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ON GENERALIZED OWA APPROACH TO SUPPORT LOCATION AND ROUTING DECISIONS

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Abstract. While modeling a transportation system one needs to take into account various negative effects of location and routing decisions for several populated spacial units. The minimization of the worst individual effect, the minimax approach, is the simplest solution concept focused on the spatial equity. In this paper we use the conditional means which generalize the worst effect by taking into account the portion of population affected (quantile). Further, aggregating conditional means for various quantiles we get a generalization of the so-called ordered weighted average (OWA) which allows us to model various preferences.

1. Introduction

In various systems which serve many users there is a need to respect the fairness rules. This applies to the desired system output (amount, quality of services) as well as to the obnoxious outcomes (like risk exposure, pollutions). The so-called minimax solution concept, where the worst individual effect (maximum individual disutility) is minimized, is usually considered as the simplest fair optimization model. The minimax approach is consistent with Rawlsian [4] theory of justice, especially when additionally regularized with the lexicographic order. On the other hand, making the locational and routing decisions to optimize the worst individual disutility may cause a large worsening of the overall (mean) performances. Therefore, several other fair decision schemes are searched and analyzed.

In this paper we use an alternative concept of the conditional mean which is a parametric generalization of the worst outcome taking into account the portion of population affected by the worst effects [3]. Namely, for a specified portion β of population we take into account the entire β portion (quantile) of the largest disutility outcomes and we consider their average as the *(worst) conditional β–mean* outcome. When parameter β approaches 0, the conditional β–mean tends to the largest outcome. On the other hand, for $\beta = 1$ the corresponding conditional mean becomes the standard mean.

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We select several conditional means for various levels of β and apply weights to aggregate them. The resulting aggregation function is a generalization of the so-called ordered weighted average (OWA) introduced by Yager [5]. While the original OWA weights the ordered individual outcomes, our aggregation weights the quantiles of the distribution of outcomes. Selecting different values of β and the corresponding weights allows us to model various preferences and thereby to achieve different solutions.

Our analysis is based on the problem of locating disposal or treatment facilities and transporting hazardous waste, introduced by Giannikos [1]. The disutility in this problem is caused by two factors: locating treatment facility close to the population center and transporting hazardous waste through the population center. In addition to the total location and transportation cost Giannikos considered several criteria based on the worst or the mean value of either of individual disutilities. Our approach aims at defining a framework allowing for more general criteria to be implemented.

2. Core model

The problem is based on a directed transportation network $\{N, A\}$, along which the waste is transported. The nodes N are divided into three disjointed sets: P – population centers not generating waste, G – population centers generating waste, and L – location candidate sites for the treatment facilities. Each population center $i \in P \cup G$ is associated a weight $w_i > 0$ expressing in relative terms the population located at site i. Each generation site $g \in G$ introduces to the network an amount of the hazardous waste denoted by D_g.

For each link (i, j) of the underlying network the unit cost c_{ij} of transporting waste from i to j is known. One of the results of the decision process is the amount of hazardous waste transported from i to j, denoted by a variable x_{ij}.

At each location candidate site $m \in L$ it is possible to locate a treatment facility with a capacity denoted by C_k $(k = 1, \ldots, K)$. The decision to build a facility of size C_k at location m is expressed by a binary variable z_{mk}. Building a treatment facility of size C_k at location m incurs a cost denoted by F_{mk}.

Having defined the decision variables and the parameters, the core model of the decision problem can be stated in the following terms:

$$\sum_{i \in \mathrm{IN}_m} x_{im} - \sum_{j \in \mathrm{ON}_m} x_{mj} \leq \sum_{k=1}^{K} C_k z_{mk} \qquad \text{for } m \in L \tag{1}$$

$$\sum_{j \in \mathrm{ON}_g} x_{gj} - \sum_{i \in \mathrm{IN}_g} x_{ig} \geq D_g \qquad \text{for } g \in G \tag{2}$$

$$\sum_{i \in \mathrm{IN}_j} x_{ij} - \sum_{i \in \mathrm{ON}_j} x_{ji} = 0 \qquad \text{for } m \in P \tag{3}$$

$$\sum_{k=1}^{K} z_{mk} \leq 1 \qquad \text{for } m \in L \tag{4}$$

$$x_{ij} \geq 0 \qquad \text{for } (i, j) \in A \tag{5}$$

$$z_{mk} \in \{0,1\} \qquad \text{for } m \in L,\ k = 1,\ldots,K \tag{6}$$

where $\text{IN}_i = \{j \in N :\ (j,i) \in A\}$ and $\text{ON}_i = \{j \in N :\ (i,j) \in A\}$. Constraints (1)–(3) ensure the flow conservation at candidate location sites, generation sites and the remaining population centers, respectively. Constraints (4) restrict the number of facilities built at each candidate location to one. Constraints (5) and (6) ensure proper values of decision variables.

To evaluate the feasible solutions of the above constraint set several outcomes can be defined. R_i denotes individual perceived risk at population center i caused by shipment of hazardous waste transported through population center i. It is calculated as:

$$R_i = \sum_{j \in \text{IN}_i} x_{ji} \quad \text{for } i \in P \cup G \tag{7}$$

Similarly, E_i denotes individual perceived disutility at population center i caused by the operation of the treatment facilities. It is calculated as:

$$E_i = \sum_{m \in L} \frac{\sum_{k=1}^{K} C_k z_{mk}}{b_{im}} \quad \text{for } i \in P \cup G \tag{8}$$

where b_{im} is a distance of candidate location site m from population center i. Note that both R_i and E_i denote in fact risk or disutility perceived by each inhabitant of the population center i, so whenever outcomes are used, the weights w_i associated with population centers should be taken into account.

Additionally, the total cost of location and transportation is denoted by T and calculated as:

$$T = \sum_{m \in L} \sum_{k=1}^{K} F_{mk} z_{mk} + \sum_{(i,j) \in A} c_{ij} x_{ij} \tag{9}$$

3. Preference model

According to the problem formulation, a smaller value of the outcome (risk or disutility) means a better effect (higher service quality or client satisfaction). Therefore, without loss of generality, we can assume that each individual outcome is to be minimized, and the location-routing problem may be stated as the following multiple criteria minimization problem:

$$\min \{\mathbf{f}(\mathbf{x},\mathbf{z}) :\ (\mathbf{x},\mathbf{z}) \in Q\} \tag{10}$$

where Q is a feasible set of solutions defined by constraints (1)–(6) and $\mathbf{f} = (f_1,\ldots,f_m)$ is a vector of the individual objective functions which measure the outcome (effect) $y_i = f_i(\mathbf{x},\mathbf{z})$ of the location-routing pattern $(\mathbf{x},\mathbf{z})$ for city i.

Recall now the weights $w_i > 0$ defined in the model to represent the population. Integer weights can be directly interpreted as numbers of unweighted individual clients located at exactly the same place (with distances 0 among them). Theoretically, one may consider that the weighted problem is transformed (dis-aggregated) to the unweighted one (with all the population weights equal to 1). Such a dis-aggregation is possible for integer as well as rational weights, but it usually dramatically increases the problem size. Therefore, we consider solution concepts which can be applied directly to the weighted problem. Since the population weights describe the distribution of outcomes, we will use the normalized weights $\bar{w}_i = w_i / \sum_{i=1}^{m} w_i$ for $i = 1, \ldots, m$ rather than the original quantities w_i.

The simplest approach depends on minimization of the objective function representing the maximum (worst) outcome $M(\mathbf{y}) = \max_{i=1,\ldots,m} y_i$ and it is not affected by the population weights at all. A natural generalization of the maximum outcome $M(\mathbf{y})$ is the (worst) conditional mean outcome defined as the mean of the specified size (quantile) of the worst (largest) outcomes. For the simplest case of the unweighted problem one may distinguish the k largest outcomes (the k worst-off clients) and define the conditional mean outcome as the mean of the k distinguished outcomes. This can be mathematically formalized as follows. First, we introduce the ordering map $\Theta : R^m \rightarrow R^m$ such that $\Theta(\mathbf{y}) = (\theta_1(\mathbf{y}), \theta_2(\mathbf{y}), \ldots, \theta_m(\mathbf{y}))$, where $\theta_1(\mathbf{y}) \geq \theta_2(\mathbf{y}) \geq \cdots \geq \theta_m(\mathbf{y})$ and there exists a permutation τ of set I such that $\theta_i(\mathbf{y}) = y_{\tau(i)}$ for $i = 1, \ldots, m$. The *(worst) conditional* $\frac{k}{m}$*–mean outcome* $M_{\frac{k}{m}}(\mathbf{y})$ is given then as

$$M_{\frac{k}{m}}(\mathbf{y}) = \frac{1}{k} \sum_{i=1}^{k} \theta_i(\mathbf{y}), \quad \text{for } k = 1, \ldots, m. \tag{11}$$

For the special cases of $k = 1$ and $k = m$ one gets $M_{\frac{1}{m}}(\mathbf{y}) = \theta_1(\mathbf{y}) = M(\mathbf{y})$ and $M_{\frac{m}{m}}(\mathbf{y}) = \frac{1}{m} \sum_{i=1}^{m} \theta_i(\mathbf{y}) = \frac{1}{m} \sum_{i=1}^{m} y_i$, respectively, thus representing the classical criteria. It turns out that the conditional $\frac{k}{m}$–mean outcome $M_{\frac{k}{m}}(\mathbf{y})$ can be found by a simple linear programming minimization [2] and this formula can be generalized to any conditional β–mean [3]:

$$M_{\beta}(\mathbf{y}) = \min \{t + \frac{1}{\beta} \sum_{i=1}^{m} \bar{w}_i d_i : \; y_i \leq t + d_i, \; d_i \geq 0, \quad \text{for } i = 1, \ldots, m\}. \tag{12}$$

For better modeling of the preferences one may consider several conditional means for various levels β_k and the simplest way of their aggregation is a weighted sum with normalized (preference) weights v_k:

$$M^{v}(\mathbf{y}) = \sum_{k=1}^{r} v_k M_{\beta_k}(\mathbf{y}). \tag{13}$$

Note that the conditional $\frac{k}{m}$–means (11) themselves as well as their weighted aggregations represent the ordered weighted averages (OWA) [5], Our weighted aggregation (13) generalizes it to taking into account the population by allocating preference weights to various quantiles of the distribution of outcomes.

4. Example

We consider a hypothetical instance of the location and routing problem described above [1]. There are ten population centers not generating waste and three population centers generating waste. Treatment facilities of capacity 30, 50 or 80 can be open in five locations. For all other parameter values the reader is referred to [1].

In the experiment presented here we add constraints for the total cost $T \leq 1100$ and for each $i \in P \cup G$ $E_i \leq 75$. Thus, we restrict our multi-criteria analysis to only risk outcomes, i.e. $y_i = R_i$ for each $i \in P \cup G$.

$\beta_k^{(1)}$	$v_k^{(1)}$	$\beta_k^{(2)}$	$v_k^{(2)}$	$\beta_k^{(3)}$	$v_k^{(3)}$	$\beta_k^{(4)}$	$v_k^{(4)}$
10	9	10	9	1	10	1	90
25	40	25	80	100	90	100	10
50	50	50	10				
100	1	100	1				

Table 1. Parameters (percentage values)

i	$\bar{w}_i$ [%]	$R_i^{(1)}$	$R_i^{(2)}$	$R_i^{(3)}$	$R_i^{(4)}$
1	5	0	0	0	0
2	6.5	0	0	0	0
3	8.5	0	0	0	0
4	6	0	1.46	0	2.33
5	5	0	1.46	0	0
6	12.5	30	33.33	30	33.33
7	9	40	33.33	35	33.33
8	7	0	3.33	0	3.33
9	9	30	33.33	35	33.33
10	8	30	33.33	35	33.33
11	7.5	0	0	0	0
12	10	0	0	0	0
13	6	0	0	0	0
	$\max R_i$	40	33.33	35	33.33
	M_{β_1}	39	33.33	35	33.33
	M_{β_2}	33.6	26.26	12.85	13.21
	M_{β_3}	24.9	26.26		
	M_{β_4}	12.45	13.23		
	mean R_i	12.45	13.23	12.85	13.21
	M^v	29.52	32.43	15.07	31.32

Table 2. Experiment results.

Table 1 shows parameter values assumed for our tests. In the first two cases the same quantile values β_k are taken but different weights v_k are applied. In the last two cases parameter values are chosen in such a way, that the preference model is in fact weighting the worst outcome (average of 1% worst outcomes) and the mean outcome. Table 2 summarizes the test results. With a restricted total cost T and individual perceived disutilities E_i different routing schemes are chosen for each set of parameter values, resulting in different values of risk R_i.

The same set of facilities is chosen in each solution: facility capacities 50, 80, 30, 50, 30 are assumed for candidate locations 14 to 18, respectively. This is probably due to the fact, that the choice of locations is mainly affected by optimization of disutilities E_i, which we only restricted from above.

5. Conclusions and further work

While modeling a transportation system (location and routing decisions) the distribution of effects among the population is an important issue and some fairness or equity rules must be respected. We analyze the use of the conditional means for several quantiles and their OWA type aggregation. Simple initial experiments shows that the usage of only few quantile levels allows us to generate various routing strategies corresponding to risk distribution preferences. One may expect even better controllability of the search process when using the reference point methodology rather than the weighting aggregation to the multiple conditional means. Such an interactive approach with aspiration and reservation levels defined for several conditional means is currently being studied.

Although the conditional means formula can be represented by auxiliary linear constraint, for larger real-life problems a search for specialized optimization algorithms customized to this criteria seems to be necessary.

References

[1] I. Giannikos. A multiobjective programming model for locating treatment sites and routing hazardous wastes. *European Journal of Operational Research*, 104:333–342, 1998.

[2] W. Ogryczak and A. Tamir. Minimizing the sum of the k largest functions in linear time. *Information Processing Letters*, 85:117–122, 2003.

[3] W. Ogryczak and M. Zawadzki. Conditional median: A parametric solution concept for location problems. *Annals of Operations Research*, 110:167–181, 2002.

[4] J. Rawls. *The Theory of Justice*. Harvard University Press, 1971.

[5] R. R. Yager. On ordered weighted averaging aggregation operators in multicriteria decision making. *IEEE Transactions on Systems, Man and Cybernetics*, 18:183–190, 1988.