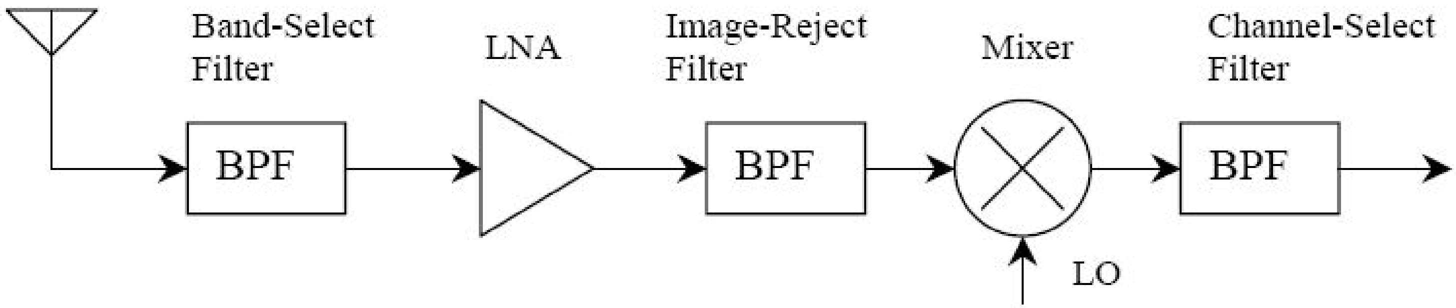
Up-/Down- conversion in frequency

Why?

- Amplification 120+ dB without parasitic feedback
- Tunable receiver without too many tuned filters
- Easier narrowband filtering

Superheterodyne receiver



IF

$$cos(\omega_1 t)cos(\omega_2 t) = 1/2\cos((\omega_1 - \omega_2)t) + 1/2\cos((\omega_1 + \omega_2)t)$$

$\omega_1 - \omega_2$ - IF or beat frequency

Intermediate frequency is usually lower than RF (easier to amplify/filter/process).

Examples (receivers):

FM radio: 10.7 MHz

AM radio: 465 kHz

analog TV: 30 MHz, 45 MHz

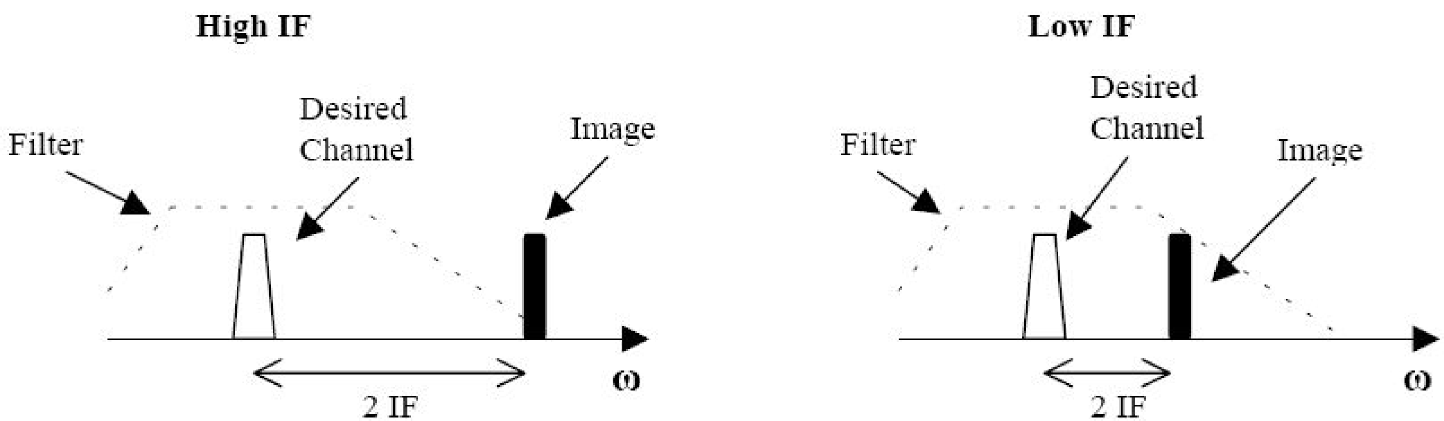
Satellite equipment: 70 MHz second, 950-1450 first IF (L-band) *double conversion receiver!!*

Terrestrial MW link: 250 MHz, 70 MHz

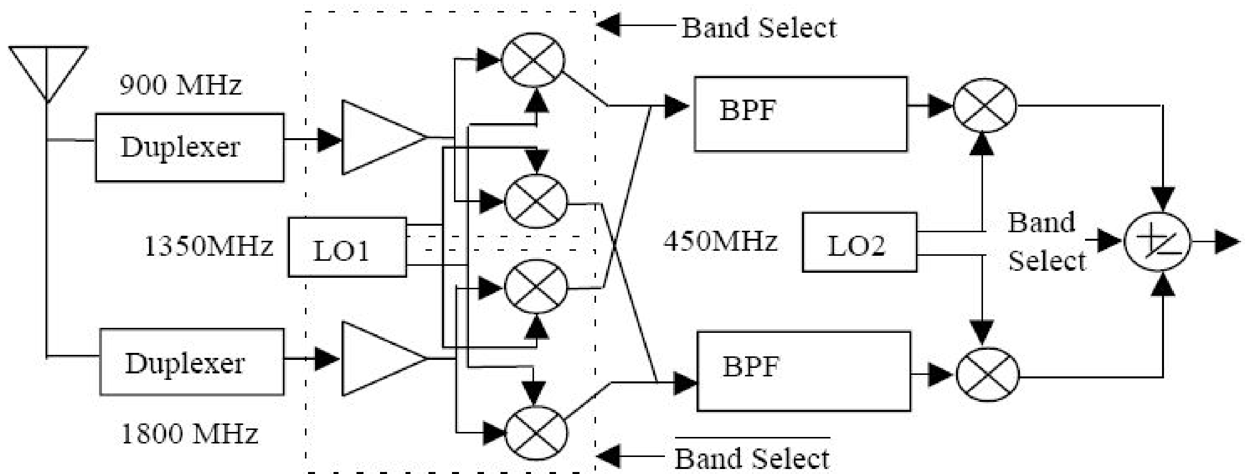
Radar: 30 MHz, 70 MHz

Image band

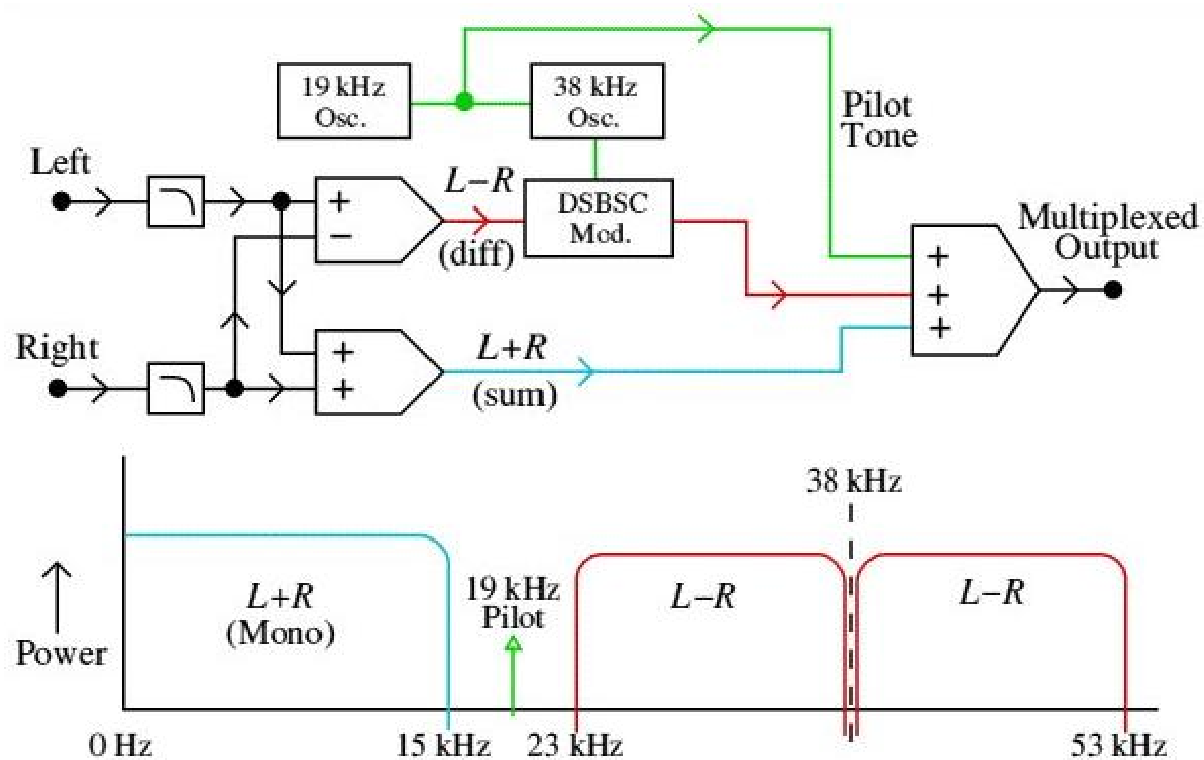
$f_{RF} = f_{LO} \pm f_{IF}$ (which one?)



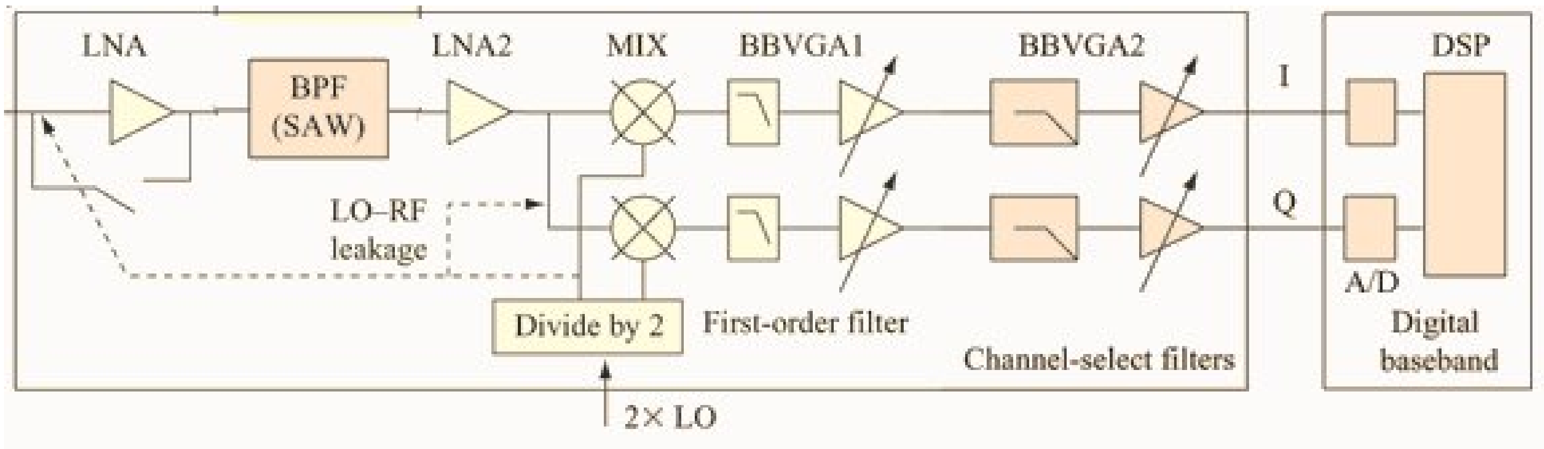
Dualband receiver example



Stereo FM example



Direct conversion receiver (homodyne)



- LO leakage ⟶ DC bias ⟶ saturation of BB amp
- Linearity (little gain control before mix)
- 2xLO - to produce phase-shifted “sin” & “cos” LO
- Synchronization of LO with carrier (radar⟶ easy, other⟶ PLL, digital cancel, ...)
- no image band!

Guess what BBVGA stands for....

Transmitter - modulation

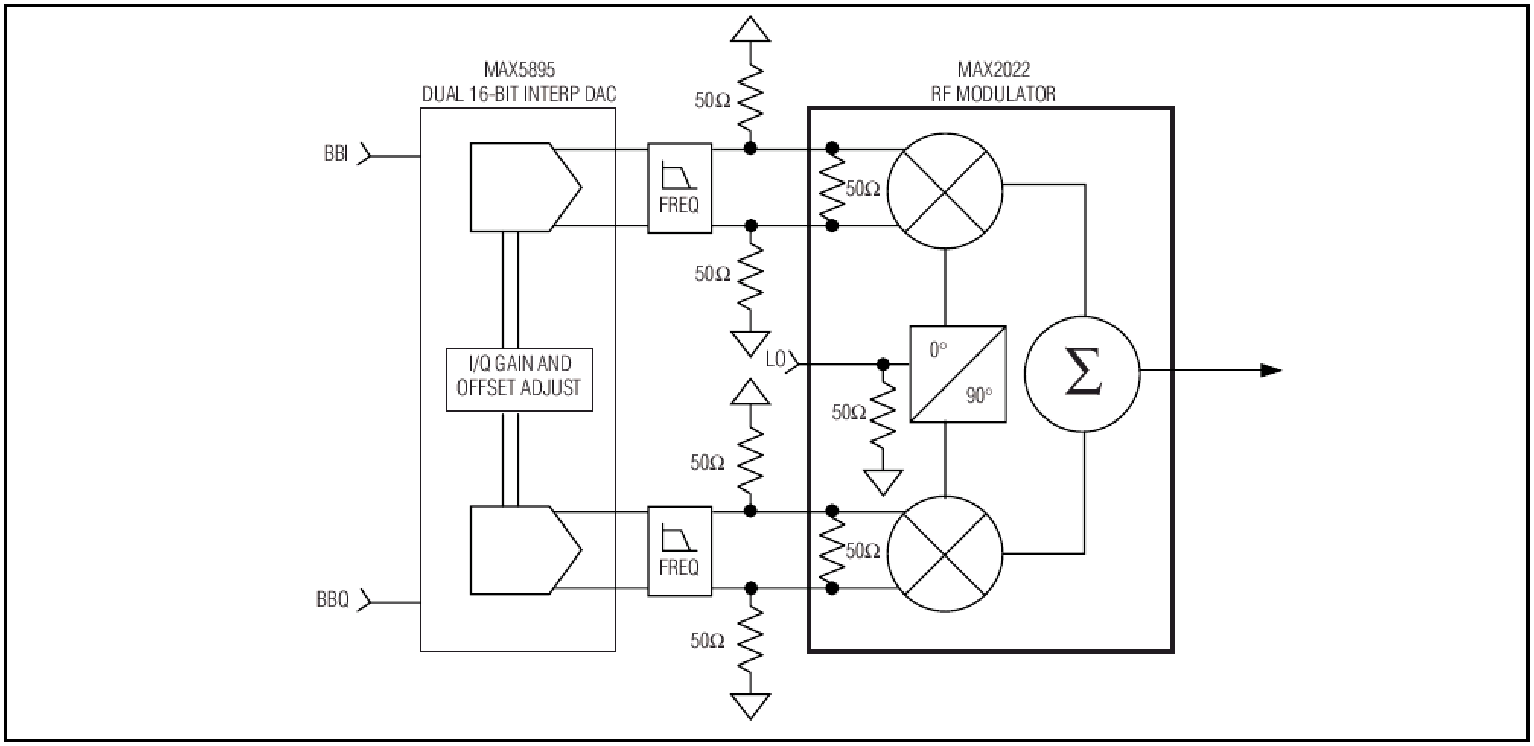
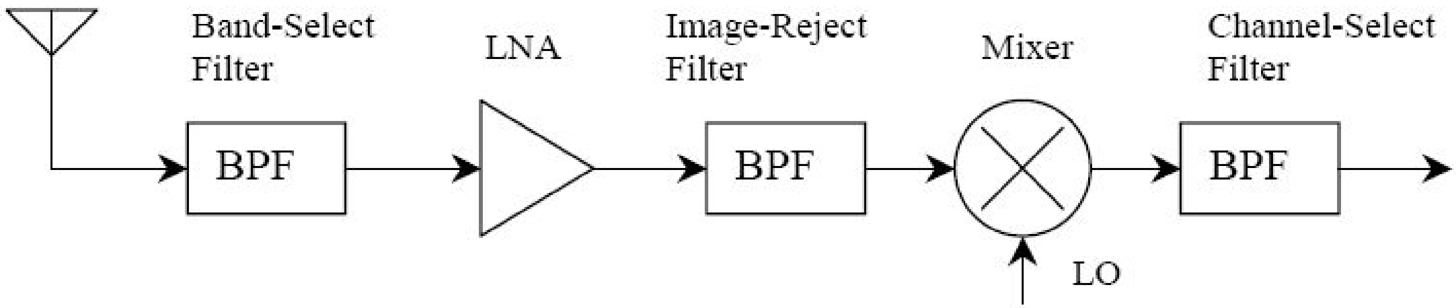


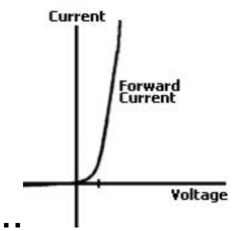
Figure 1. MAX5895 DAC Interfaced with MAX2022

Why do we need barbecue?

Mixer



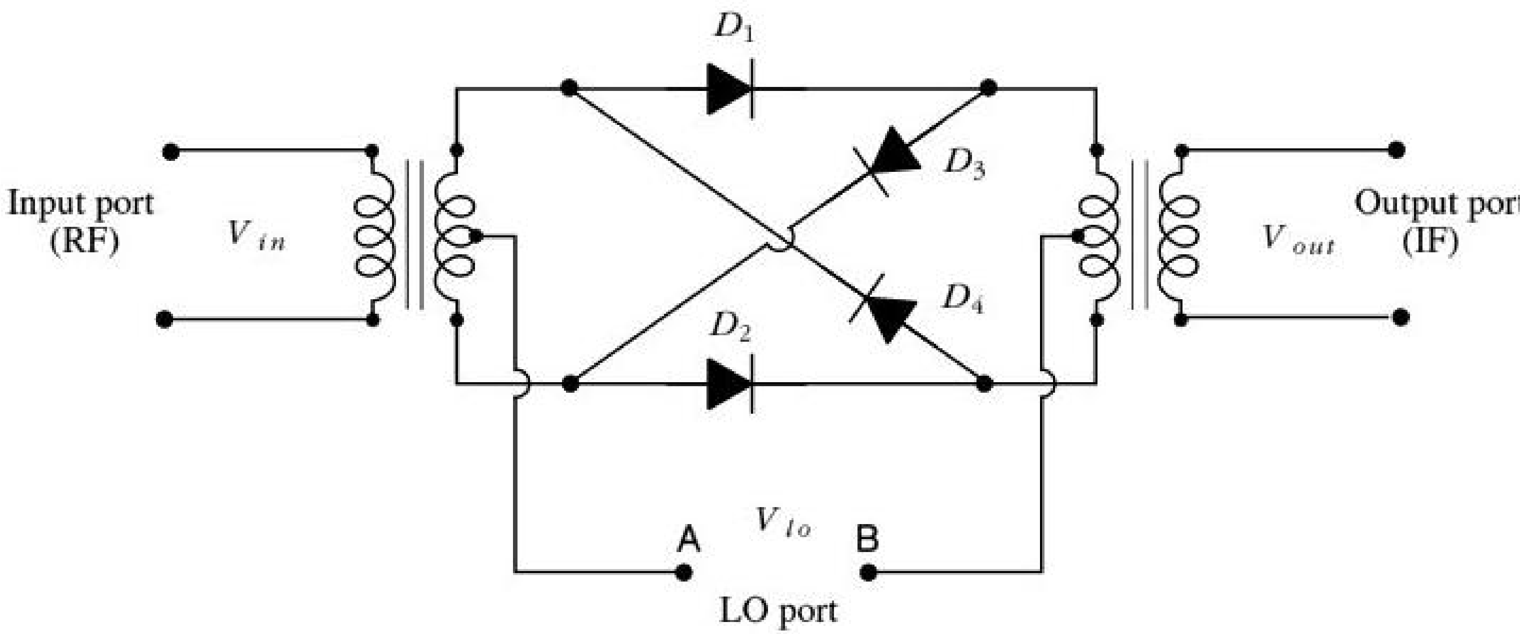
How to multiply two signals? $\longrightarrow$ use any element with x^2 type nonlinearity...



$$(cos(\omega_1 t) + cos(\omega_2 t))^2 = cos(\omega_1 t)^2 + cos(\omega_2 t)^2 + 2cos(\omega_1 t)cos(\omega_2 t)$$

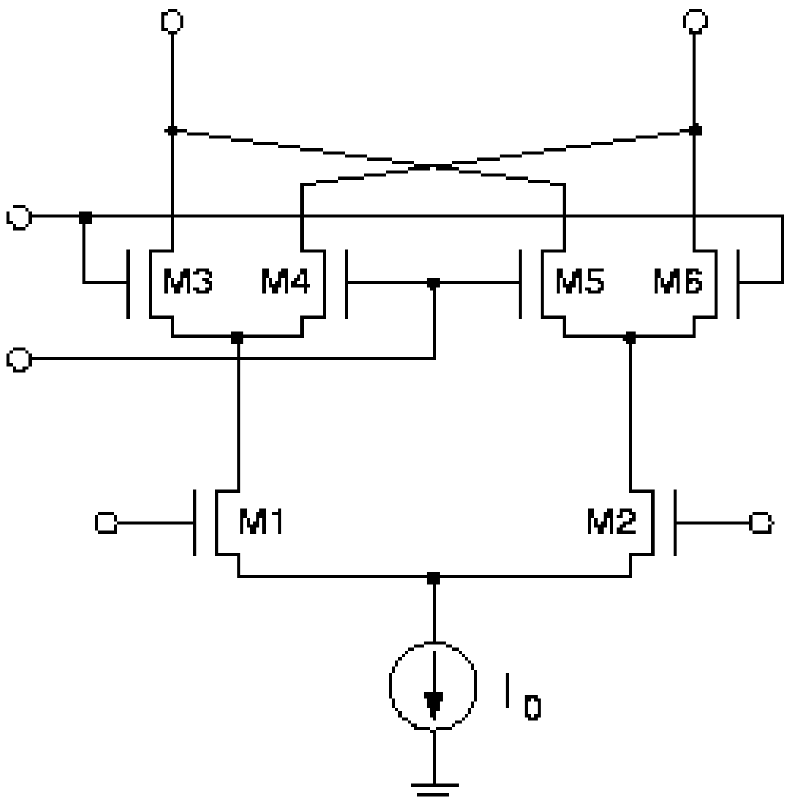
... and filter out $cos^2(\omega t) = 1/2 + 1/2cos(2\omega t)$ components
(simple but widely used version)

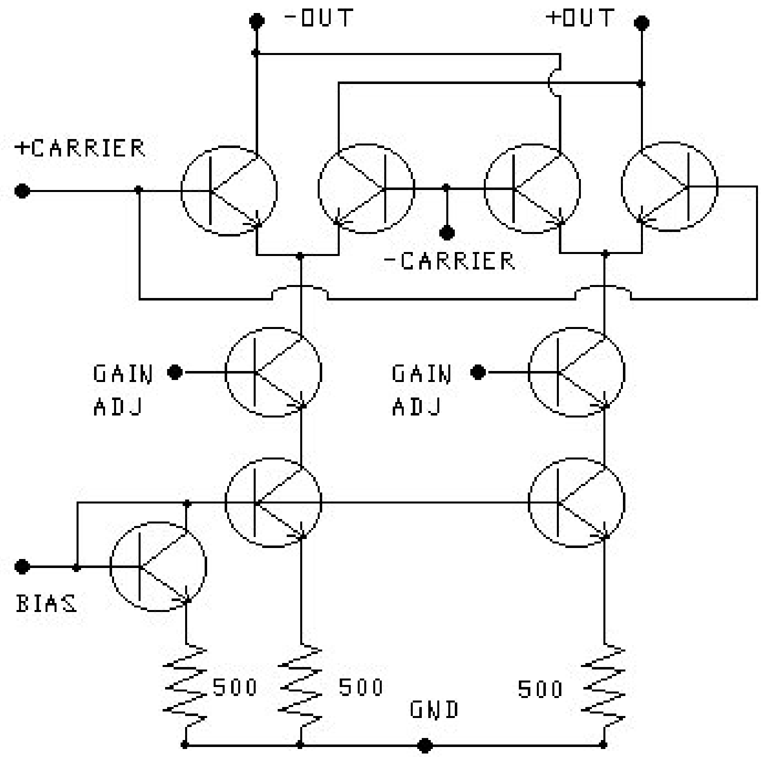
Diode mixer



(Balanced mixer: works symmetrically vs. the positive and negative half-period)

Balanced mixer circuits

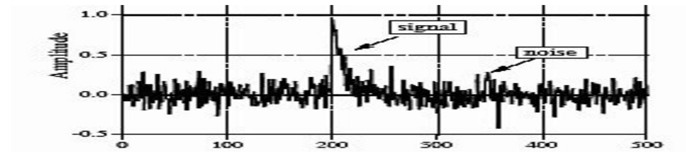




Try to see the similarity in principle to the 4-diode mixer;
guess input roles in MOS;
find bad input markings in bipolar.

Matched filter

Task: detect a signal $s(t)$ in noise.
Is There Anybody Out There? (and *when?*)



Idea: use LTI filter that maximizes p*eak*SNR=(signal instanteneous power)/(noise power) ratio *at some instant* (assume e.g. at $t = 0$)

Derivation: assume noise is white with PSD N_0; after a filter $H(j\omega)$ noise power is

$$P_N = N_0/4\pi \int_{-\infty}^{+\infty} |H(j\omega)|^2 d\omega$$

After the $H(j\omega)$ filter the signal spectrum becomes

$$Y(j\omega) = S(j\omega)H(j\omega)$$

thus the signal at $t = 0$ (by inv. Fourier transf.)

$$y(0) = 1/2\pi \int_{-\infty}^{+\infty} S(j\omega)H(j\omega)e^{j\omega 0} d\omega$$

We want to maximize $|y(0)|^2/P_n$

Now we put our spectra into the Schwarz inequality:

$$\left|\int S(j\omega)H(j\omega)d\omega\right|^2 \leq \int |S(j\omega)|^2 d\omega \int |H(j\omega)|^2 d\omega$$

so

$$pS/N = \frac{|y(0)|^2}{P_n} = \frac{|\int S(j\omega)H(j\omega)d\omega|^2}{N_0/2\int |H(j\omega)|^2 d\omega} \leq 2/N_0 \int |S(j\omega)|^2 d\omega$$

If we ~~guess~~ find $H(j\omega)$ such that the above $\leq$ becomes $=$, nobody will find anything better.. For complex numbers $|X|^2 = X \cdot X^*$ so

$$\frac{\int S(j\omega)H(j\omega)d\omega \int S^*(j\omega)H^*(j\omega)d\omega}{\int H(j\omega)H^*(j\omega)d\omega} \leq \int S(j\omega)S^*(j\omega)d\omega$$

if we put $H(j\omega) = S^*(j\omega)$ we got it!$\longrightarrow$ *(please recall from circuit theory how we compensate reactive power to maximize power drained from the source)*

Matched filter - conclusions

$H(j\omega) = S^*(j\omega) \longrightarrow H(t) = S(-t)$

We may modify it a little bit:

- delay in time by length t_s of $s(t)$ $\longrightarrow$ so that $h_d(t) = h(t - t_s)$ is causal
- scale it by any constant (equality holds)

As now $|y(0)|^2 = 1/2\pi \int_{-\infty}^{+\infty} |S(j\omega)|^2 d\omega = E$ = signal energy of $s(t)$ (Parseval), pSNR at the output of an ideal matched filter equals

$$pSNR = \frac{2E}{N_0}$$

... but other signal parameters (BW, t_s ...) may be used for whatever we want. (technical reasons, sidelobes.....)

Matched filter - variations

- ...with non-white noise:

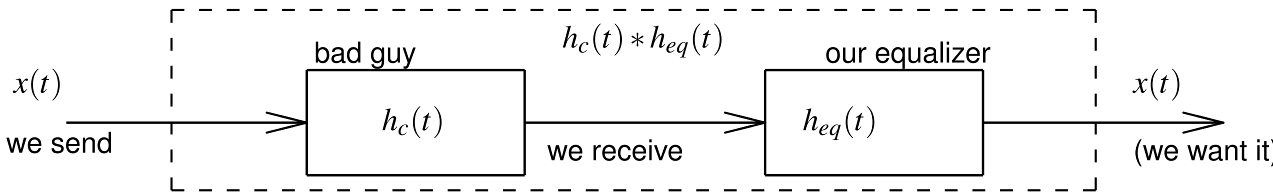
$$H(j\omega) = \frac{S^*(j\omega)}{|N(j\omega)|^2} = \frac{1}{N(j\omega)} \cdot \frac{S^*(j\omega)}{N^*(j\omega)}$$

(whitening + matched to a spoiled $s(t)$)

- Mismatched filter - a matched filter modified a bit, e.g. to reduce sidelobes:
  - modification by windowing
  - modification by optimization techniques

  $\longrightarrow$ **mis**matched filter is not optimal for $pSNR$, but by losing a little bit of $pSNR$ we may make it optimal in some other sense

Equalizer or Inverse Filter



Somebody (~~a bad guy~~ the channel) disturbs our signal as a filter $h_c(t)$; we want to compensate for it with $h_{eq}(t)$ so that $h_c(t) * h_{eq}(t) = \delta(t)$

In spectral domain: $H_c(j\omega) \cdot H_{eq}(j\omega) = 1 \longrightarrow H_{eq}(j\omega) = \frac{1}{H_c(j\omega)}$.

How to know $h_c(t)$? Estimate it using a known signal (e.g. preamble). We may also estimate $h_{eq}(t)$ using *adaptive filter*.
DANGERS:

- If $h_c(t)$ is FIR, $h_{eq}(t)$ is IIR
- When $H_c(j\omega) = 0$ we have $1/0$ (trick: use $H_{eq}(j\omega) = \frac{1}{H_c(j\omega)+k}$ with small $k > 0$.
- Zeros of $H_c(s)$ become POLES of $H_{eq}(s)$ (tricks needed to ensure stability).

Filters - summary

- Matched filter - maximize pSNR, $h(t) = s(-t)$
  - mismmatched filter - matched filter with some modifications
- Whitening filter - change non-white noise to white, $H(j\omega) = 1/N(j\omega)$
- Equalization filter or *inverse filter* - compensate the $F(j\omega)$ of a channel: $H(j\omega) = 1/F(j\omega)$