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Two-dimensional signal processing in FMCW radars

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ABSTRACT

The use of an FMCW radar for range and velocity measurement is outlined in the paper. An FMCW radar emits a continuous wave, frequency modulated by a sawtooth function. Object range information is deduced from the frequency of homodyne-demodulated return signal. Object velocity can be found from the change of return signal phase between the modulation sweeps. Thus, the measurement method involves finding, using FFT, the maximum of the two-dimensional spectrum of an one-dimensional signal. In the presented paper, the use of advanced algebraic methods is proposed in order to determine the velocity unambiguously in a wide range. An outline of a digital two-dimensional spectrum analyzer implemented with modern digital signal processors is also presented.

1. INTRODUCTION

The idea of an FMCW radar has been described long ago [1]. Recently, radars of this type gain much popularity, as they make use of the emerging technologies of high power solid state microwave transmitters and of very fast digital circuits, capable of generating and processing complex signals in real time.

An FMCW radar emits a continuous microwave signal, frequency modulated with low frequency waveform, e.g. of a sawtooth shape and of period T (see fig. 1 – continuous line). For a simplified analysis it is usually assumed [2] that the signal received after reflection from a stationary object is a copy of the transmitted signal, delayed by propagation time

$$\tau = \frac{2R}{c} \tag{1}$$

where R is an object range, and c is the speed of light. In the homodyne reception scheme, the received signal is mixed with an attenuated transmitted signal. After low-pass filtering a low (differential) frequency signal is obtained, called video signal in the sequel. The video signal is approximately sinusoidal, and its frequency f_w, constant in the time interval $T - \tau$, equals the change of the transmitter frequency during time τ,

$$f_w = \alpha\tau \tag{2}$$

where $\alpha = \Delta f/T$ is a modulation waveform slope, and $\Delta f = f_{mx} - f_{min}$ is the maximum frequency deviation. As it can be seen from (2) and (1), the measurement of a stationary object range R is equivalent to the determining of video signal frequency during the $T - \tau$ interval.

If an object with an initial range R_0 (at $t_0 = 0$) moves with some radial velocity v, the delay (1) won't be constant. Under the condition $v \ll c$, it will be almost linear function of time

$$\tau \approx \frac{2}{c}\left(R_0 + vt\right) \tag{3}$$

As the delay change is relatively slow, it can be noticed only in the phase change of the video signal. If the signal is analyzed in K modulation periods, the Doppler frequency can be estimated from the phase changes, thus allowing for the object velocity to be computed.

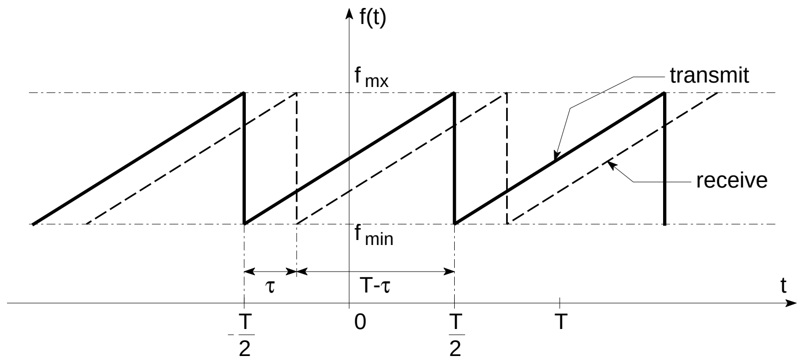

Fig. 1. Frequency of the transmitted and received signals

The analyzed problem is equivalent to estimating the unknown differential frequency f_w and the Doppler frequency f_d of an approximately sinusoidal video signal $x(t)$, observed in some time interval with additive noise. If we assume the noise is white and Gaussian, with zero mean, it is well-known ([3]) that the optimum estimator (reaching the Cramer–Rao bound) of an unknown frequency of a sinusoidal signal, observed in finite time interval is the maximum location of the Fourier spectrum absolute value.

In order to determine the f_w frequency the *range spectrum* of a video signal $x(t)$ in the constant interval of $T - \tau_{mx}$ (where τ_{mx} is a maximum possible echo delay) has to be computed. The phase change of the video signal $x(t)$, effecting from the object movement, is very slow (compared to the modulation period T, typically in order of 1 ms for an X-band radar). Thus, to estimate f_d – Doppler frequency – the video signal $x(t)$ has to be analyzed in a longer interval of $K \cdot T$, i.e. for several modulation periods. In order to determine the object velocity, absolute value of the spectrum of discrete signal (sampled with a period of T) must be computed. This spectrum is called *velocity spectrum*, and its maximum appears at the estimated Doppler frequency f_d.

The measurement of object range and velocity with an FMCW radar involves two-dimensional signal processing, as the two-dimensional spectrum of an one-dimensional video signal $x(t)$ has to be computed.

2. SPECTRAL ANALYSIS

FMCW radar transmitter emits the signal $u(t) = U\cos\phi(t), \ -\infty < t < \infty$ whose frequency

$$f(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} = f_{min} + \alpha(t - kT), \quad kT - \frac{T}{2} < t < kT + \frac{T}{2}, \quad (k = 0,\ \pm 1,\ \ldots) \tag{4}$$

is a periodical function of time shown on Fig. 1.

The received signal $u_0(t) = U_0 \cos\phi_0(t), \ -\infty < t < \infty$, reflected from an object is delayed by the propagation time τ. Upon mixing $u_0(t)$ with a local oscillator signal (an attenuated copy of $u(t)$) and low-pass filtering, a video signal $x(t) = \cos\phi_w(t), \ -\infty < t < \infty$ is obtained. The video signal differential phase $\phi_w(t) = \phi(t) - \phi_0(t)$ can be described by an equation

$$\phi_w(t_k) = \begin{cases} 2\pi f_0\tau - \pi\alpha[(T-\tau)^2 + 2(T-\tau)t_k], & -T/2 < t_k < -T/2+\tau \\ 2\pi f_0\tau - \pi\alpha[\tau^2 - 2\tau t_k] & -T/2+\tau < t_k < T/2 \end{cases} \tag{5}$$

where $t_k = t - kT, \ \ k = 0, \ \pm 1, \ \ldots$.

When the instrumental range of a radar equals R_{mx}, the signal reflected from an object at this range is delayed by τ_{mx}. Thus, the effective time of video signal observation in one modulation period equals $T - \tau_{mx}$.

Substituting (3) into (5) and deleting unimportant quadratic components, we get

$$\phi_w(t_k) \ \approx \ 2\pi[f_0\tau_0 + kf_dT + (f_w + f_d)t_k] \quad -T/2+\tau < t_k < T/2 \tag{6}$$

where $f_d = \frac{2v}{c} f_0$ is a Doppler (velocity) frequency of a signal, and $f_w = \alpha\tau_0$ is a video frequency value corresponding to a stationary object at range R_0. It can be seen that the object movement shifts the f_w frequency by a value of f_d. The range measurement is disturbed with an error dependent on the object velocity. This error may be corrected after determining the velocity value.

The range information is contained in the Fourier spectrum of a signal $x(t)$

$$X_r(\omega, \ k) = \int_{\frac{-T}{2}+\tau_{mx}}^{T/2} x(t_k)e^{-j\omega t_k}dt_k \tag{7}$$

computed for one (k-th) modulation period. Its absolute value $|X_r(\omega, \ k)|$ attains the maximum for the angular frequency $\omega = \pm 2\pi(f_w + f_d)$. The spectrum $X_r(\omega, \ k)$ is a function of k, as the angle $2\pi k f_d T$ in (6) changes with k. The information on the object movement is contained in this change. When $X_r(\omega, \ k)$ is treated as a discrete function of k, (with sampling period T), its spectrum $X(\omega, \theta)$, observed in K consecutive modulation periods $(k = 0, 1, \ldots, K-1)$, is described by

$$X(\omega, \ \theta) = \sum_{k=0}^{K-1} X_r(\omega, \ k)e^{-jk\theta} \tag{8}$$

where $\theta = \omega_v T$, and ω_v is *velocity frequency* (as opposed to *range frequency* ω). The object velocity measurement requires computation of the spectrum (8) and finding the frequency θ_d, where the absolute value of the spectrum attains its maximum. This frequency is equal to $\theta_d = 2\pi f_d T$ where f_d is the sought Doppler frequency. Since the spectrum $X(\omega, \theta)$ is a periodic function of θ, the measurement of f_d will be ambiguous outside the range of $(-\frac{1}{2T} < f_d < \frac{1}{2T})$ — the maximum unambiguously measured velocity is equal to $v_{mx} = \frac{c}{4Tf_0}$.

Two-dimensional spectrum (8) is usually determined with digital methods. The signal $x(t)$ is sampled with the frequency f_p, then the range spectrum (7) is computed with N-point DFT for every modulation period, using fast Fourier transform algorithms. Finally, spectrum (8) is computed using K-point DFT. Thus, range cell size equals $\Delta R = 2R_{mx}/N$, when velocity cell size is equal to $\Delta v = 2v_{mx}/K$.

Example 1

Let's take for example a radar with the following parameters: $f_0 = 10$ GHz, $T \approx 1$ ms, $R_{mx} = 7,5$ km, $\tau_{mx} = 0,05$ ms, $f_p = 1$ MHz, $N = 1024$, $K = 16$, $v_{mx} = 7,5$ m/s. With these parameter values, range and velocity cell sizes are: $\Delta R \approx 14,65$ m, $\Delta v \approx 0,94$ m/s. Figure 2 shows the two-dimensional spectrum of a video signal reflected by an object at range $R_0 = 3$ km, moving towards radar with radial velocity of $v = 4$ m/s. Signal to noise ratio of 0 dB was assumed.

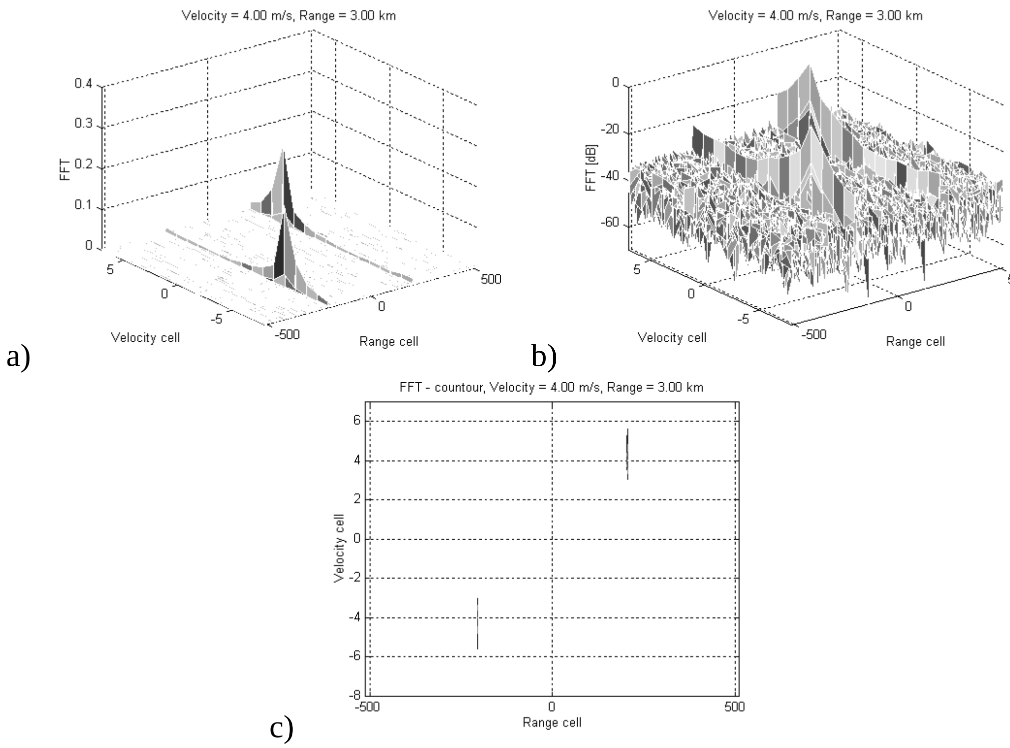

Fig. 2. Video signal from Example 1 – amplitude spectrum: (a) linear scale; (b) logarithmic scale; (c) contour map.

3. EXTENDING THE UNAMBIGUOUS VELOCITY MEASUREMENT RANGE

Unambiguous Doppler frequency measurement is possible up to one half of the modulation period inverse $1/T$ (see (7)). This limitation can be, however, lifted using different (properly chosen) modulation periods and different sizes of the range transforms. If, for example, K_1 modulation periods of length T_1 are used, together with K_2 periods of length T_2, then Doppler frequency *remainders* modulo $1/T_1$ and modulo $1/T_2$ may be obtained. From these remainders the Doppler frequency ambiguity may be resolved with use of Chinese Remainder Theorem [4] with proper choice of period lengths.

With use of M different frequencies of modulation waveform repetition, $F_i = 1/T_i$, $i = 1, 2, \ldots, M$, chosen so that they are integer and relatively prime multiples of some frequency F, (i.e. $F_i = m_i F$, where m_i fulfill the Chinese Remainder Theorem conditions), Doppler frequency may be resolved from the remainders $f_i = f_d \pmod{F_i}$ in the range $[-f_{vmx}/2,\ f_{vmx}/2]$ where $f_{vmx} = m_1 \cdot m_2 \cdot \ldots \cdot m_M \cdot F$.

Example 2

Let us assume that the required range of unambiguous velocity measurement with a radar described in Example 1 equals $v_{mx} = \pm 120$ m/s, (16 times the range of Example 1 – $v_{mx} = \pm 7,5$ m/s). This time, two different modulation periods are used, $T_1 = 0,81$ ms $= 13/16$ m/s and $T_2 = 1$ ms $= 13/13$ ms. In the transmitted waveform there are $K_1 = 16$ periods with period length T_1 and $K_2 = 13$ periods with period length T_2. This way, in both cases the Doppler frequency bin sizes are equal, as

$$\Delta f_v = \frac{1}{K_1 \cdot T_1} = \frac{1}{K_2 \cdot T_2} = \frac{1}{16 \cdot \frac{13}{16}\text{ms}} \cong 76,9 \text{ Hz} \tag{9}$$

The velocity cell equals $\Delta v = 1,15$m/s. Maximum frequency of $f_{vmx} = 76,9\text{Hz} \cdot 13 \cdot 16 = 16$ kHz gives the required velocity range of ± 120 m/s .

Fig. 3 shows the contour plot of a two-dimensional spectrum of complex video signal (with quadrature reception), when an object at range $R_0 = 6$ km moves with radial velocity of $v = 20,5$ m/s. Velocity spectrum has its maximum for $K_1 = 16$ in cell numbered $x_1 = 2$, and for $K_2 = 13$ in cell numbered $x_2 = 5$. Using Chinese Remainder Theorem, the true location of the cell can be found. For the chosen modulation parameters $K_1,\ K_2$ there exist numbers $g_1,\ g_2$, that allow to resolve the true value of unambiguous cell number, x

$$x = g_1 \cdot x_1 + g_2 \cdot x_2 (\text{mod}\ m_1 m_2) \tag{10}$$

where, in this case, $m_1 = K_1 = 16$, $m_2 = K_2 = 13$, $g_1 = 65$, $g_2 = 144$. Finally, the unambiguous cell number is $x = 850(\text{mod}\ 208) = 18$, and the estimated velocity equals $\hat{v} = 18 \cdot \Delta v = 18 \cdot 1,15 = 20,7$ m/s.

a) b)

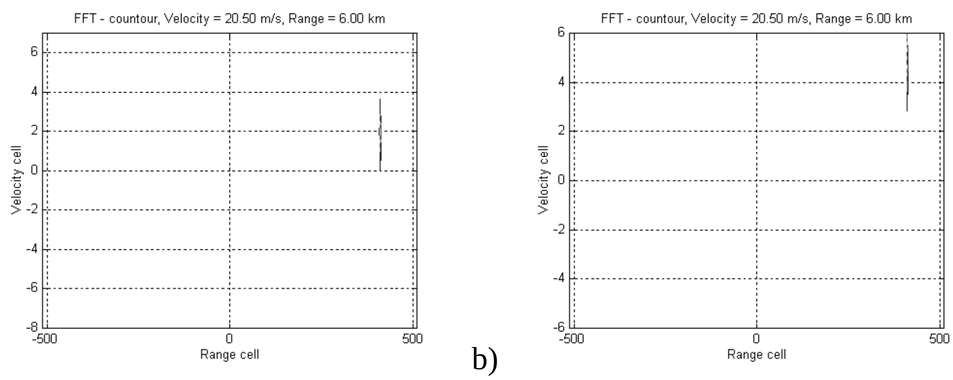

Fig. 3. Contour plot of amplitude spectrum a) $T_1 = 13/16$ ms ($K_1 = 16$) , b) $T_2 = 13/13$ ms ($K_2 = 13$)

4. TWO-DIMENSIONAL SIGNAL ANALYZER FOR AN FMCW RADAR

The basic computational block of digital signal processor in an FMCW radar is a two-dimensional spectrum analyzer. This block may be built with a digital signal processor capable of addressing big and fast static data memory. Let us exemplify this requirement: the storage needed for holding two tables of 16 modulation periods (1024 samples in each period), necessary for the two-dimensional FFT, with quadrature reception, equals **64K real words**. Neither 16-bit processors, nor the older 24-bit ones (DSP5600x) are capable of addressing such amount of data memory.

Required computing power is in order of one hundred MFLOPS, thus the processor speed must be taken into account too. Among the newest generation DSP's — TMS320C8x, TMS320C6xx, DSP5630x, ADSP2106x — the last one (known under the name "SHARC") is the slowest (25 ns instruction cycle), but has the biggest internal memory, excellent multiprocessing mechanisms, and the floating-point arithmetic unit. The internal memory of an ADSP21062 may be configured as 128K of 16-bit words or 64K of 32-bit words, which makes it possible to avoid using external memory throughout the FMCW radar system. Using this type of processor, a flexible and scalable system may be easily built. Thanks to the possible modular architecture, it may be adapted to different spectral analysis algorithms [6].

Two-dimensional FFT may be implemented by superposition of two one-dimensional transforms, so the classical 1024-point FFT with windowing may be used for assessing the number of required processors. The range spectrum calculation (with modulation period of 1 ms) will use about 65% of single 40-MHz SHARC processor time. The velocity spectrum is computed with 512 short (about 16-point) FFT's. This will need about 85% of single processor time. Thus, the whole two-dimensional FFT will need the computing power of two cooperating SHARC processors. One or two additional processors will be required for further processing — object detection and parameter estimation. For proper synchronization and data flow control between processors, a multitasking, multiprocessor real-time operating system must be employed. Such a system, adequate for an FMCW radar requirements, was proposed in [7].

5. CONCLUSIONS

The presented analysis of an FMCW radar theoretical fundamentals and numerical complexity of a two-dimensional signal analyzer shows that the digital signal processing algorithms may be implemented in software if sufficiently fast digital signal processors are used. Required range of unambiguous velocity measurement is attained with proper choice of different modulation periods, by implementing an algorithm derived from Chinese Remainder Theorem.

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