1 Laboratory Nr 2: Simulation of simple sequences of dependent random variables. Time series.

1. Select two conjugate, complex numbers z_1 and z_2 from inside of the unit circle.
2. Perform multiplication $(z-z_1)\times(z-z_2)$ obtaining polynomial z^2-az-b.
3. Generate $NN = 2000$ observations of so called *white noise* $\{W_n\}$ (i.e. sequence of independent identically distributed random variables) having :
   (a) normal $N(0,1)$ distribution,
   (b) exponential distribution and center it (that is subtract expectation).
4. Generate $NN = 2000$ observations of so called autoregression process $AR(2)$ that is outcome of the following difference equation $X_n = aX_{n-1} + bX_{n-2} + W_n$; $n = 0,1,2,...NN$ assuming some initial values!!! with sequence $\{W_n\}$ generated in points a. and b..
5. Find estimators of two functions useful in identification of the time series
   (a) estimators $\hat{K}(k)$ of parameters $K(k)$ (values of covariance function) for $k = 0,1,2,...,32$ and plot$\hat{K}(k)$. Namely

   $$\hat{K}(k) = \frac{1}{NN-k}\sum_{i=1}^{NN-k}\left(X_i - \bar{X}_N\right)\left(X_{i+k} - \bar{X}_N\right),$$

   where $\bar{X}_N = \frac{1}{NN}\sum_{i=1}^{NN} X_i$
   (b) estimators $\hat{R}(k)$ of so called partial correlation function $R(k)$ i.e. $r-$th solutions of the system of linear equations (so called Yul-Walker equations)

   $$\begin{aligned}
   \rho_1 &= a_{1,r} + a_{2,r}\rho_1 + \ldots + a_{r,r}\rho_{r-1},\\
   \rho_2 &= a_{1,r}\rho_1 + a_{2,r} + \ldots + a_{r,r}\rho_{r-2},\\
   &\ldots\ldots\ldots\\
   \rho_r &= a_{1,r}\rho_{r-1} + a_{2,r}\rho_{r-2} + \ldots + a_{r,r},
   \end{aligned}$$

   where $\rho_i = K(i)/K(0)$, for $r = 1,2,\ldots$. More precisely we solve above mentioned system of equations for $r = 1,2,\ldots$ and are interested in solutions $a_{r,r}$.
6. Find an estimator of $\hat{f}(\lambda)$ the spectral density function, i.e. calculate: $\hat{f}(\lambda) = \frac{1}{2\pi}\sum_{n=-32}^{32}\hat{K}(n)\exp(-in\lambda)$ assuming that $\hat{K}(-k) = \hat{K}(k)$; $k = 0,1,2\ldots,32$. Plot this function.

7. Repeat points 5. and 6. for moving average precess $MA(2)$ i.e. process given by difference equation: $Y_n = W_n - a_1 W_{n-1} - b_1 W_{n-2}$, $n = 0, 1, 2, ...$ defining W_{-2} and W_{-1} and takin any values for a_1 and b_1.

8. Repeat points 5. and 6. for the process $ARMA(2,2)$ i.e. solution of the deferential equation: $Y_n = aY_{n-1} + bY_{n-2} + W_n - a_1 W_{n-1} - b_1 W_{n-2}$, $n = 0, 1, 2, ...$ a_1 and b_1 any, and defining values of $Y's$ and $W's$ for negative indices

9. Write short description of what have you done and observed and submit it in pdf format by E-mail: pjsz@hotmail.com or in case of trouble pszablowski@elka.pw.edu.pl .