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DINAS: A COMPUTER-ASSISTED ANALYSIS SYSTEM FOR MULTIOBJECTIVE TRANSSHIPMENT PROBLEMS WITH FACILITY LOCATION

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Scope and Purpose—The classical interactive procedures for multiple criteria decision analysis assume the so-called rational behavior of decision makers: they know the decision problem, and they are consistent and coherent in the decision process. However, as has been stressed by many researchers and practitioners, decision makers usually learn the decision problem during the interactive session with the decision support system, and there are numerous examples in which people systematically violate the consistency and coherence of their preferences. Therefore, the so-called aspiration-based interactive decision support schemes seem to be much more interesting for practical implementations. This interactive process explicitly depends on the aspiration levels stated and modified by the decision maker, and thereby makes operational the concept of the adaptive dependence of the decision process on learning and context. The paper presents an implementation of an aspiration-based interactive procedure to solve various multiobjective transshipment problems with facility location.

Abstract—DINAS is an interactive system to aid solving various multiobjective transshipment problems with facility location using IBM-PC XT/AT or compatibles. DINAS utilizes the so-called aspiration-based (or reference point) approach to interactive handling of multiple objectives. In this approach the decision maker forms his/her requirements in terms of aspiration and reservation levels, i.e. specifies acceptable and required values for given objectives, whereas the system searches for a satisfying efficient solution by optimization of a special scalarizing achievement function. A sophisticated solver has been developed to provide DINAS with solutions to these single-objective problems. This numerical kernel of the system is, however, hidden from the user. Therefore the interactive analysis of the multiobjective problem can be performed with DINAS by a decision maker who is familiar with neither computer techniques nor mathematical programming.

1. INTRODUCTION

Most real-life decision problems cannot be modeled with a single objective function. This causes growing interest in techniques taking into account multiple objectives. Various methods for multiple criteria decision making have been summarized in many books (see e.g. [10]). The progress in computer technology made during the last decade allows us to consider as implementable even very complex multiple criteria optimization techniques requiring solutions to sequences of single objective problems. Even more important is the increased accessibility of computers. The use of computers is no longer limited to a narrow group of specialists concentrated around professional computer centers. Powerful microcomputers became standard productivity tools for businessmen

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and other decision makers. It fruits in many attempts to build interactive decision support systems where mathematical models and methods play only the supporting role in decision analysis being performed by a human decision maker.

The classical interactive procedures for multiple criteria decision analysis assume the so-called rational behavior of decision makers: they know the decision problem, and they are consistent and coherent in the decision process. Usually the existence of some individual or group utility function is assumed and the interactive process depends on its identification. However, as was stressed by many researchers and practitioners, decision makers usually learn the decision problem during the interactive session with the decision support system, and there are numerous examples in which people systematically violate the consistency and coherence of their preferences. Therefore the so-called quasisatisficing or aspiration-based interactive decision schemes (see [4]) seem to be much more interesting for practical implementation.

The quasisatisficing framework of multiple criteria decision making was developed mainly by Wierzbicki (see [11]). This approach deals with the so-called scalarizing achievement functions which when optimized generate efficient decisions relative to the current list of objectives. It is slightly similar to a utility function and, in fact, can be used as an approximation to a class of utility functions. It is, however, explicitly dependent on aspiration levels stated and modified by the decision maker, and thereby makes operational the concept of adaptive dependence of utility function on learning and context. Completeness, computational robustness and controllability of the interactive scheme are important here, rather than consistency and coherence. Several successful implementations of the aspiration-based multiple criteria optimization systems have been reported (see e.g. [2]).

In this paper we present an implementation of an aspiration-based interactive procedure to solve some mixed integer programming problems. Namely, the dynamic interactive network analysis system (DINAS), a scientific transferable tool which allows various multiobjective transshipment problems with facility location to be solved. The system is based on a sophisticated numerical solver taking advantage of the specific structure of the problem (SON and VUB techniques). This numerical kernel of the system is, however, hidden from the user. Therefore the interactive analysis of the multiobjective problem can be performed with DINAS by a decision maker who is familiar with neither computer techniques nor mathematical programming.

The distribution–location type problems belong to the class of most significant real-life problems based on mathematical programming. They are usually formalized as the so-called transshipment problems with facility location. A network model of the transshipment problem with facility location consists of nodes connected by a set of direct flow arcs. The set of nodes is partitioned into two subsets: the set of fixed nodes and the set of potential nodes. The fixed nodes represent "fixed points" of the transportation network, i.e. points that cannot be changed, whereas the potential nodes are introduced to represent possible locations of new points in the network. Some groups of the potential nodes may represent different versions of the same facility to be located (e.g. different sizes of warehouse etc.). For this reason, potential nodes are organized in the so-called selections, i.e. sets of nodes with multiple choice requirements. Each selection is defined by the list of included potential nodes as well as by a lower and upper number of nodes to be selected (located).

A homogeneous good is considered to be distributed along the arcs among the nodes. Each fixed node is characterized by two quantities: supply and demand on the good, but for mathematical statement of the problem only the difference supply-demand (the so-called balance) is important. Each potential node is characterized by a capacity which bounds the maximal flow of the good through the node. The capacities are also given for all the arcs but not for the fixed nodes.

A few linear objective functions are considered in the problem. The objective functions are introduced into the model by given coefficients associated with several arcs and potential nodes. They are called cost coefficients independently of their real character. The cost coefficients for potential nodes are, however, understood in a different way than those for arcs. The cost coefficient connected to an arc is treated as the unit cost of the flow along the arc whereas the cost coefficient connected to a potential node is considered as the fixed cost associated with the activity (locating) of the node rather than as the unit cost.

Summarizing, the following groups of input data define the transshipment problem under consideration:

—objectives
—fixed nodes with their balances
—potential nodes with their capacities and (fixed) cost coefficients
—selections with their lower and upper limits on number of active potential nodes
—arcs with their capacities and cost coefficients.

The problem is to determine the number and location of active potential nodes and to find the flows (along arcs) so as to satisfy the balance and capacity restrictions and, simultaneously, optimize the given objective functions. A mathematical model of the problem is described in detail in [8]. For the purpose of this presentation of the DINAS functionality, however, it is enough to state that the problem under consideration is a multiobjective mixed integer linear program:

$$\text{optimize} \quad \mathbf{q} \tag{1}$$

subject to

$$\mathbf{q} = \mathbf{F}(\mathbf{x}) \tag{2}$$

$$\mathbf{x} \in Q \tag{3}$$

where $\mathbf{q}$ represents the acheivement vector, $\mathbf{F} = (F_1, \ldots, F_k)$ represents the vector of k objective functions, optimize means minimize or maximize for several objective functions, respectively to their nature, Q denotes the feasible set of the program, and $\mathbf{x}$ is a vector of decision variables.

2. OVERVIEW OF THE SYSTEM

DINAS enables a solution to the problem (1)–(3) using an IBM-PC XT/AT or compatible. The basic version of the DINAS system can process problems consisting of:

—up to seven objective functions
—a transportation network with up to one hundred fixed nodes and three hundred arcs
—up to fifteen potential locations.

DINAS consists of three programs prepared in the C programming language:

—the interactive procedure for efficient solutions generation
—the solver for single-objective problems
—the network editor for input data and results examination.

DINAS is a menu-driven system with very simple commands. Operations available in the DINAS interactive procedure are partitioned into three groups and three corresponding branches of the main menu (see Table 1): PROCESS, SOLUTION and ANALYSIS.

The PROCESS branch contains basic operations connected with the processing of the multiobjective problem and generation of several efficient solutions. There are problem definition operations included such as calling the network editor for input or modification of the problem (PROBLEM) and converting the edited problem with error checking (CONVERT). Further, in this branch the basic optimization operations are available: PAY-OFF and EFFICIENT. The last command in this branch is the QUIT operation which allows the decision maker (DM) to leave the system.

The PAY-OFF command must be executed as the first step of the multiobjective analysis. It

Table 1. DINAS main menu

Process	Solution	Analysis
PROBLEM	SUMMARY	COMPARE
CONVERT	BROWSE	PREVIOUS
PAY-OFF	SAVE	NEXT
EFFICIENT	DELETE	LAST
QUIT		RESTORE

Table 2

Optimized function	Objective values			
	Invest	Satisf	Dist	Prox
Invest	186	100	2.61	4976
Satisf	401	368	2.17	6385
Dist	413	279	2.03	8782
Prox	398	187	2.12	8854

performs optimization of each objective function separately. Namely, the following p single-objective programs are solved:

$$\text{optimize} \quad \{F_p(\mathbf{x},\mathbf{y}) + 1/k \sum_{i=1}^{k} \rho_i F_i(\mathbf{x}) : \mathbf{x} \in Q\} \quad p = 1, 2, \ldots, k \tag{4}$$

where F_i denotes the ith objective function and ρ_i are arbitrarily small numbers (positive if the corresponding objective function F_i is to be minimized, and negative otherwise). This means that some small regularization term is added to all the objective functions to guarantee efficiency of all generated optimal solutions. As a result of these computations one gets the so-called pay-off matrix. The pay-off matrix is a well-known device in multiple criteria decision making (see [10]). It is displayed as a table containing values of all the objective functions (columns) obtained while solving several single-objective problems (rows), thereby it helps to understand the conflicts between different objectives (compare Table 2).

The execution of the PAY-OFF command also provides the DM with two reference vectors: the utopia vector and the nadir vector. The utopia vector represents the best values of each objective considered separately, and the nadir vector expresses the worst values of each objective noticed during several single-objective optimizations. The utopia vector is, usually, not attainable, i.e. there are no feasible solutions with such objective values. Coefficients of the nadir vector cannot be, in general, considered as the worst values of the objectives over the whole efficient (Pareto-optimal) set. They usually estimate these values but they express only the worst values of each objective noticed during optimization of the other objective functions.

Due to the regularization technique, used while computing the pay-off matrix [compare (4)], each generated single-objective optimal solution is also an efficient solution to the multiobjective problem. So, after calculation of the pay-off matrix there is already available a number of efficient solutions connected with several rows of the pay-off matrix. The pay-off matrix calculation is, usually, the most time-consuming operation of the multiobjective analysis. Therefore DINAS automatically saves the computed pay-off matrix on the problem file.

Having executed the PAY-OFF command one can start the interactive search for a satisfying efficient solution. DINAS utilizes aspiration and reservation levels to control the interactive analysis. More precisely, for several objectives the DM specifies the values he/she wishes to approach as the aspiration levels, and the worst acceptable values as the reservation levels. All the operations connected with editing the aspiration and reservation levels as well as with computation of a new efficient solution are performed within the EFFICIENT command.

The system searches for a satisfying efficient solution using an achievement scalarizing function as a criterion in single-objective optimization. Namely, DINAS computes the optimal solution to the following problem:

minimize

$$\underset{1 \leqslant p \leqslant k}{\text{maximum}} \quad u_p(\mathbf{q}, \mathbf{q}^a, \mathbf{q}^r) + \rho/k \sum_{p=1}^{k} u_p(\mathbf{q}, \mathbf{q}^a, \mathbf{q}^r) \tag{5}$$

subject to

$$\mathbf{q} = \mathbf{F}(\mathbf{x}), \mathbf{x} \in Q \tag{6}$$

where ρ is an arbitrarily small number, and u_p is a function which measures the deviation of results from the DM's expectations with respect to the pth objective depending on a given aspiration level

$\mathbf{q}^a$ and reservation level $\mathbf{q}^r$. The function u_p is a strictly monotone function of the objective vector $\mathbf{q}$ with value $u_p = 0$ if $\mathbf{q}_p = \mathbf{q}_p^a$ and $u_p = 1$ if $\mathbf{q}_p = \mathbf{q}_p^r$. In our system, we use a piece-wise linear function u_p defined as follows:

$$u_p(\mathbf{q}, \mathbf{q}^a, \mathbf{q}^r) = \begin{cases} -a_p|\mathbf{q}_p - \mathbf{q}_p^a|/|\mathbf{q}_p^r - \mathbf{q}_p^a|, & \text{if } \mathbf{q}_p \text{ is better than } \mathbf{q}_p^a \\ |\mathbf{q}_p - \mathbf{q}_p^a|/|\mathbf{q}_p^r - \mathbf{q}_p^a|, & \text{if } \mathbf{q}_p \text{ is between } \mathbf{q}_p^a \text{ and } \mathbf{q}_p^r \\ b_p|\mathbf{q}_p - \mathbf{q}_p^r|/|\mathbf{q}_p^r - \mathbf{q}_p^a| + 1, & \text{if } \mathbf{q}_p \text{ is worse than } \mathbf{q}_p^r \end{cases}$$

where $a_p \ll 1$ and $b_p \gg 1$ $(p = 1, 2, \ldots, k)$ are given positive parameters. Such a function u_p can be considered as a measure of the DM's dissatisfaction from the achievement with respect to an individual objective. The coefficients a_p and b_p allow to model a small premium for achievement better than the aspiration level, and a high penalty for achievement worse than the corresponding reservation level (the so-called soft bound), respectively.

Due to using the maximum operator for aggregation of the individual functions u_p (Chebychev norm) and an additional regularization term [compare (5)] the computed optimal solution to the problem (5)–(6) is always an efficient solution to the original multiobjective model (even if the given aspiration levels are attainable). DINAS stores the efficient solutions in a special solution base. All the efficient solutions generated (or input from a file) during a session get consecutive numbers and are automatically put into the solution base. However, at most, nine efficient solutions can be stored in the solution base. When the tenth solution is put into the base then the oldest solution is automatically dropped from it. On the other hand any efficient solution can be saved on a separate file and restored during the same or a subsequent session with the problem.

DINAS is armed with many operations helping to manage the solution base. There are two kinds of operations connected with the solution base: operations on a single efficient solution, and operations on the entire solution base. Operations addressed to a single solution are connected with the current solution. The newest generated efficient solution is automatically assumed to be the current solution but any efficient solution from the solution base can be manually assigned as the current solution.

The SOLUTION branch of the main menu (see Table 1) contains additional operations connected with the current solution. One can examine in detail the current solution using the network editor (BROWSE) or analyze only short characteristics such as objective values and selected locations (SUMMARY). Values of the objective functions are presented in three ways: as a standard table, as bars in the aspiration/reservation scale and as bars in the utopia/nadir scale. The bars show percentage level of each objective value with respect to the corresponding scale. One may also save the current solution on a separate file in order to use it during the next runs of the system with the same problem (SAVE). There is also available a special command to delete the current solution from the solution base if one finds it as quite useless (DELETE).

The ANALYSIS branch of the main menu groups commands connected with operations on the solution base. The main command, COMPARE, allows the DM to perform a comparison between all the efficient solutions included in the solution base or in some subset of the base. In the comparison only short characteristics of the solutions are used, i.e. objective values in the form of tables and bars as well as tables of selected locations. Moreover, some commands included in this branch (PREVIOUS, NEXT and LAST) allow the selection of any efficient solution from the solution base as the current solution. One can also restore some efficient solution (saved earlier on a separate file) to the solution base (RESTORE).

A special solver has been prepared to provide the multiobjective analysis procedure with solutions to single-objective problems. The solver is hidden from the user but it is the most important part of the DINAS system. It is a numerical kernel of the system which generates efficient solutions. The concept of the solver is based on the branch and bound scheme with a pioneering implementation of the simplex special ordered network (SON) algorithm proposed by Glover and Klingman [3] with implicit representation of the simple and variable upper bounds (SUB & VUB) suggested by Schrage [9]. The mathematical detailed background of the solver is provided by Ogryczak *et al.* [8].

DINAS is equipped with a special network editor. It is a full-screen editor specifically designed for input and edit of the data of network problems analyzed with DINAS. The DINAS interactive procedure works with a special file containing whole information defining the problem and the editor enables the preparation of this file. The essential data of the problem can be divided into two groups:

—logical data defining the structure of a transportation network (e.g. nodes, arcs, selections)
—numerical data describing the nodes and arcs of the network (e.g. balances, capacities, coefficients of the objective functions).

The general concept of the editor is to edit the numerical data while defining or examining the logical structure of the network. More precisely, the essence of the editor concept is a dynamic movement from some current node to its neighbor nodes according to the network structure. The input data are inserted by a special mechanism of windows, while visiting several nodes. Apart from the windows with local information some special windows containing a list of nodes and a graphic scheme of the network can be activated at any moment to ease movement across the network.

3. SAMPLE PROBLEM

In the next section we present an outline of the basic multiobjective analysis performed with the DINAS system on a small artificial problem of health service districts reorganization. Such real-life problems connected with reorganization of the primary health service in a district of Warsaw were successfully solved with the MPSX/370 package by Ogryczak and Malczewski [6]. Recently such an analysis connected with location of new pediatric hospitals in Warsaw macroregion using the DINAS system on an IBM-PC AT microcomputer has been completed (see [5]). To illustrate the interactive procedure and the system capabilities we will present an analysis of a test problem constructed as a small artificial part of this real-life model.

The problem of health service districts reorganization connected with location of new health-care centers can be formulated as follows. The region under consideration is assumed to consist of some number of geographically defined subareas or spatial units with known distribution of the population. A number of health-care centers is available in the region but their capabilities of offering health services is not sufficient. Therefore some new facilities are located. The problem depends on determination of the locations and capacities of some new centers as well as on assignment of individuals to the centers (new and old). The proposed solution should be optimal with respect to a few objective functions and simultaneously it must be accepted by the competent decision maker.

To set the stage, we consider as the region a part of city divided by five major highways into 12 subareas. For each of these areas the demand on health-care services is identified in thousands of visits (treatments) per year. Within the region there are two health-care centers offering services: Pond and Hill. They can offer 100 and 90 thousands of visits per year, respectively. Thus, the total supply of services amounts to 190 while the total demand on services in the region was counted as 240. Therefore some new health-care centers should be located within the region.

There are four potential locations considered for the new centers: Ice, Fiord, Bush, and Oasis. The locations are divided into two subsets associated with the corresponding two subregions:

$$\text{North} = \{\text{Ice}, \text{Fiord}\},$$
$$\text{South} = \{\text{Bush}, \text{Oasis}\}.$$

The distance between two potential locations in the same subregion are relatively small whereas each of them can meet the demands on health services. Therefore the locations belonging to the same subregion are considered as exclusive alternatives, i.e. no more than one location from the subregion can be used. However, different designed capacities of health centers are associated with several locations.

One must decide which potential health-care centers have to be built so as to meet the total demand on health services. The decision should be optimal with respect to the following criteria:

—minimization of the average distance per visit
—maximization of the overall proximity to centers
—minimization of the investment cost
—maximization of the population satisfaction.

The first two criteria are connected with distances between health-care centers and areas assigned to them. Taking into account the urban morphology and the transportation network, it has been accepted that the city-block metric is the best approximation to the real distances. Therefore we define the distance between an individual and the health center as the rectangular distance between the center of the corresponding area and the location of the health-care center. Certain connections between the areas and the health centers are eliminated as unacceptable due to some transportation inconveniences.

The overall proximity to the health care services is defined as a sum of all the individual proximity coefficients. The individual proximity is assumed to be inversely proportional to square of the distance to the health-care center. More precisely, the individual proximity coefficients are defined according to the following formula (compare [1]):

$$p_{ac} = 1/(d_{ac} + \varepsilon)^2 \tag{7}$$

where d_{ac} denotes the distance between the corresponding area a and the health-care center c, and ε is an arbitrarily small positive number.

The investment cost and the population satisfaction level are assumed to be a sum of fixed costs and a sum of fixed satisfaction levels connected with several possible locations, respectively.

The problem of health service districts reorganization connected with location of new health-care centers, stated above, can be easily formulated as a multiobjective transshipment problem with facility location. The areas and existing health-care centers are, certainly, fixed nodes of the network under consideration. Similarly, all the potential locations of new health centers are treated as potential nodes. Arcs represent all the possible assignments of patients to the health-care centers. A flow along the arc from a center c to an area a expresses a number of visits in the area a serviced by the center c. In order to balance the problem in terms of supply and demand an artificial node Tie with supply equal to the overall demand is introduced. There are also defined additional arcs from the artificial node to each health-care center (existing or potential). Capacity of the existing health-care centers (Pond and Hill) are then represented as capacities of the arcs from Tie to the corresponding fixed nodes. A scheme of the network is presented in Fig. 1.

In the transshipment problem with facility location objective functions are considered as sums of linear functions of flows along several arcs and fixed costs connected with the used locations. In our model objective functions can be divided into two groups. Functions Investment (cost) and Satisfaction (level) are independent of the assignment decisions and thereby they have no coefficients connected with flows along arcs (i.e. these coefficients are equal to 0). On the other hand, functions (average) Distance and (overall) Proximity depend only on assignment decisions and they do not contain fixed terms connected with locational decisions. Fixed coefficients of the functions Investment and Satisfaction can be directly taken from original data. The linear coefficients of the function Proximity are calculated from the original distances according to formula (7). The linear coefficients of the function Distance are defined as quotients of the corresponding distances by the sum of all the demands.

There are four potential nodes which represent the potential locations of the health-care centers, i.e. Ice, Fiord, Bush, Oasis. As we have already mentioned the locations belonging to the same subregion are considered as exclusive alternatives, i.e. no more than one location from the subregion can be used. Therefore we introduce into the network model selections which represent such a type of requirements. In our model there are two selections associated with to subregions: North and South. Both the selections have lower numbers equal to 0 and upper numbers equal to 1. This guarantees that, at most, one potential node in each selection is active.

The last group of data is connected with the arcs. The arcs are characterized by their capacities and objective functions coefficients. The cost coefficients have been already discussed while considering the objective functions. Capacities of the arcs from the artificial node Tie to the nodes representing health-care centers (Pond, Hill, Ice, Fiord, Bush, Oasis) express capacities of the corresponding centers. The arcs connecting the nodes representing health-care centers with the nodes representing the areas have essentially unlimited capacities. However, in practice, flows along these arcs are also bounded by capacities of the corresponding health-care centers and we can use them as arcs capacities.

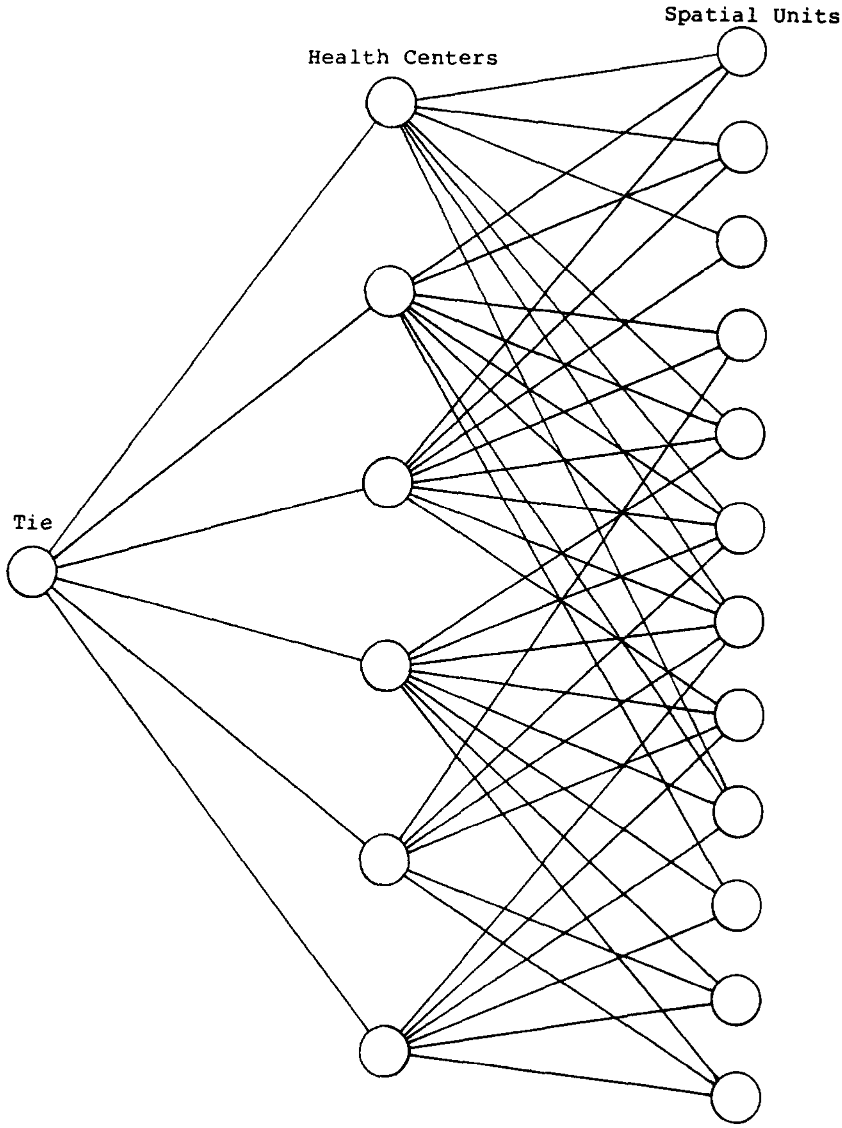

Fig. 1. A network scheme for the sample problem.

4. SAMPLE SESSION

Having defined and converted the problem as the first step of the multiobjective analysis one must execute the PAY-OFF command. In effect, we get the pay-off matrix presented in Table 2. It gives values of all the objective functions (columns) obtained while solving several single-objective problems (rows) and thereby helps in understanding the conflicts between different objectives. Here, and thereafter, the objective functions are denoted by abbreviations of the corresponding names.

Execution of the PAY-OFF command also provides us with two reference vectors: the utopia vector and the nadir vector (see Table 3). The utopia vector represents the best values of each objective considered separately, and the nadir vector expresses the worst values of each objective noticed during optimization of the other objective functions. The utopia vector is, obviously, not attainable, i.e. there are no feasible solutions with such objective values.

While analyzing Tables 2 and 3 we find that the objective values vary significantly depending on selected optimization. Only for the average distance we notice the relative variation less than 30% whereas for the other objectives it even oversteps 100%. Moreover, we recognize a strong conflict between the investment cost and all the other objectives. While minimizing the investment cost we obtained the worst values for all the other objectives. On the other hand, while optimizing another objective function we obtained a double investment cost in comparison to its minimal value.

Table 3

	Objective values			
	Invest	Satisf	Dist	Prox
Utopia	186	368	2.03	8854
Nadir	413	100	2.61	4976

Table 4

	Invest	Satisf	Dist	Prox
Aspiration	186	300	2.08	8500
Reservation	250	200	2.50	7000

Coefficients of the nadir vector cannot be considered as the worst values of the objectives over the entire efficient (Pareto-optimal) set. They usually estimate these values but they express only the worst values of each objective noticed during optimization of another objective function. With further analysis we will show that these estimations can sometimes be exceeded.

Due to the special regularization technique used while computation of the pay-off matrix [see (4)], each generated single-objective optimal solution is aso an efficient soution to the mutiobjective probem. So, we have already available in the solution base four efficient solutions connected with several rows of the pay-off matrix. Using different comands of DINAS we can examine in detail these solutions. In particular, we can recognize the locations structure for several solutions. The first solution which minimizes the investment cost is based on only one new health-care center located at Bush. Each other solution uses two new centers which explains their significantly higher investment costs. They are based on the following locations: Ice and Oasis, Fiord and Oasis, Bush and Fiord, respectively.

Having computed the utopia vector we can start the interactive search for a satisfying efficient solution. As we have already mentioned, DINAS utilizes aspiration and reservation levels to control the interactive analysis. At the beginning of the interactive analysis we compute the so-called neutral solution. For this purpose we accept the utopia vector as the aspiration levels and the nadir vector as the reservation levels. In effect, we get the fifth efficient solution based on the location of two new health-care centers: Bush and Ice. The investment cost of this solution is rather high (invest = 386) whereas the other objectives obtain middling values (satisf = 276, dist = 2.26, prox = 6457).

Apart from the solution connected with minimization of the investment cost all the other solutions are based on the location of two new health-care centers which implies their high investment costs. Therefore we try to find an efficient solution with a small investment cost (one new center) and relatively good values of the other objectives. For this purpose we define the aspiration and reservation levels as it is given in Table 4.

In effect, we get the sixth efficient solution based on the location of one new health-care center at Oasis. The investment cost is small (invest = 201), the satisfaction level has a middling value (satisf = 192), while the average distance is very large (dist = 2.58) and the overall proximity is even less than the corresponding coefficient of the nadir vector (prox = 4933). The system automatically corrects the nadir vector by putting the new worst value as the proper coefficient.

To avoid too small values of the overall proximity we modify the reservation level for this objective putting 8000 as the new value. After repeating the compution we get the seventh efficient solution based solely on the new health-care center located at Ice. Due to the very convenient form of solution presentation in DINAS we can easily examine performances (in terms of objective values) of the new solution in comparison with the previous one. The overall proximity, the average distance and the investment cost are slightly better (prox = 5287, dist = 2.53 and invest = 200) while the overall satisfaction level is a few percent worse (satisf = 176).

After analysis of two last efficient solutions we make a supposition that it is necessary to relax requirements on the satisfaction level to make it possible to find an efficient solution with good values of the average distance and the overall proximity under a small investment cost. So, we

Table 5

	Invest	Satisf	Dist	Prox
Aspiration	200	300	2.10	8800
Reservation	400	200	2.50	8400

Table 6

	Invest	Satisf	Dist	Prox
Solution 1	186	100	2.61	4976
Solution 2	401	368	2.17	6385
Solution 3	413	279	2.03	8782
Solution 4	398	187	2.12	8854
Solution 5	386	276	2.26	6457
Solution 6	201	192	2.58	4933
Solution 7	200	176	2.53	5287
Solution 8	212	87	2.38	7691
Solution 9	413	279	2.04	8791

Table 7

	Bush	Fiord	Ice	Oasis
Solution 1	yes	no	no	no
Solution 2	no	no	yes	yes
Solution 3	no	yes	no	yes
Solution 4	yes	yes	no	no
Solution 5	yes	no	yes	no
Solution 6	no	no	no	yes
Solution 7	no	no	yes	no
Solution 8	no	yes	no	no
Solution 9	no	yes	no	yes

change the reservation level associated with the function satisf on 100. The system confirms our supposition. We get the eighth efficient solution based solely on the new health-care center located at Fiord. The solution guarantees quite large overall proximity (prox = 7691) and relatively small average distance (dist = 2.38) under small investment cost (invest = 212). On the other hand, the satisfaction level has a value even less than the corresponding coefficient of the nadir vector (satisf = 87). Despite the latter, the solution seems to be very interesting compared to the other efficient solutions based on the location of a single health-care center.

Further research for a satisfying efficient solution based on only one new health-care center has finished without success. Namely, for different values of the aspiration and reservation levels the same efficient solutions have been generated. So, to complete the analysis we try to examine other efficient solutions. For this purpose we relax requirements on the investment cost. Among others, while using the aspiration and reservation levels given in Table 5, we get the ninth efficient solution. It is based on the same location of the new centers as the third solution (Fiord and Oasis) and thereby gives the same investment cost (invest = 413) and satisfaction level (satisf = 279). However, the average distance and the overall proximity differs slightly (dist = 2.04 and prox = 8791) due to another allocation scheme.

Finally, we examine all the generated efficient solutions using special comparison tools available in DINAS. The solutions are listed in Tables 6 and 7. Careful analysis of these solutions leads us to the following conclusion. The investment cost cannot be regarded as a typical objective function since its values depend on the number of new health-care centers rather than on their locations. It only partitions all the efficient solutions into gwo groups: solutions based on a single new health-care center and solutions based on the location of two new centers. Therefore, it is necessary to look for a good solution based on one new center that can be expanded later to a better solution by adding the second new center. In our opinion, the first new health-care center should be located at Fiord (Solution 8). It is the only efficient solution (based on a sole new center) which gives acceptable values of the average distance and the overall proximity (compare Tables 6 and 7). This solution also gives the worst value of the satisfaction level. However, further development of this solution by adding the new health-care center at Oasis (Solution 9) leads to quite a high value of the satisfaction level and makes further significant improvements with respect to the average distance and the overall proximity (see Table 6). Both the proposed solutions have the highest investment costs in the corresponding groups of solutions but variation of this objective among solutions of the same group is so mall that it canot be considered as a serious weakness.

Due to the easy way DINAS has for modification of the problem we can perform an additional analysis with some canceled objective functions. While repeating the multiobjective analysis with the omitted objective function Invest we get the ninth solution as the neutral solution which confirms optimality of this solution with respect to good values of all three objectives.

Using the BROWSE command we can examine in detail the selected solutions with the network editor. It turns out that in the eighth efficient solution the new health-care center Fiord is completely loaded whereas in the old centers we can notice some small free capacities. This suggests that the old health-care centers have nonoptimal location with respect to the considered objective functions. The ninth efficient solution confirms this observation. The new additional health-care center at Oasis takes some area from the region of Hill and from the region of Pond. So, in this solution both the new health-care centers use their entire capacities whereas the old centers use only 50–70% of their capacities.

5. FINAL COMMENTS

DINAS has already been used successfully while analyzing two real-life problems: routes optimization for building materials transportation and location of new pediatric clinics. The former was a three-objective transportation problem without facility location. It was originally a two-commodity transshipment problem but we managed to model it as a single-commodity one. The latter problem was more complex (compare [5]). It contained 5 objective functions, almost 300 arcs and almost 100 nodes, including 8 potential ones.

Initial experiences with the DINAS system confirm the appropriateness of the methodology used for solving multiobjective transshipment problems with facility location. The interactive scheme is very easy and supported by many analysis tools. Thereby, a satisfactory solution can usually be reached in a few interactive steps.

As has been shown in this paper, application of DINAS is not limited to typical transshipment problems. DINAS can be successfully used to solve different distribution–location problems. The problem of health service reorganization connected with location of new health-care centers presented in the paper is only an example, among many others, of real-life decision problems which can be solved with DINAS or similar tools.

When real-life problems are solved with DINAS on IBM-PC XT/AT microcomputers the single-objective computations take, obviously, much more time than when using some standard optimization tools (like the MPSX/370 package) on a mainframe. However, our experiences with both these approaches (see [5, 6]) allow us to suppose that DINAS, in general, will take much less time for performing the entire multiobjective analysis.

DINAS is available in executable form to educational and scientific institutions. Inquiries for software should be directed to: System and Decision Sciences Program, International Institute for Applied Systems Analysis, A-2361 Laxenburg, Austria.

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