

# Hash function

**A function which maps strings of bits to fixed-length strings of bits, satisfying the following two properties:**

- it is computationally infeasible to find for a given output an input which maps to this output,**
- it is computationally infeasible to find for a given input a second input which maps to the same output.**

**Collision-resistant hash-function: A hash-function satisfying the following property:**

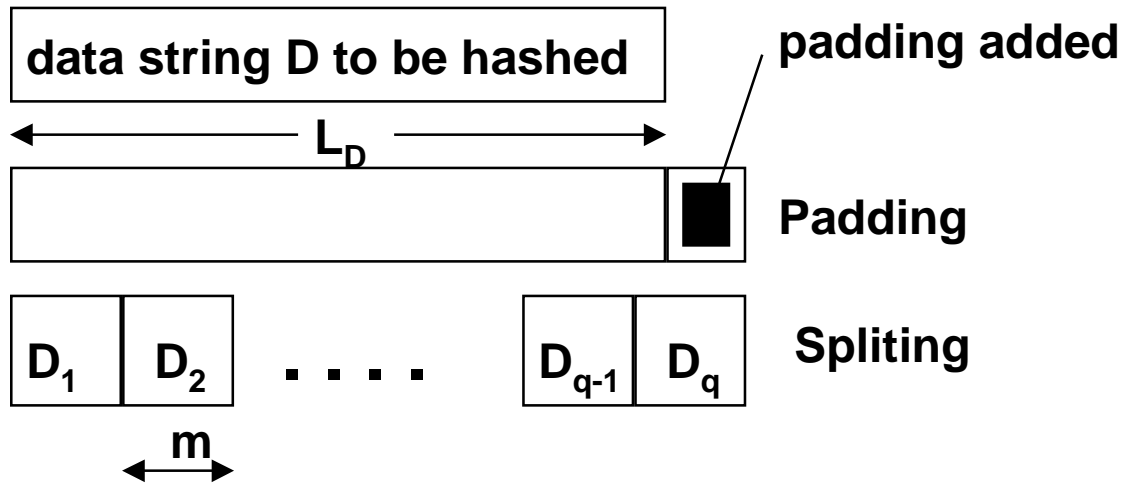
- it is computationally infeasible to find any two distinct inputs which map to the same output.**

# ISO/IEC 10118-3 IT ST Hash functions

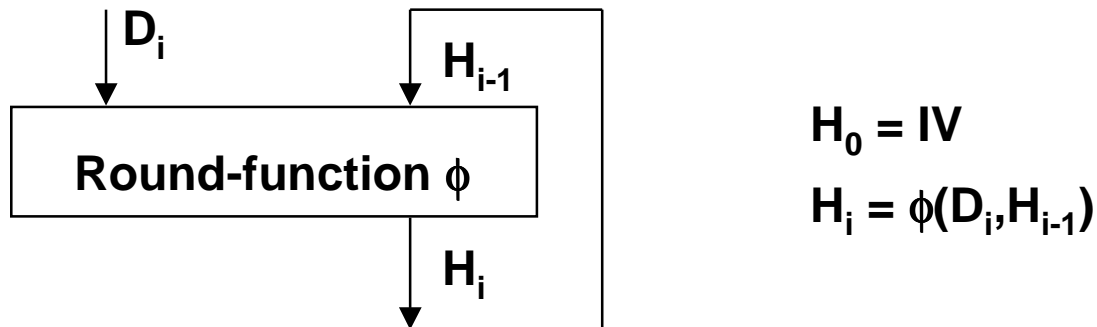
## Part 3: Dedicated hash-functions

**Step 1 - padding.** The length of data string should be a multiple of  $L_X$ .

**Step 2 - splitting.**



**Step 3 - iteration**



**Step 4 - truncation.** The hash-code  $H$  is derived by taking the leftmost  $L_H$  bits of the final  $L_Y$ -bit output string  $H_q$ .

**Annex A 10118-1**  
(Normative)  
**Padding Methods**

The calculation of a hash-code, as specified in other parts of ISO/IEC 10118, may require the selection of a padding method. The padding method will always output a padded data string whose length (in bits) is a multiple of  $L1$ . Three methods are presented in this annex.

NOTE - Hash-functions using Padding Method 1 may be subject to trivial forgery attacks, where an adversary can add or delete a number of trailing '0' bits of the data string without changing the hash-code. This means that Padding Method 1 shall only be used in environments where the length of the data string  $D$  is known to the parties beforehand, or where data strings with a different number of trailing '0' bits have the same semantics. Theoretical results (see, for example, [1]) also indicate that, in environments not satisfying the above condition, Padding Method 3 may offer certain advantages over Padding Method 2.

The padding bits (if any) need not be stored or transmitted with the data. The verifier shall know whether or not the padding bits have been stored or transmitted, and which padding method is in use.

### **A.1 Method 1**

The data for which the hash-code is to be calculated are appended with as few (possibly no) '0' bits as are necessary to obtain the required length.

### **A.2 Method 2**

The data for which the hash-code is to be calculated are appended with a single '1' bit. The resulting data are then appended with as few (possibly none) '0' bits as are necessary to obtain the required length.

Note – Method 2 always requires the addition of at least one padding bit.

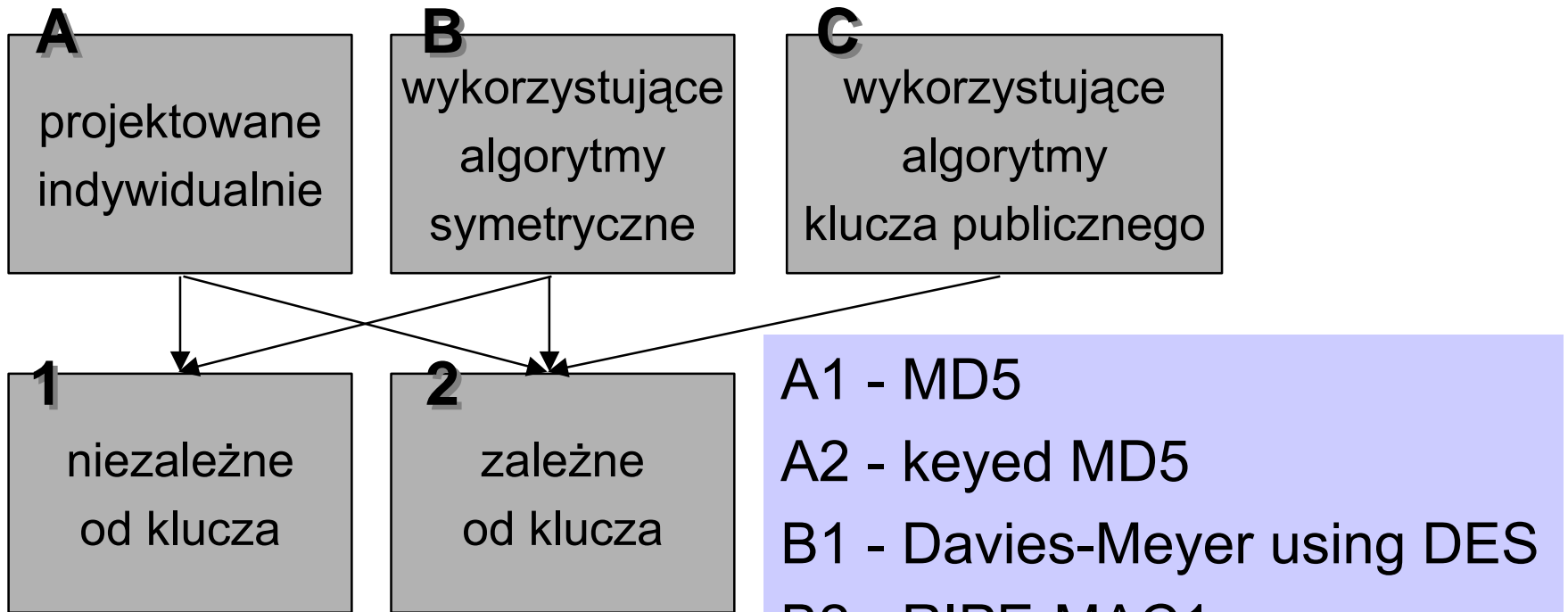
### **A.3 Method 3**

This padding method requires the selection of a parameter  $r$  (where  $r \leq L$ ), e.g.  $r = 64$ , and a method for encoding the bit length of the data  $D$ , i.e.  $LD$ , as a bit string of length  $r$ . The choice for  $r$  will limit the length of  $D$ , in that  $LD < 2r$ .

The data  $D$  for which the hash-code is to be calculated is padded using the following procedure.

1.  $D$  is concatenated with a single '1' bit.
2. The result of the previous step is concatenated with between zero and  $L-1$  '0' bits, such that the length of the resultant string is congruent to  $L-1-r$  modulo  $L$ . The result will be a bit string whose length will be  $r$  bits short of an integer multiple of  $L$  bits (in the case  $r = L$ , the result will be a bit string whose length is an exact multiple of  $L$  bits).
3. Append an  $r$ -bit encoding of  $LD$ , using the selected encoding method, yielding the padded version of  $D$ .

# Typy funkcji skrótu

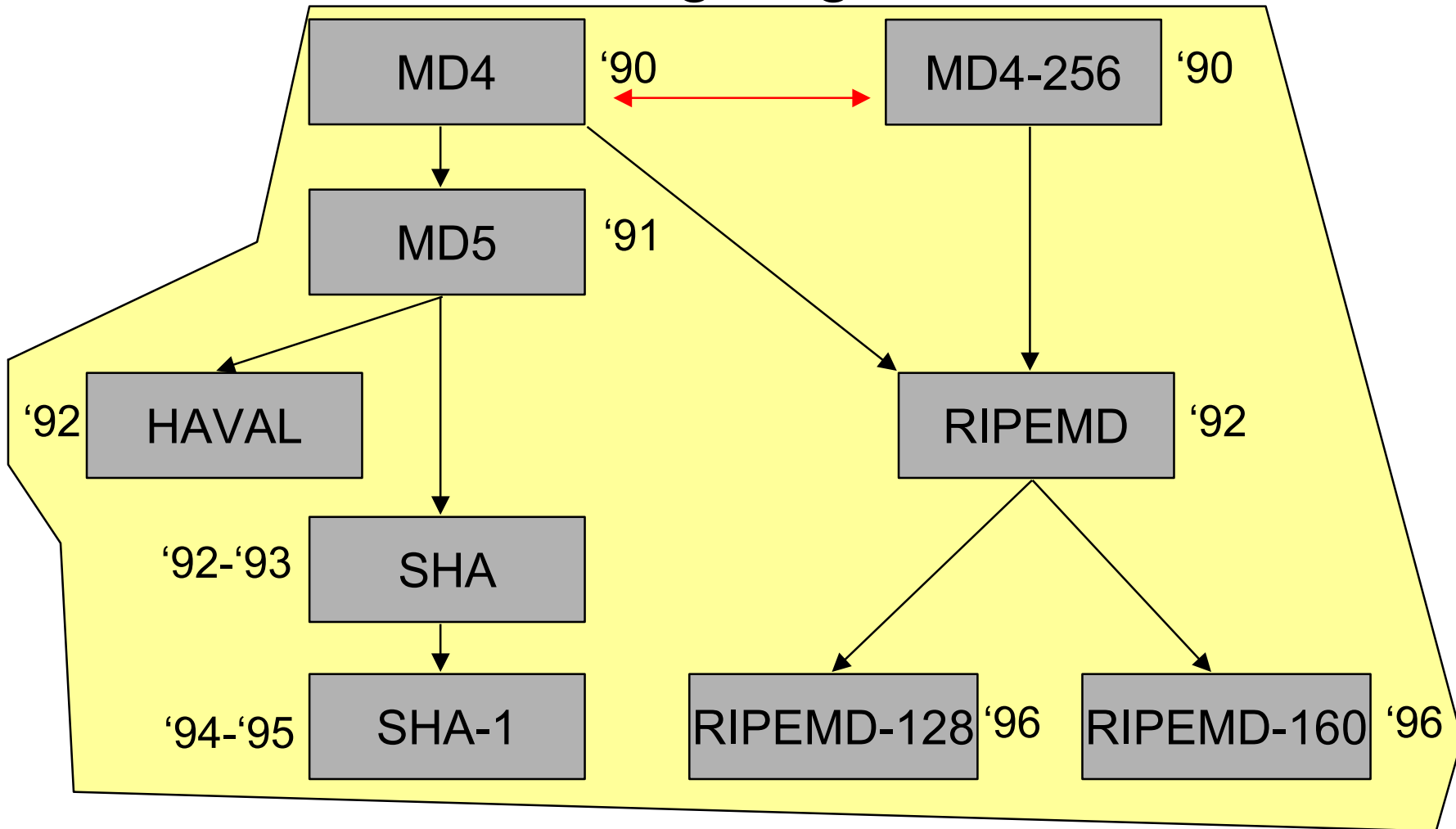


A, B, C - budowa

1, 2 - funkcjonowanie

# Rodzina funkcji skrótów MD4

MD- Message Digest



# ISO/IEC 9797 IT ST Data Integrity Mechanism

## MAC - Message Authentication Code

### MAC calculation

#### 1. Padding

#### 2. Blocking $D_1, D_2, \dots, D_q$ - the n-bit blocks of the data

#### 3. Iteration

$$I_1 \leftarrow D_1$$

$$I_i = A(I_{i-1}, K) \quad i=2,3, \dots, q$$

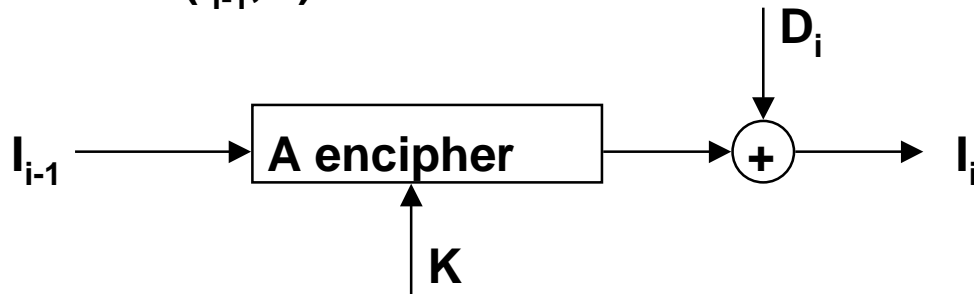
A - cryptographic algorithm using key K (may be DEA)

K - should be randomly or pseudorandomly generated

#### 4. Truncation.

The MAC - the m leftmost bits of the final n-bit block,  $m < n$

$$H = A(I_{i-1}, K)$$



#### 5. Optional processes

## ISO/IEC 10118-2 IT ST Hash functions

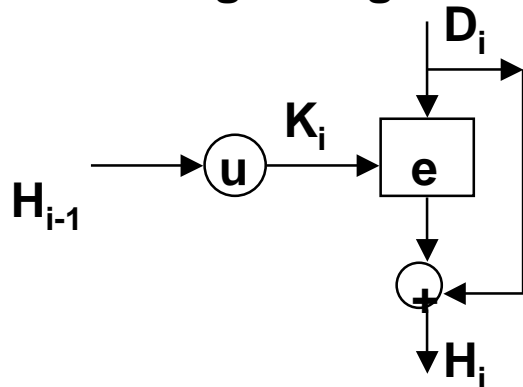
### Part 2: Hash-functions using an n-bit block cipher algorithm

Step 1 - splitting.  $D_1, D_2, \dots, D_q$  - the n-bit blocks of the data

Step 2 - padding.

Step 3 - iteration.

a. A single length hash-code

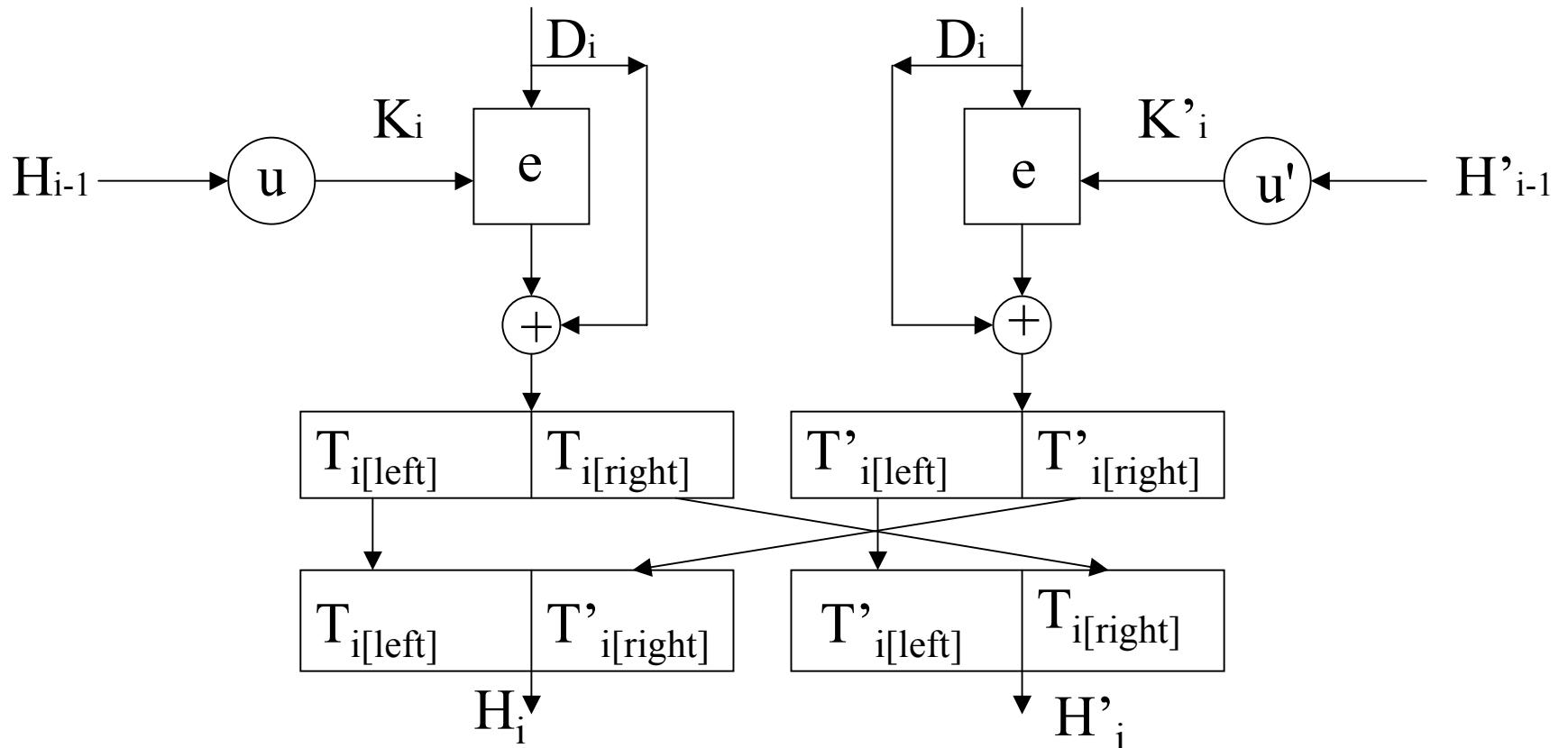


$H_0$  - a block equal to IV

$K_i = u(H_{i-1})$

$H_i = eK_i(D_i)$

## b. A double length hash-code



$H_0, H'_0$  - two blocks equal to IV and IV' respectively

$$K_i = u(H_{i-1})$$

$$K'_i = u'(H'_{i-1}),$$

$$T_i = eK_i(D_i) \text{ xor } D_i$$

$$T'_i = eK'_i(D_i) \text{ xor } D_i$$

} for  $i = 1, 2, \dots, q$

$$H_i = T_{i[\text{left}]} || T'_{i[\text{right}]}$$

$$H'_i = T'_{i[\text{left}]} || T_{i[\text{right}]}$$

$T_{[\text{left}]}$  -  $n/2$  leftmost bits of  $T$  when  $n$  - even,  $(n+1)/2$  -  $n$  - odd

$T_{[\text{right}]}$  -  $n/2$  rightmost bits of  $T$  when  $n$  - even,  $(n+1)/2$  -  $n$  - odd

Step 4 - truncation

# ISO/IEC 10118-4 IT ST Hash functions

## Part 4: Hash functions using modular arithmetic

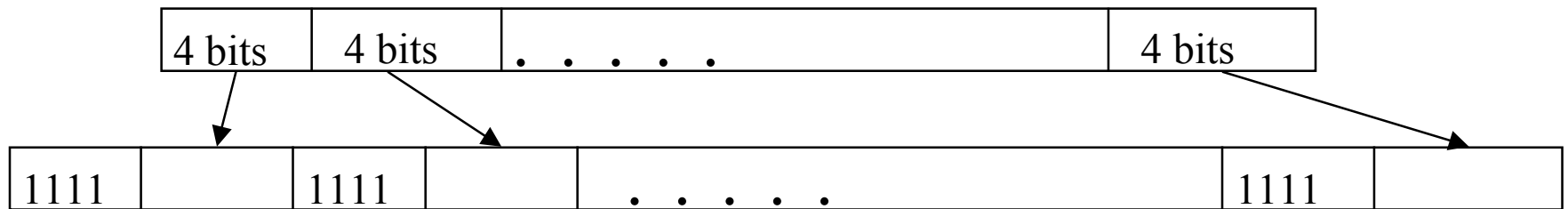
**Step 1 - splitting.**

**Step 2 - padding.**

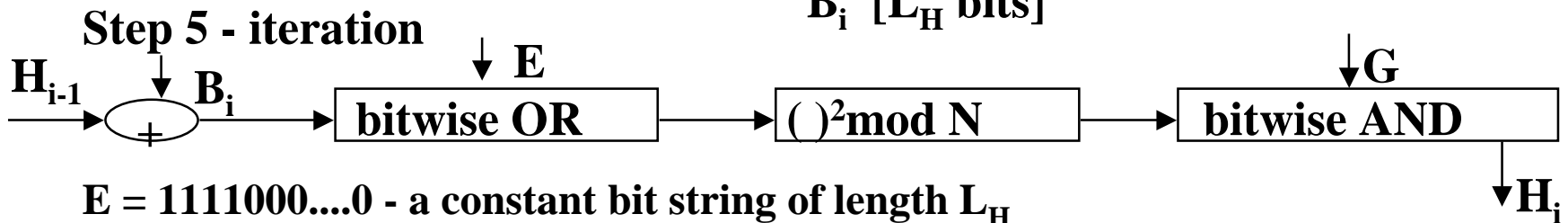
**Step 3 - additional substring - contains the length of the original data string  $D$  headed by binary zeroes**

**Step 4 - expansion**

$D_i$  [ $L_H/2$  bits]



$B_i$  [ $L_H$  bits]



$E = 1111000\dots0$  - a constant bit string of length  $L_H$

$G = 1111\dots1$  - a constant bit string of 1-s of length  $L_H$

$N$  - modulus, either a prime number or a product of two primes. Length:  $L_H+1 \leq L_N \leq L_H+16$

$H_0 = IV$     $H_i = (((H_{i-1} \text{ xor } B_i) \text{ or } E)^2 \text{ mod } N) \text{ and } G$

**Step 6 - the hash-code  $H$  is the final block  $H_{m+1}$**

**a. Dedicated Hash-Function 1 - RIPEMD** (Race Integrity Primitives Evaluation Message Digest)

**Initializing Value:  $Y_0=67452301$   $Y_1=EFCDAB89$   $Y_2=98BADCFE$   $Y_3=10325476$**

**Each 512-bit data block  $D_i$  is treated as a sequence of 16 words,  $Z_0, Z_1, \dots, Z_{15}$ .**

**The round-function  $\phi$  operates as follows.**

1.  $\phi(Z, Y)$
2.  $X_i := Y_i$   $X'_i := Y_i$   $i=0,1,2,3$
3. For  $i:=0$  to 47
  - a.  $W := S^{t_i}(X_0 + g_i(X_1, X_2, X_3) + Z_{ai} + C_i)$ ;
  - b.  $X_0 := X_3$ ;  $X_3 := X_2$ ;  $X_2 := X_1$ ;  $X_1 := W$ ;
  - c.  $W := S^{t_i}(X'_0 + g_i(X'_1, X'_2, X'_3) + Z_{ai} + C'_i)$ ;
  - d.  $X'_0 := X'_3$ ;  $X'_3 := X'_2$ ;  $X'_2 := X'_1$ ;  $X'_1 := W$ ;

where:

$S^n()$  - circular left shift by  $n$  bit positions,

$g_i(X_0, X_1, X_2) = (X_0 \text{ and } X_1) \text{ or } (\text{not } X_0 \text{ and } X_2)$ , ( $0 \leq i \leq 15$ )

$g_i(X_0, X_1, X_2) = (X_0 \text{ and } X_1) \text{ or } (X_0 \text{ and } X_2) \text{ or } (X_1 \text{ and } X_2)$ , ( $16 \leq i \leq 31$ )

$g_i(X_0, X_1, X_2) = X_0 \text{ xor } X_1 \text{ xor } X_2$ , ( $32 \leq i \leq 47$ )

$C_i, C'_i, t_i, a_i$   $i=0,1 \dots, 47$  - constants defined in Standard.

4. Let

$W := Y_0$   $Y_0 := Y_1 + X_2 + X'_3$   $Y_1 := Y_2 + X_3 + X'_0$   $Y_2 := Y_3 + X_0 + X'_1$   $Y_3 := W + X_1 + X'_2$

5. The 128-bit string  $Y_0, Y_1, Y_2, Y_3$  then represents the output of the round-function  $\phi$ .

## b. Dedicated Hash-Function 1 - Secure Hash Algorithm SHA-1 (US NIST)

Initializing Value:

$$Y_0=67452301 \quad Y_1=EFCDAB89 \quad Y_2=98BADCFE \quad Y_3=10325476 \quad Y_4=C3D2E1F0$$

Each 512-bit data block  $D_i$  is treated as a sequence of 16 words,  $Z_0, Z_1, \dots, Z_{15}$ .

The round-function  $\phi$  operates as follows.

1.  $\phi(Z, Y)$

2. For  $i=16$  to  $79$  let  $Z_i := S^1(Z_{i-3} \text{ xor } Z_{i-8} \text{ xor } Z_{i-14} \text{ xor } Z_{i-16})$

3.  $X_i := Y_i \quad i=0, 1, 2, 3, 4$

4. For  $i=0$  to  $79$  do the following two steps

a.  $W := S^5(X_0) + f_i(X_1, X_2, X_3) + X_4 + Z_i + C_i;$

b.  $X_4 := X_3; \quad X_3 := X_2; \quad X_2 := S^{30}(X_1); \quad X_1 := X_0; \quad X_0 := W;$

where:

$f_i(X_0, X_1, X_2) = (X_0 \text{ and } X_1) \text{ or } (\text{not } X_0 \text{ and } X_2), \quad (0 \leq i \leq 19)$

$f_i(X_0, X_1, X_2) = X_0 \text{ xor } X_1 \text{ xor } X_2, \quad (20 \leq i \leq 39)$

$f_i(X_0, X_1, X_2) = (X_0 \text{ and } X_1) \text{ or } (X_0 \text{ and } X_2) \text{ or } (X_1 \text{ and } X_2), \quad (40 \leq i \leq 59)$

$f_i(X_0, X_1, X_2) = X_0 \text{ xor } X_1 \text{ xor } X_2, \quad (60 \leq i \leq 79)$

$C_i \quad i=0, 1, \dots, 79$  - constants defined in Standard.

5. Let  $Y_i := Y_i + X_i \quad i=0, 1, \dots, 4.$

6. The 160-bit string  $Y_0, Y_1, Y_2, Y_3, Y_4$  then represents the output of the round-function  $\phi$ .

# Paradoks dnia urodzin

Ile średnio osób musi być w pomieszczeniu

aby znalazła się druga  
osoba obchodząca  
urodziny tego samego  
dnia co ja?

**183**

$\sim 365/2$

aby znalazły się dwie  
osoby obchodzące  
urodziny tego samego  
dnia?

**23**

Birthday Attack

# Birthday Attack

Wyobraźmy sobie tablicę, na której napisano  $n$  różnych dni roku. Każdy student wchodzi skreśla dzień swoich urodzin.

Liczmy prawdopodobieństwo, że  $r$ . student nie zastanie swojego dnia przekreślonego:

dla 1. - prawdopodobieństwo  $1=n/n$

dla 2. -  $(n-1)/n$

dla  $r$ . -  $p=n(n-1)(n-2)\dots(n-r+1) / n^r = n! / [(n-r)! n^r]$

Dla  $n=365$ ,  $r=23$ ,  $p=0,493$  - skreślenie już jest dla 23 studentów i wynosi  $q=1-p$

# Ataki na funkcję skrót

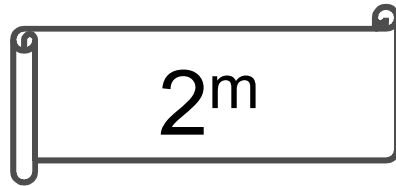
Rozważamy funkcję skrót  $H$  dającą skrót o dł.  $m$

(niezależne od algorytmu)

dla danego  $M$  szukamy  $M'$ ,  
dla którego  $H(M)=H(M')$

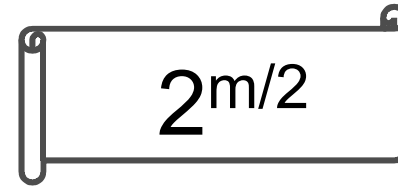
szukamy dowolnych  $M$  i  $M'$ ,  
dla których  $H(M)=H(M')$

birthday/square root attack



dla  $m=64 \sim 1,845e+19$

**58.5 mln lat**



dla  $m/2=32 \sim 4,295e+9$

**5 dni**

przy zał. 10.000 prób na sekundę

# Przykład wykorzystania Birthday Attack [Yuval]

Zał. wykorzystujemy podpis cyfrowy oparty na funkcji skrótu

1. Ania przygotowuje dwie wersje kontraktu, który ma być podpisany z Basią - jedną fałszywą i drugą prawdziwą.
2. Ania dokonuje niewidocznych okiem zmian edytorskich w dokumentach - generuje w ten sposób  $2^{m/2}$  wersji każdego z kontraktów.
3. Ania dokonuje skrótów dokumentów i szuka wspólnej pary.
4. Basia podpisuje „prawdziwy” kontrakt. „Fałszywy” kontrakt również staje się obowiązujący.

# Rzut monetą

1. Asia losuje liczbę  $x$  i zlicza  $h=H(x)$
2. Asia wysyła  $h$  do Bartka
3. Bartek zgaduje czy  $x$  jest parzyste, czy też nie i wysyła swoją odpowiedź do Asi
4. Jeśli Bartek zgadł, rezultat rzutu to orzeł, jeśli nie - reszka. Asia obwieszcza wynik i wysyła  $x$  do Bartka
5. Bartek sprawdza, czy  $h=H(x)$

# Szybkość implementacji

