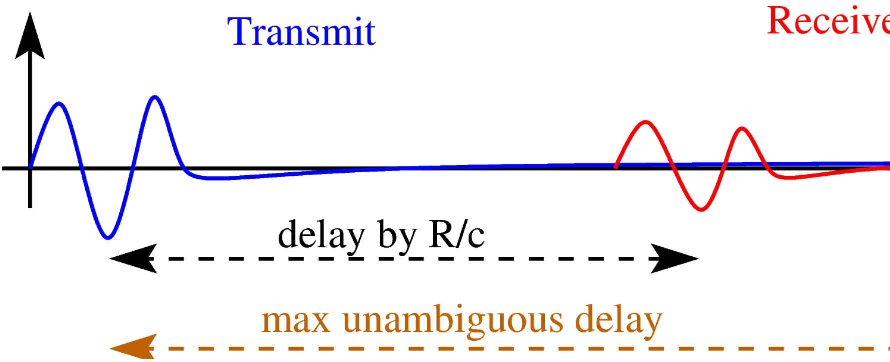


Image – a 2-D signal – and its 2-D DFT

- $T_{s\,1},\ T_{s\,2}$ – pixel dimensions; $f_{s\,1},\ f_{s\,2}$ – resolution (dpi, lines/mm)
- Fourier spectrum:

$$X(e^{j\theta_1},\ e^{j\theta_2}) = \sum_{n_1=-\infty}^{+\infty} \sum_{n_2=-\infty}^{+\infty} x(n_1,\ n_2)e^{-jn_1\theta_1}\ e^{-jn_1\theta_2}$$

$\theta_k = \omega_k \cdot T_{sk} = \frac{2\pi f_k}{f_{sk}}$ – normalized angular frequency

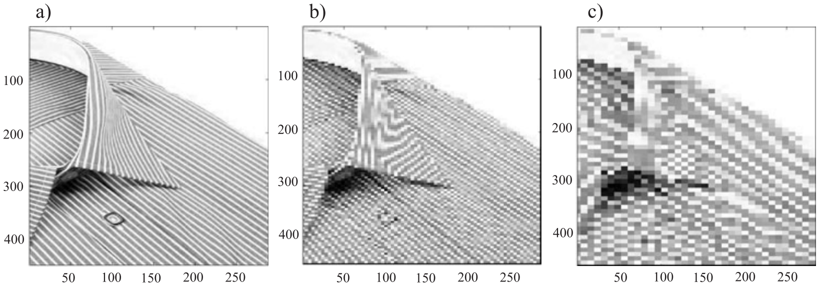
- finite picture $\longrightarrow$ represented by a discrete spectrum

$$X(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x(n_1,\ n_2)e^{-j2\pi k_1 n_1/N_1}\ e^{-j2\pi k_2 n_2/N_2}$$

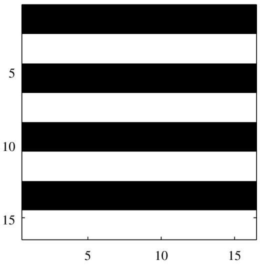
- reconstruction by a discrete Fourier series

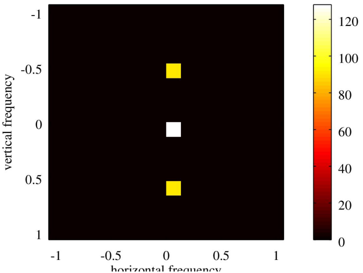
$$x(n_1, n_2) = \frac{1}{N_1 N_2} \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} X(k_1\ , k_2)e^{j2\pi k_1 n_1/N_1}\ e^{j2\pi k_2 n_2/N_2}$$

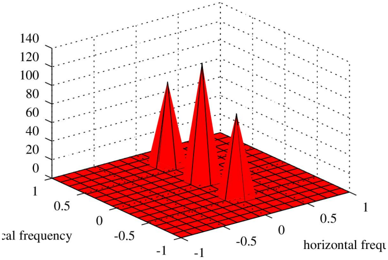
image sampling example



2-D DFT example







2-D LTI systems

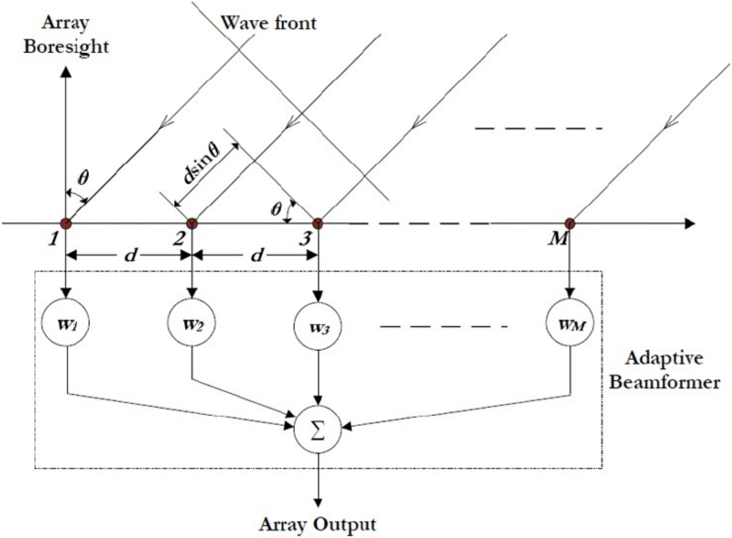
Linear and *(Time = shift in n_k)* Invariant (we extend the definition for 1-D systems)

- allows for analysis by impulse response
  (unit impulse: $\delta(m, n) = 1$ if $m = n = 0, \; = 1$ otherwise)
  impulse response is sometimes called Point Spread Function – PSF (especially in optics)
- causality – meaningful if one of dimensions is time-related
- delay - in spatial dimension, zero-delay filter is the best
- 2-D convolution (linear filtering)

$$y(n_1, n_2) = \sum_{m_1=-\infty}^{+\infty} \sum_{m_2=-\infty}^{+\infty} h(m_1,\ m_2) \cdot x(n_1 - m_1,\ n_2 - m_2)$$

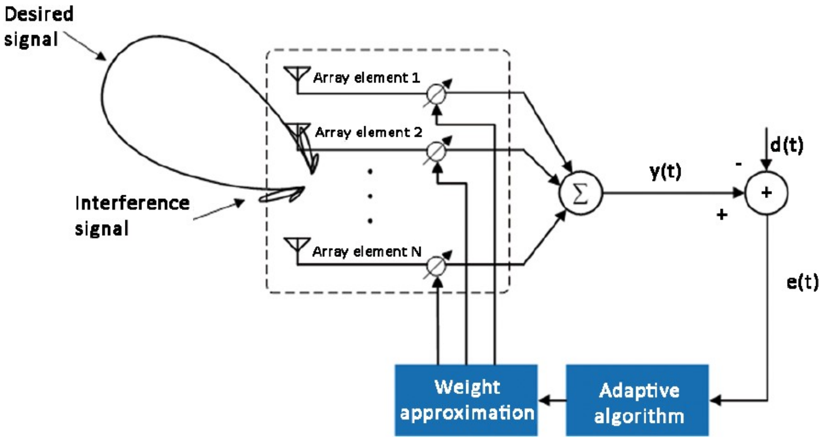
- if $h(m_1,\ m_2) = h_1(m_1) \cdot h_2(m_2)$, we may decompose 2-D filtering into 2x(1-D) (important for long impulse responses)

Beamforming

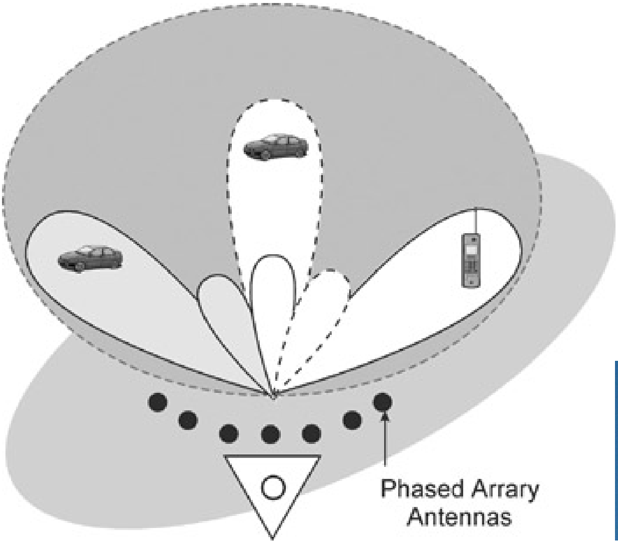


$$x_k(t) = A(t)cos(\Omega t + \frac{nd}{\lambda} sin\theta) \longrightarrow x_k^{bb}(t) = A(t)e^{j\frac{nd}{\lambda} sin\theta}$$

Beamforming

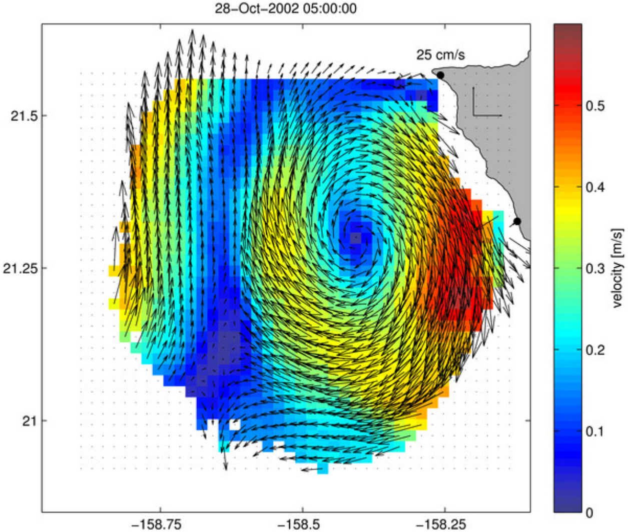


Application - beamforming in SDMA



Application - ocean currents monitoring (WERA)

WERA is a HF radar using electromagnetic waves between 6 and 30 MHz (50 m to 10 m wave length) to measure surface current velocities, ocean wave height (spectra) and wind.



Images - practical remarks

We concentrate on monochrome images (B/W photos, print, raw images in medical, radar, sonar, satellite technology). Color images add some complexity, unimportant for the signal processing basics.
Understanding frequency concept:

- Horizontal frequency - changes in the horizontal direction (trunks in the forest)
- Vertical frequency - changes in the vertical direction (horizon)
- High frequencies - sharp edges, small features
- Low frequencies - soft edges, large areas of same intensity

Filtering

Why:

- resizing by interpolation/decimation (LP)
- removing noise or interference (LP)
- sharpening images (slightly HP)
- detecting edges (HP)

Filtering methods:

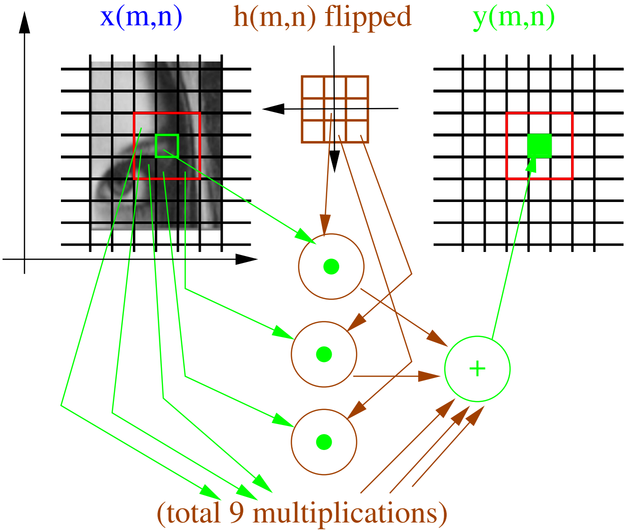
- Spatial (image) domain: convolution=weighted average of neighboring pixels
- Frequency domain: masking out (zeroing) parts of the 2D spectrum
- Nonlinear filters: some nonlinear manipulation on the set of neighboring pixels (e.g median)

How to behave at the image boundary with spatial filters?

- assume zeros outside (effect:dark areas at the boundaries)
- repeat last data row/column
- circular symmetry (effect: sky under the ground?)
- mirror symmetry (best for photo type images)

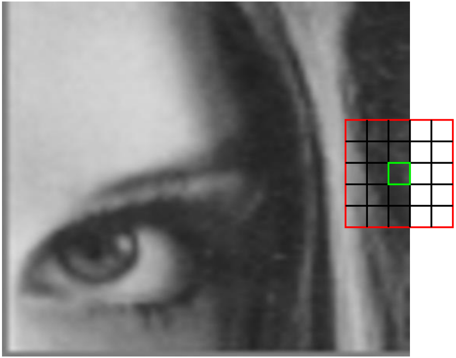
Spatial filter (2D convolution)

Operation on some vicinity of the current pixel.



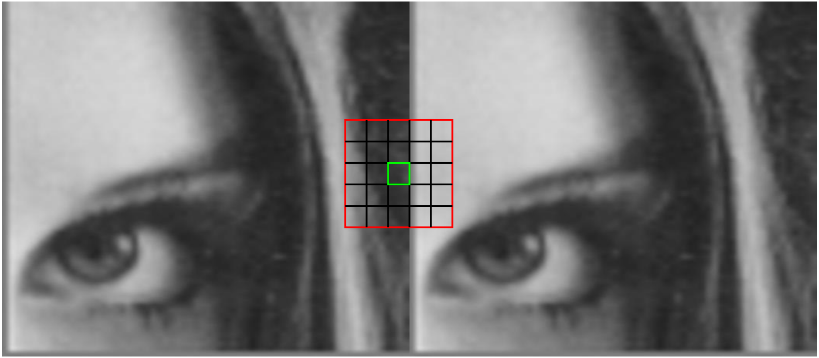
$$y(m,n) = \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} x(i,\ j) \cdot h(m-i,\ n-j)$$

Edge problem



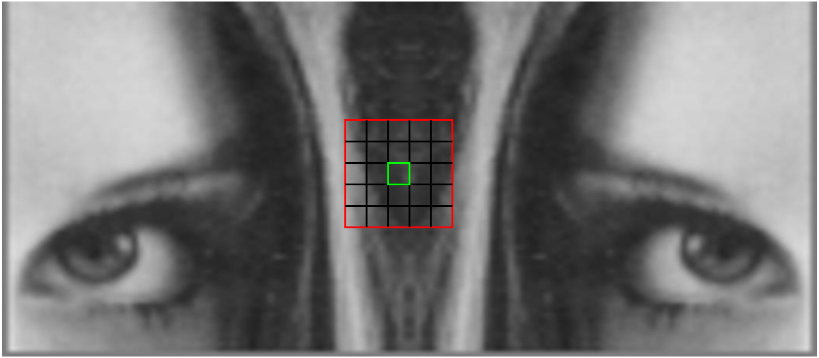
Assume zero outside

Edge problem



Assume circular copy

Edge problem



Assume flipped (mirror) copy

Spectral domain filtering (by FFT)

Operation on whole picture.

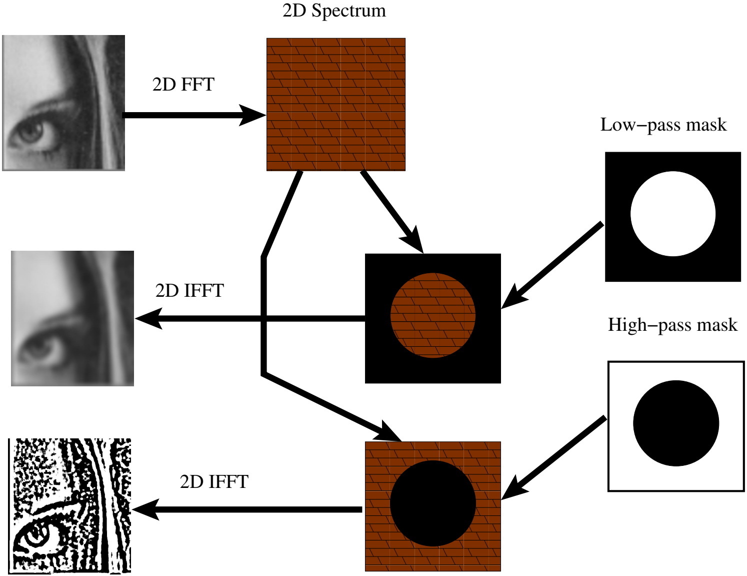


Image denoising

- AWGN - linear filtering
- impulsive noise (“salt and pepper”) - nonlinear methods

Median filter (nonlinear)

- removing impulses
- preserving edges
- method: replace a pixel with median from its vicinity (3x3,5x5, ...)

Median filter

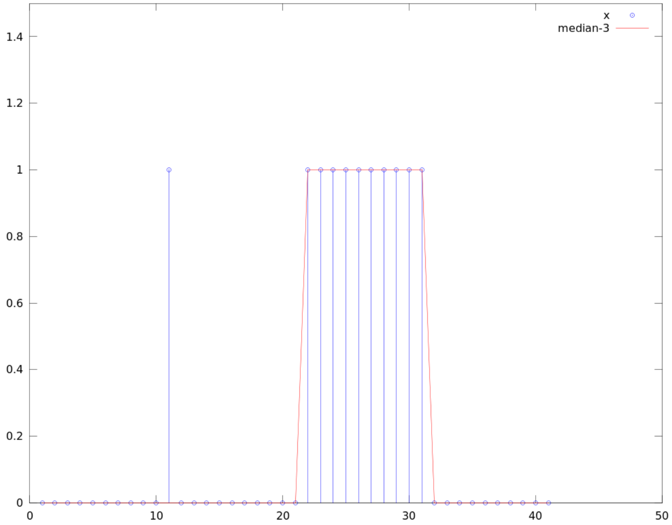
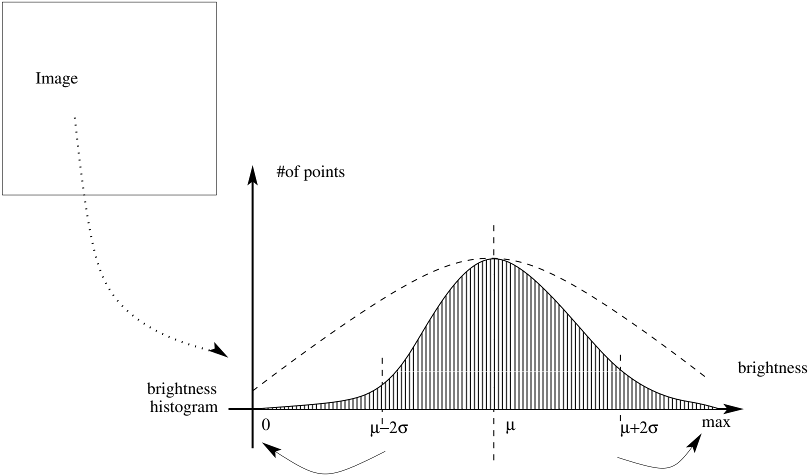


Image brightness normalization



Other (non-image) uses

- WERA array radar: spatial freq = direction, time freq = Doppler
- Smart antenna: spatial filter = selection of a user direction

Prepare for the lab!

- Understanding frequency in 2D
- Calculate a simple 2D Fourier transform: practice on 2x2 or 4x4 pictures:

0	0	0	0
1	1	1	1
0	0	0	0
1	1	1	1

0	1	0	1
0	1	0	1
0	1	0	1
0	1	0	1

1	0	1	0
0	1	0	1
1	0	1	0
0	1	0	1

- Apply a linear filter to above pictures.
- Understand a median. Why it is a nonlinear operation?