#### ESPTR: Radar Basics

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# Prototype



#### RADAR - echolocation

RAdio Detection And Ranging detection—transmit some energy and watch it return ranging—and measure the round-trip time

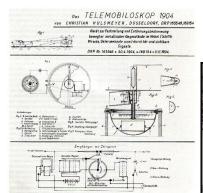
### Electromagnetic version

- 1865 James Clerk Maxwell theory of electromagnetic waves
- 1886 Heinrich Hertz experimental proof
- 1904 Christian Hülsmeyer *Telemobiloskop*: ship collision avoidance apparatus, patented in Germany and UK; demonstration at the Rhine river in Cologne, DE.

...

1939-1945 Chain Home, Klein Heidelberg and other installations

### Telemobiloskop





#### Chain Home

Frequency:	20 to 30 MHz
Peak Power:	350 kW (750 kW)
p.r.f.:	25 and 12.5 p.p.s.
Pulse Length:	20 us

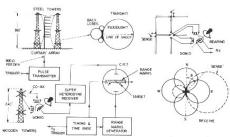
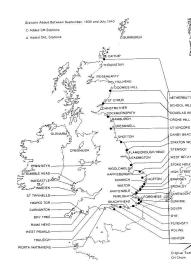
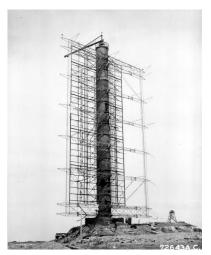


Fig. 1. Principles of CH (Chain Home) R.D.F. system



## Klein Heidelberg Parasit



Range 400 km, accuracy 1 - 2 km and 1 degree

### Radar equation

Transmit-reflect-receive-detect:

http://commons.wikimedia.org/wiki/File:Radarops.gif

Received power: radar range equation

$$P_r = \frac{P_t G_t A_r \sigma F^4}{(4\pi)^2 R_t^2 R_r^2}$$

 $P_t$  transmitter power

 $G_t$  gain of the transmitting antenna  $G=4\pi A_{eff}/\lambda^2$ 

 $A_r$  effective aperture (area) of the receiving antenna

 $\sigma$  radar cross section, or scattering coefficient, of the target

F pattern propagation factor

 $R_t$  distance from the transmitter to the target

 $R_r$  distance from the target to the receiver.

 $2x \text{ range } \longrightarrow 2^4 = 16x \text{ power needed } \dots$ 

## Signal model

Transmit:

$$x_T(t) = A_T(t)e^{j\phi_T(t)}$$

Receive:

$$x_R(t) = A_T(t - R(t)/c)e^{j\phi_T(t - R(t)/c)}$$

simple case:  $\phi_T(t) = \omega t + \phi_M(t)$ ,  $R(t) = R_0 + vt$ 

$$\mathbf{x}_{R}(t) = A_{T}(t - R_{0}/c - vt/c)e^{j(\omega(t - R_{0}/c - vt/c) + \phi_{M}(t - R_{0}/c - vt/c))}$$

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Baseband:

$$x_{R}(t) = A_{T}(t - R_{0}/c - vt/c)e^{j\phi_{M}(t - R_{0}/c - vt/c)} \qquad e^{-j\omega(R_{0}/c)}e^{-j\omega vt/c}$$



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#### Detection



→compare signal with threshold

$$P_r > P_{noise} \cdot D \longrightarrow \text{declare a target}$$

with integration (by a matched filter) over  $t_i$  seconds

$$\frac{P_t G_t A_r \sigma F^4 t_i B}{(4\pi)^2 R_t^2 R_r^2} > kTBD$$

so the minimum detected object RCS

$$\sigma_{min} = \frac{(4\pi)^2 R_t^2 R_r^2 kTD}{P_t G_t A_r F^4 t_i}$$

We sometimes express  $\sigma$  in dBsm (dB w.r.t. square meter).



#### Detection threshold

- $x(t) < D \longrightarrow H_0$  Hypothesis 0: only noise
- $x(t) > D \longrightarrow H_1$  Hypothesis 1: noise + signal

Maximize  $P_d$  (detection), keep  $P_{fa}$  (false alarm) low.

The threshold *D* set above:

- ► Noise (thermal etc)
- Clutter (unwanted echoes)
- Multipath
- Jamming (intentionally malicious transmitters)
- ► Interferences (other equipment, e.g. other radars)

Improvements: matched filter (S $\uparrow$ ), interference cancellation (C $\downarrow$ )

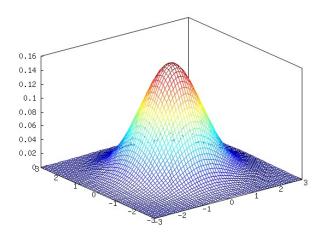
Typical: SNR  $\approx$  13 dB Adaptation: CFAR



#### Noise and clutter

You can fight noise, jamming and interferences by using more power or better matched filtering.
You cannot fight clutter or multipath this way.

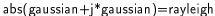
## H0: Noise distribution (complex)

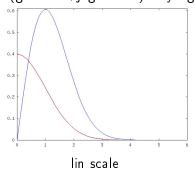


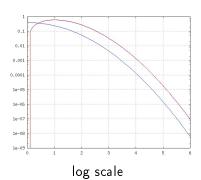
Two-dimensional (imag / real) gaussian distribution



## H0: Noise distribution (abs)

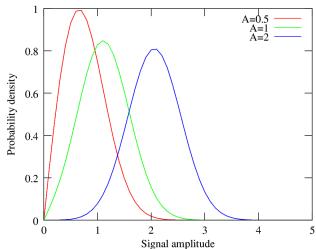




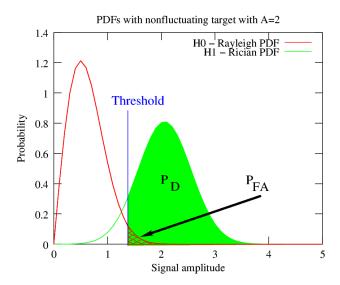


## H1: Signal + noise distribution (abs)

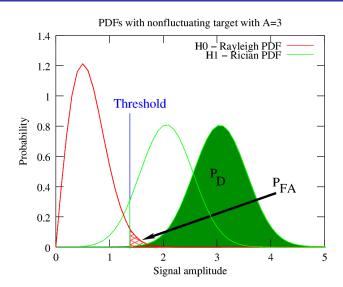
abs((gaussian+j\*gaussian)+constant A)=rician
Rician PDF with s=0.5



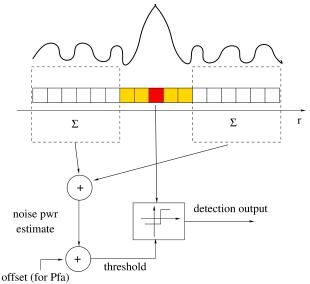
## Neyman-Pearson detector



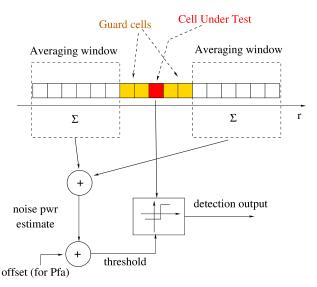
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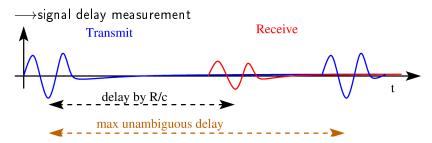
#### CFAR- Constant False Alarm Rate



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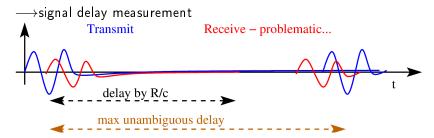
### Range measurement



- Max unambiguous range limited by modulation period
- Min range limited by transmit signal entering the receiver (in pulsed radar)
  - Antenna separation
  - ► T/R switch + receiver safety (ionised gas + pin diode)



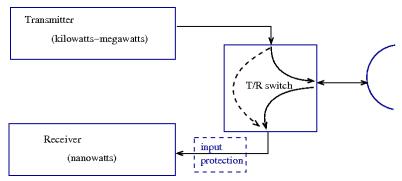
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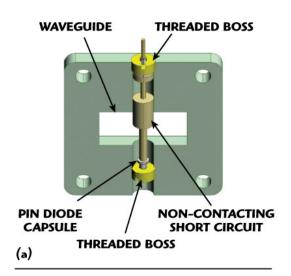


## Transmit/receive switch



Make friends with good microwave engineers . . .

### Receiver input protection



## Velocity measurement



→ Doppler shift measurement

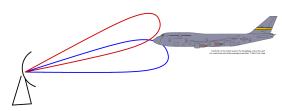
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- ► Min velocity: ground/meteo clutter
- ► Max velocity (frequency): inverse of modulation period (PRI)



### Angle measurement





- azimuth
- elevation

#### Methods

- Scanning: mechanical, electronic
- Monopulse techniques (multielement antenna)
  - Power ratio
  - Sigma-Delta (power)
  - Phased arrays

