Up-/Down- conversion in frequency

Why?

• Amplification 120+ dB without parasitic feedback
• Tunable receiver without too many tuned filters
• Easier narrowband filtering
Superheterodyne receiver

- Band-select filter: fixed or roughly tunable to the needed channel or band
- Image-reject filter: fixed – see the “image band” problem in following slides
- LO = Local Oscillator ("heterodyne"): tunable
- Channel select filter: fixed, good quality
Mixing RF $\omega_1$ with LO $\omega_2$:

$$\cos(\omega_1 t) \cos(\omega_2 t) = \frac{1}{2} \cos((\omega_1 - \omega_2)t) + \frac{1}{2} \cos((\omega_1 + \omega_2)t)$$

$\omega_1 - \omega_2$ - IF (intermediate frequency) or beat frequency
Intermediate frequency is usually lower than RF (easier to amplify/filter/process).

Examples (receivers): RF $\rightarrow$ IF

**FM radio:** 88-108Mhz $\rightarrow$ 10.7 MHz

**AM radio:** 150-300kHz, 530-1600kHz, 2.3-26MHz $\rightarrow$ 465 kHz

**analog TV:** 30 MHz, 45 MHz

**Satellite equipment:** 10.7-12.75 GHz $\rightarrow$ 950-1450 MHz first IF (L-band), $\rightarrow$ 70 MHz second IF *double conversion receiver!!*

**Terrestrial MW link:** 2.4, 24, 60-70GHz $\rightarrow$ 250 MHz, 70 MHz

**Radar:** 9.5 GHz $\rightarrow$ 70 MHz, 30 MHz
Image band

\[ f_{RF} = f_{LO} \pm f_{IF} \] (which one?)
Dualband receiver example
Stereo FM example – coding

```
<table>
<thead>
<tr>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 Hz</td>
</tr>
<tr>
<td>15 kHz</td>
</tr>
<tr>
<td>23 kHz</td>
</tr>
<tr>
<td>38 kHz</td>
</tr>
<tr>
<td>53 kHz</td>
</tr>
</tbody>
</table>
```

```
19 kHz Osc.  38 kHz Osc.  DSBSC Mod.  Pilot Tone  Multiplexed Output
```

```
L-R (diff)  L-R (sum)
```

```
Left
```

```
Right
```

```
L+R (Mono)
```

```
L+R
```

```
19 kHz Pilot
```

```
38 kHz
```

```
53 kHz
```
Stereo FM example – decoding

Many implementations exist!
Direct conversion receiver (homodyne)

- LO leakage $\rightarrow$ DC bias $\rightarrow$ saturation of BB amp
- Linearity (little gain control before mix)
- 2xLO - to produce phase-shifted “sin” & “cos” LO
- Synchronization of LO with carrier (radar $\rightarrow$ easy, other $\rightarrow$ PLL, digital cancel, ...)
- no image band!

*Guess what BBVGA stands for....*
RF band and baseband (BB)

Mathematically: conversion to baseband (BB) $\rightarrow$ demodulating BP signal (shifting by $-F_c$)

- $F_c$ is moved to zero
- we ignore the band at $-2F_c$ (implementation: LP filter)

$$A \cos(F_c t) \text{ is now } A \cos(0t) = A, \quad A \sin(F_c t) \text{ is now } jA \cos(0t) = jA$$

$$B \cos((F_c + F_d) t) \text{ becomes } B \exp(+j2\pi F_d t)$$

$$C \cos((F_c - F_d) t) \text{ becomes } C \exp(-j2\pi F_d t)$$

- real-valued signal from $F_c - F_0$ to $F_c + F_1$ $\rightarrow$ complex-valued signal from $-F_0$ to $+F_1$.
- phase, amplitude, and offset from $F_c$ retained $\rightarrow$ complete representation of RF signal.

Simulating (and, in real world, processing) of BB signal is easier than RF signal
You need a sampling frequency equal $F_1 - (-F_0)$, which is a two-sided bandwidth of the BB signal or a simple bandwidth of the RF signal – usually MUCH less that 2 times $F_c + F_1$. 
Transmitter - modulation

The correspondence between BB and RF can be proven: there is an explicit equation to reconstruct RF signal from BB signal.

Why do we need barbecue?

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Figure 1. MAX5895 DAC Interfaced with MAX2022
How to multiply two signals? → use any element with $x^2$ type nonlinearity...

$$(\cos(\omega_1 t) + \cos(\omega_2 t))^2 = \cos(\omega_1 t)^2 + \cos(\omega_2 t)^2 + 2\cos(\omega_1 t)\cos(\omega_2 t)$$

... and filter out $\cos^2(\omega t) = 1/2 + 1/2\cos(2\omega t)$ components
(simple but widely used version)
Diode mixer

(Simple mixer: works only in the positive half-period)
Single-balanced mixer

(Simple mixer: with symmetrical diodes LO ↔ RF leakage minimized)
Balanced diode mixer

(Balanced mixer: works symmetrically vs. the positive and negative half-period)
Balanced mixer circuits

Try to see the similarity in principle to the 4-diode mixer; guess input roles in MOS; find bad input markings in bipolar.
**Matched filter**

Task: detect a signal $s(t)$ in noise. 

*Is There Anybody Out There?* (and when?)

Idea: use LTI filter that maximizes peakSNR=(signal instantaneous power)/(noise power) ratio *at some instant* (assume e.g. at $t = 0$)

Derivation: assume noise is white with PSD $N_0$; after a filter $H(j\omega)$ noise power is

$$P_N = N_0 / 4\pi \int_{-\infty}^{+\infty} |H(j\omega)|^2 d\omega$$

After the $H(j\omega)$ filter the signal spectrum becomes

$$Y(j\omega) = S(j\omega)H(j\omega)$$

thus the signal at $t = 0$ (by inv. Fourier transf.)

$$y(0) = 1/2\pi \int_{-\infty}^{+\infty} S(j\omega)H(j\omega)e^{j\omega 0} d\omega$$
We want to maximize $|y(0)|^2 / P_n$

Now we put our spectra into the Schwarz inequality:

$$\left| \int S(j\omega)H(j\omega) d\omega \right|^2 \leq \int |S(j\omega)|^2 d\omega \int |H(j\omega)|^2 d\omega$$

so

$$pS/N = \frac{|y(0)|^2}{P_n} = \frac{\int S(j\omega)H(j\omega) d\omega}{N_0/2 \int |H(j\omega)|^2 d\omega} \leq 2/N_0 \int |S(j\omega)|^2 d\omega$$

If we guess find $H(j\omega)$ such that the above $\leq$ becomes $=$, nobody will find anything better..

For complex numbers $|X|^2 = X \cdot X^*$ so

$$\frac{\int S(j\omega)H(j\omega) d\omega \int S^*(j\omega)H^*(j\omega) d\omega}{\int H(j\omega)H^*(j\omega) d\omega} \leq \int S(j\omega)S^*(j\omega) d\omega$$

if we put $H(j\omega) = S^*(j\omega)$ we got it! $\rightarrow$ (please recall from circuit theory how we compensate reactive power to maximize power drained from the source)
Matched filter - conclusions

\[ H(j\omega) = S^*(j\omega) \rightarrow H(t) = S(-t) \]

We may modify it a little bit:

- delay in time by length \( t_s \) of \( s(t) \) so that \( h_d(t) = h(t - t_s) \) is causal
- scale it by any constant (equality holds)

As now \( |y(0)|^2 = 1/2\pi \int_{-\infty}^{+\infty} |S(j\omega)|^2 d\omega = E = \) signal energy of \( s(t) \) (Parseval),
pSNR at the output of an ideal matched filter equals

\[ pSNR = \frac{2E}{N_0} \]

... but other signal parameters (BW, \( t_s \) ...) may be used for whatever we want. (technical reasons, sidelobes.....)
**Matched filter - improvement factor**

How big a peak can we have after a MF?

S/N before & after MF:

\[ SNR_{in} = \frac{x^2}{BN_0} \quad SNR_{out} = \frac{2E}{N_0} \]

and

\[ E = x^2 t_i \]

so we get improvement by:

\[ IF_{MF} = \frac{SNR_{out}}{SNR_{in}} = \frac{2E}{N_0} \frac{B'}{\sqrt{x^2}} = 2B t_i \]

An alternative explanation:
A signal with a bandwidth B may have a minimum duration of \(1/B\). So with a best (matched) filter we can squeeze a pulse from \(t_i\) to \(1/B \rightarrow B t_i\) times....
Matched filter - variations

- ...with non-white noise:

\[ H(j\omega) = \frac{S^*(j\omega)}{|N(j\omega)|^2} = \frac{1}{N(j\omega)} \cdot \frac{S^*(j\omega)}{N^*(j\omega)} \]

(whitening + matched to a spoiled \( s(t) \))

- Mismatched filter - a matched filter modified a bit, e.g. to reduce sidelobes:
  - modification by windowing
  - modification by optimization techniques
  \( \rightarrow \text{mis} \)matched filter is not optimal for \( pSNR \), but by losing a little bit of \( pSNR \) we may make it optimal in some other sense
Equalizer or Inverse Filter

Somebody (a bad guy the channel) disturbs our signal as a filter $h_c(t)$; we want to compensate for it with $h_{eq}(t)$ so that $h_c(t) * h_{eq}(t) = \delta(t)$

In spectral domain: $H_c(j\omega) \cdot H_{eq}(j\omega) = 1 \rightarrow H_{eq}(j\omega) = \frac{1}{H_c(j\omega)}$.

How to know $h_c(t)$? Estimate it using a known signal (e.g. preamble). We may also estimate $h_{eq}(t)$ using adaptive filter.

DANGERS:

- If $h_c(t)$ is FIR, $h_{eq}(t)$ is IIR
- When $H_c(j\omega) = 0$ we have $1/0$ (trick: use $H_{eq}(j\omega) = \frac{1}{H_c(j\omega)+k}$ with small $k > 0$.
- Zeros of $H_c(s)$ become POLES of $H_{eq}(s)$ (tricks needed to ensure stability).
Filters - summary

- Matched filter - maximize pSNR, \( h(t) = s(-t) \)
  - mismatched filter - matched filter with some modifications
- Whitening filter - change non-white noise to white, \( H(j\omega) = 1/N(j\omega) \)
- Equalization filter or inverse filter - compensate the \( F(j\omega) \) of a channel: \( H(j\omega) = 1/F(j\omega) \)