



## On Multiple Criteria Decision Support for Suppliers on the Competitive Electric Power Market

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**Abstract.** As an active participant of a competitive energy market, the generator (the energy supplier) challenges new management decisions being exposed to the financial risk environment. There is a strong need for the decision support models and tools for energy market participants. This paper shows that the stochastic short-term planning model can be effectively used as a key analytical tool within the decision support process for relatively small energy suppliers (price-takers). A self-scheduling method for the thermal units on the energy market is addressed. A schedule acquired for given preferences can be used as a desired pattern for bidding process. The uncertainty of the market prices is modeled by a set of possible scenarios with assigned probabilities. Several risk criteria are introduced leading to a multiple criteria optimization problem. The risk criteria are well appealing and easily computable (by means of linear programming) but they meet the formal risk aversion standards. The aspiration/reservation based interactive analysis applied to the multiple criteria problem allows us to find an efficient solution (generation scheme) well adjusted to the generator preferences (risk attitude).

**Keywords:** energy market, decision support, risk, multiple criteria optimization, short-term planning, unit commitment

### Introduction

Very complex systems are often beyond efficient direct control and management. Therefore, in many countries various large scale technical systems, such as electric power industry, or telecommunication sector, are undergoing unprecedented changes related to deregulations. The trends are toward deregulating whole industries, that have traditionally been a regulated monopoly, in order to allow for economic competition. On the technical site, a whole industry sector, such as the electric power industry, is a large, hierarchically coordinated system which has to provide various services to customers and meet strict global performance objectives (power demand, frequency, voltage levels, safety of delivery) and which has to manage various system-wide resources required to assure proper operation of the system. These resources may belong to independent subsystems, which may be defined according to administrative divisions among particular companies.

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Under deregulation, the elements of the system are undergoing drastic restructuring and transformation from cost-conscious, regulated utilities to competitive market participants. In the deregulated framework for control and management, instead of operating according to central rules and plans established by a hierarchical control structure in a centralized system, the system operates through cooperative behavior of many interactive subsystems which are organized as the competitive market participants. The subsystems may have their own independent interests, values, different tasks, operations and services.

The market mechanisms can be implemented through pool or bilateral arrangements. A pool facilitates the market by providing a forum for matching supply and demand, where all transactions are cleared at the same price. These prices may be obtained through some sort of iterative bidding process. In a bilateral market, supply and demand are matched through individual contracts between buyer and seller, and each transaction is cleared at a different price based on details agreed in the contract. A typical electric power system may consist of one or many power pools that are operated at the same level of hierarchy. Within each power pool the lower level of hierarchy may be formed by independent utilities. Each utility owns some resources, such as generators (generating units) or transmission lines, it may enjoy some operating autonomy and may become a competitive market participant. In this paper we focus on a generator (manager of a power generation utility) as an active participant of the energy markets.

Bidding and clearing process is still a new phenomenon for the energy market participants. Apart from the return maximization or the risk reduction, their operational decisions must take into account various other objectives. A generator is then enforced to serve as a decision maker (DM) dealing with new goals and decision processes as well as new types of necessary information [16,38]. Under deregulation, the key obligation and responsibility of obtaining an optimal schedule of units, satisfying all the plants technical requirements (unit commitment) is transferred from the Independent System Operator to individual generators (decentralized unit commitment). The change in planning process enforces the generators to take into account various risk factors [39]. There is a strong need for a decision support system (DSS) dedicated to electricity market participants. An appropriate system should offer all the necessary functionality to support decision processes in several energy market segments: long-term production planning and contracts administration, as well as short-term planning and spot market bids preparation. Furthermore, the system should provide solutions consistent with the generator's preferences and risk attitude.

Various formal methods were introduced for strategic decisions related to the entire power system including priority lists, dynamic programming, integer programming (branch and bound methods), Benders decomposition, Lagrangian relaxation [6,8,12,24,30,31,37,40]. Methods of stochastic optimization and multiple criteria programming were applied to power generation planning with both supply-side [3] and demand-side management [28]. Especially, decisions related to the environmental impact of the power production were analyzed with the use of multiple criteria optimization and related decision support techniques [15,50]. On the other hand, the single generator decisions

considered for a long horizon are hard to formal modeling and effective optimization for the sake of their mostly qualitative attributes as well as a strong uncertainty factor.

In the recent years various methods of scheduling and bidding in a competitive market have been addressed. The main goal of the research is to provide methods maximizing suppliers profit from selling energy in the spot market and through bilateral contracts. Paper [4] proposes mixed-integer linear programming approach that allows a rigorous modeling of thermal unit technical constraints. Methods for price risk management in hydropower systems are detailed in [12,29]. Decision analysis tools for genco dispatchers are analyzed in [39]. Paper [17] presents method for solving the unit commitment problem by simulation of a competitive market. Procedures for bidding and market clearing are described under the assumption of perfect competition. Discussion on long-term project valuation and power portfolio management in a competitive market is presented in [42].

Self-scheduling and bidding strategies were analyzed under various specific modeling assumptions like, availability of the complete information about competitors [26], the limited number of suppliers/buyers [23,47], the linear form of the supply function [5,43] and many others [13,29,35,41,53]. One of the crucial assumption is related to the price making capability of a single supplier. Typical game theory approaches [10,11] cover price-making suppliers. However, while dealing with a perfectly competitive market of relatively small suppliers one may accept the price-taker assumption. This assumption allows one to simplify significantly the model. In particular, from the perspective of a producer who does not influence significantly the market prices, the prices themselves can be modeled as uncertain quantities with the scenarios approach. Under the price-taker assumption self-scheduling problem can be decomposed by thermal units. But in some circumstances, units cannot be considered independently, because of generation and transmission constraints relating commitment decisions for several units.

High volatility of energy market forces participants to be able to make tactical and operational decisions repeatedly in a short time under risk. Mathematical models and formal methods are necessary to achieve satisfactory results on time. Therefore, while designing models and algorithms for a DSS to be used by a single generator, we especially focus on the short-term planning problems for the energy market. Other markets, e.g., ancillary service market are not addressed. We also assume the supplier is a price-taker, small producer with few generators, which are technically related. The paper shows that the stochastic short-term planning model can be effectively used as a key analytical tool within the decision support process concerning self scheduling task. As a result of presented model optimization, one obtains the best generation schedule in the meaning of provided preferences. The schedule can be used as a basis for the construction of a bidding strategy. It is a complex and hardly formalized process, although there are some initial results in its formalization [46]. Therefore, we leave this issue out of the scope of our paper assuming that the schedule is used by the decision maker (analyst) to develop the bidding strategy. The latter assumption is consistent with the basic decision support paradigm of avoiding any attempts to replace a decision maker with any (stiff) automatic optimization tool [50].

There are two basic short-term planning decision problems: unit commitment and generation dispatch. Unit commitment is the process of deciding when and which generating units to start-up and to shut-down [1]. Generation dispatch is the process of deciding what the individual power outputs should be of the scheduled generating units at each time point. The short-term self-scheduling planning model, we consider, covers both types of decision processes simultaneously. One needs to find out the start-up and shut-down schedule as well as the hourly production of each unit. Nevertheless, such a deterministic planning model can be applied only to search for an optimal schedule of the units operations when the final results of the bidding process are known and the generator needs to maximize profit while providing a reliable supply of the agreed load and fulfilling all the technical constraints. To make the planning model capable to support operational decisions including the bids preparation process, uncertainty of market prices is introduced to the model thus transforming it into a stochastic one. The uncertainty is modeled by a set of possible scenarios with assigned probabilities which allow us to build several risk criteria leading to a multiple criteria optimization problem. The aspiration/reservation based interactive analysis is applied to the multiple criteria problem thus allowing to find an efficient solution (generation schedule) well adjusted to the generator preferences (risk attitude). This “optimal” generation schedule may be used while dealing with operational decisions.

The paper is organized as follows. In the next section the short-term planning model is formulated. The technical constraints result in a mixed-integer linear program while the market price uncertainty leads to the stochastic objective function. Section 3 introduces the risk measures to be used as multiple criteria for modeling the generator’s risk attitude. Apart from typical dispersion measures some extreme events risk measures are available. In section 4 the aspiration/reservation based interactive technique to handle multiple criteria is described. It is followed by an illustrative example analyzed in section 5. Finally, in section 6 we outline the structure of the entire DSS, we are developing, for active participants of the energy market.

## **1. Short-term planning model**

The short-term planning and scheduling problems are considered for a horizon corresponding to a few cycles of the market auction (typically a few days). The generator seeks an optimal schedule of the units operation while the decisions on the units operation may strongly affect the financial result (the return). The main scheduling decisions are related to the units commitment in a sense that it must be decided when and which generating units to start-up and to shut-down. Opposite to the traditional unit commitment problem where the generator has no option beside of providing a reliable supply of the required load, the decentralized unit commitment decisions assume the generator to be responsible for meeting all the technical constraints. The failure of completing of any technical requirement may result in infeasible schedules and severe financial losses. Hence, in the planning model they are implemented as stiff constraints for the optimization process [16].

The unit commitment problem can be modeled as a mixed-integer linear program [9]. It covers some number of generation units  $j \in \mathcal{J}$  scheduled for several time units (hours)  $h \in \mathcal{H}$  within the horizon. To model the main scheduling (commitment) decisions we introduce two set of binary (integer 0/1) variables  $v_{jh}$  and  $r_{jh}$  defined for each generation unit and for each hour (time unit) within the horizon:

$$v_{jh} = \begin{cases} 1, & \text{if unit } j \text{ is committed at hour } h, \\ 0, & \text{otherwise,} \end{cases}$$

$$r_{jh} = \begin{cases} 1, & \text{if unit } j \text{ is concluded start-up at hour } h, \\ 0, & \text{otherwise.} \end{cases}$$

To preserve the logic of running and start-up changes the binary variables have to satisfy the following constraints:

$$v_{jh} - v_{j,h-1} \leq r_{jh} \leq v_{jh} \quad \forall j, h, \quad (1)$$

$$v_{j,h-1} + \sum_{t=0}^{T_j} r_{j,h+t} \leq 1 \quad \forall j, h, \quad (2)$$

$$\sum_{h=0}^{T_j^0} r_{jh} = 0 \quad \forall j, h, \quad (3)$$

where  $T_j$  – start-up time of unit  $j$ ,  $T_j^0$  – the earliest time when unit  $j$  can be committed.

Inequalities (1) guarantee that  $v_{jh} - v_{j,h-1} = 1$  (unit  $j$  committed at hour  $h$  while not being committed at hour  $h - 1$ ) implies  $r_{jh} = 1$  (start-up of unit  $j$  concluded at hour  $h$ ), and  $r_{jh} = 1$  implies  $v_{jh} = 1$  (unit  $j$  is committed at hour  $h$ ). Constraints (2) enforce the minimum start-up time while inequalities (3) define the initial conditions by introducing the earliest time after the beginning of the planning horizon, when unit  $j$  can be committed. Inequalities (1)–(3) represent only basic unit commitment constraints [16] but the discrete variables enable easy modeling of various additional requirements. For instance, the requirement that unit 1 and unit 2 are allowed to work only when at least one of units 3 or 4 is working can be modeled with the following inequality [51]:  $v_{1,h} + v_{2,h} \leq 2v_{3,h} + 2v_{4,h}$ .

To model the generation dispatch decisions, we introduce (continuous) variables  $P_{jh}$  representing power output of unit  $j$  at hour  $h$ . The power output (if not equal to 0) need to be always between the minimum and the maximum power output of the corresponding unit. This is enforced by the constraints:

$$P_j^l v_{jh} \leq P_{jh} \leq P_j^u v_{jh} \quad \forall j, h, \quad (4)$$

where  $P_j^u$  – maximum power output of unit  $j$ ,  $P_j^l$  – minimum power output of unit  $j$ .

Again, inequalities (4) represent only the simplest constraints related to the generating units characteristics. Nevertheless, they can be extended to take into account more specific generation characteristics.

Finally, we need to model the generation costs corresponding to various generation schedules. The generation costs are defined by the following quantities:

- $K_0$  – overall fixed cost of the generator;
- $b_j$  – start-up cost of unit  $j$ ;
- $K_{jh}$  – (variable) generation cost of unit  $j$  at hour  $h$ .

While quantities  $K_0$  and  $b_j$  represent some constant data, the generation cost  $K_{jh}$  is a variable representing the cost depending on the scheduled generation level at unit  $j$ . That means,  $K_{jh} = K_j(P_{jh})$  where  $K_j$  is a function representing variable generation costs of unit  $j$ . We assume piecewise linear convex functions:

$$K_j(P_{jh}) = \begin{cases} 0, & \text{if } P_{jh} = 0, \\ A_j^k P_{jh} + B_j^k, & \text{if } P_{jh} \in I^k, \end{cases}$$

where  $A_j^k$  – slope of the  $k$ th linear segment of variable cost,  $B_j^k$  – intercept of the  $k$ th linear segment of variable cost,  $I^k$  – power output interval for the  $k$ th linear segment.

Due to its convexity, the variable cost function can be expressed as

$$K_j(P_{jh}) = \max_k \{A_j^k P_{jh} + B_j^k v_{jh}\}.$$

Hence, under the natural assumption on the cost minimization, the variable generation cost  $K_{jh}$  can be defined in the model by the following inequalities:

$$K_{jh} \geq A_j^k P_{jh} + B_j^k v_{jh} \quad \forall j, k, h. \quad (5)$$

While we have introduced constant start-up cost, one may easily consider start-up cost as function of the boiler temperature. Our model can be also extended to consider various costs for cold, warm and hot start-ups, respectively.

For better modeling of thermal units, nonconvex (piecewise linear) cost functions may be introduced by the use of some auxiliary binary variables [4,51], thus preserving the mixed integer structure of the entire model.

When the auction cycle is completed and the market-clearing prices are known it is possible to calculate the total return of the generator implementing a specific generation schedule:

$$z = \sum_h \sum_j (c_h P_{jh} - K_{jh} - b_j r_{jh}) - K_0, \quad (6)$$

where  $z$  – variable representing the total return,  $c_h$  – market-clearing price at hour  $h$ .

Formula (6) represents the return from selling the generated power over the entire planning horizon at market-clearing prices. It includes revenues from selling reduced by the costs of production and start-ups.

Formula (6) can be applied to search for an optimal schedule of the units operations when the final results of the bidding process are known and the generator needs to maximize profit while providing a reliable supply of the agreed power supply (load) and fulfilling all the technical constraints. The corresponding optimization problem depends

on maximization of  $z$  subject to generation constraints (1)–(5) and the load requirements which in the simplest form can be written as

$$\sum_j P_{jh} = D_h, \quad (7)$$

where  $D_h$  denotes the required power load at hour  $h$ . Such a short term planning model is a Linear Programming (LP) problem including some integer decision variables (Mixed-Integer LP). When considered for the entire power system it leads to a large-scale problems requiring special techniques [6] to be solved. However, the problems related to a single generator managing several generation units can be effectively solved with a standard general purpose Mixed-Integer LP solvers.

The short-term planning model, formulated above, is based on maximization of the overall return  $z$  as a function of the generation decisions. All the model parameters (data) have been assumed to be known in advance. In particular, the energy prices have been assumed known leading to the deterministic return. In the decision process, we consider, all the data are related to the future which causes their uncertainty. Uncertainty of the energy prices is crucial while supporting decisions of a market participant since it directly introduces the risk factor into the return measurement. Therefore, we suggest, the stochastic planning model with uncertain energy prices as an optimization kernel of the decision support system.

We use the scenario analysis approach [3,36] for incorporating uncertainty into the model. One can consider a set  $S$  of possible energy price scenarios. Each scenario  $s \in S$  has assigned the weight  $p_s$  that reflects the probability of its occurrence. Hence, the overall return is a discrete random variable  $Z$  defined by its realizations  $z_s$  under several scenarios  $s \in S$ . That means  $z_s$  represents the overall return under a given scenario of energy prices. It is a function of the generation decisions given by the deterministic return formula (6) with the price coefficients defined according to the specific scenario  $s$ , i.e.,

$$z_s = \sum_h \sum_j (c_h^s P_{jh} - K_{jh} - b_j r_{jh}) - K_0 \quad \forall s \in S, \quad (8)$$

where  $c_h^s$  denotes the energy price in hour  $h$  under scenario  $s$  while all the other parameters are given as in the deterministic model.

Although the number of all price scenarios can be potentially extremely huge, in practice, a limited number of scenarios can be used as a representative set [29]. In most of historical demand scenarios there are time zones, where demand trends are similar, e.g., peak demand, increasing demand curve zone. We can even group all hours in a day into few demand time zones, so that the number of reasonably and remarkably different demand scenarios can be reduced. Moreover, when experts deal with scenarios, they consider rather limited number of demand scenarios.

## 2. Risk and criteria

A common approach to optimize uncertain return is to focus on its expected value (the mean). In the case of our scenario analysis the expected return

$$\bar{z} = \mathbb{E}\{Z\} = \sum_{s \in S} z_s p_s \quad (9)$$

can easily be used as a combined optimization criterion. Such a simple deterministic equivalent of the stochastic decision problem melds scenarios corresponding to all degrees of return (or loss) and probability of occurrence. Due to the nature of mathematical averaging the mean value criterion equally treats a guaranteed return as well as a lottery of possible high losses or high returns resulting in the same expectation. In other words, the mean value criterion (9) itself is not capable to model typical risk attitudes.

Following the seminal work by Markowitz [27], the problem of optimization under risk is modeled as a mean–risk bicriteria optimization problem where the mean  $\bar{z}$  is maximized and some risk measure  $\rho(Z)$  is minimized. In the original Markowitz model [27] the risk is measured by the standard deviation or variance:  $\sigma^2(Z) = \mathbb{E}\{(\bar{z} - Z)^2\}$ . Several other risk measures have been later considered thus creating the entire family of mean–risk (Markowitz-type) models. While the original Markowitz model forms a quadratic programming problem, many attempts have been made to linearize the optimization procedure (cf. [44] and references therein). The LP solvability is very important for our application where the feasible set of generation decisions is defined by the mixed integer LP constraints (1)–(5).

The mean–variance model is frequently criticized as not consistent with axiomatic models of preferences for choice under risk. Namely, except for the case of returns meeting the multivariate normal distribution, the mean–variance model may lead to inferior conclusions with respect to the stochastic dominance order. The concept of stochastic dominance order [48] is based on an axiomatic model of risk-averse preferences. In stochastic dominance, uncertain returns (random variables) are compared by pointwise comparison of functions constructed from their distribution functions. The first function  $F_Z^{(1)}$  is given as the right-continuous cumulative distribution function of the rate of return  $F_Z^{(1)}(\eta) = F_Z(\eta) = \mathbb{P}\{Z \leq \eta\}$ . The second function is derived from the first as

$$F_Z^{(2)}(\eta) = \int_{-\infty}^{\eta} F_Z(\xi) d\xi \quad \text{for real numbers } \eta,$$

and defines the (weak) relation of *second degree stochastic dominance* (SSD)

$$Z' \succeq_{\text{SSD}} Z'' \iff F_{Z'}^{(2)}(\eta) \leq F_{Z''}^{(2)}(\eta) \quad \text{for all } \eta.$$

If  $Z' \succeq_{\text{SSD}} Z''$ , then  $Z'$  is preferred to  $Z''$  within all risk-averse preference models where larger outcomes are preferred. It is therefore a matter of primary importance that a model for the uncertain return optimization be consistent with the SSD relation, in the sense that  $Z' \succeq_{\text{SSD}} Z''$  implies that the performance measure of  $Z'$  takes a value not worse than that of  $Z''$ .



Function  $F_Z^{(2)}$  used to define the SSD relation, can also be presented as follows [33]:

$$F_Z^{(2)}(\eta) = \mathbb{P}\{Z \leq \eta\} \mathbb{E}\{\eta - Z | Z \leq \eta\} = \mathbb{E}\{\max\{\eta - Z, 0\}\} \quad (10)$$

thus expressing the mean below-target deviations (the expected shortages) for each target return  $\eta$ . Hence, the SSD relation can be seen as a multidimensional (continuum-dimensional) risk measurement scheme. It is consistent with a very intuitive understanding of the notion of risk as a possible failure of achieving some targets.

The simplest scalar risk measure induced by the stochastic dominance is the mean below-target deviation for the specific target value  $\tau$ :

$$\bar{\delta}_\tau(Z) = \mathbb{E}\{\max\{\tau - Z, 0\}\} = F_Z^{(2)}(\tau). \quad (11)$$

In the case of returns represented by their realizations under several scenarios, as we deal with, the mean below-target deviation is a convex piecewise linear function of realizations  $z_s$ :  $\bar{\delta}_\tau(Z) = \sum_{s \in S} \max\{\tau - z_s, 0\} p_s$ . Hence, for any target  $\tau$ , the mean below-target deviation is LP computable with respect to values  $z_s$ .

The mean below-target deviations are very useful for decisions with clearly defined minimum acceptable returns. In the energy planning problem, we consider, such a critical target is defined by the zero return level. We use the mean below-zero deviation

$$\bar{\delta}_0(Z) = \mathbb{E}\{\max\{-Z, 0\}\} \quad (12)$$

as a risk criterion expressing the mean loss.

Certainly one may consider some other specified return levels as possible targets to define risk criteria with the corresponding mean below-target deviations. Alternatively, when the expected return is already used as a performance measure, then one may consider extending the concept of shortage by using the mean itself as a target. This results in the risk measure known as the downside mean semideviation from the mean

$$\bar{\delta}(Z) = \mathbb{E}\{\max\{\bar{z} - Z, 0\}\} = F_Z^{(2)}(\bar{z}). \quad (13)$$

The downside mean semideviation from the mean is always equal to the upside one [33] ( $\mathbb{E}\{\max\{\bar{z} - Z, 0\}\} = \mathbb{E}\{\max\{Z - \bar{z}, 0\}\}$ ) and we will call it simply the mean semideviation. Actually, the mean semideviation is a half of the mean absolute deviation from the mean (the MAD measure),  $\bar{\delta}(Z) = (1/2)\mathbb{E}\{|\bar{z} - Z|\}$ . Hence, the corresponding mean-risk approach is equivalent to the so-called MAD model [21] which is an LP computable Markowitz-type model. For returns given with a discrete random variable represented by its realizations  $z_s$ , the mean semideviation (13) is LP computable.

The expected value of risk does not accentuate the extreme events and their consequences, thus misrepresenting what would have been a perceived unacceptable risk [14]. The expected value of shortage to a specific target or to the mean return when used as a risk measure assumes the decision maker to have a constant trade-off for a unit deviation from the target. This assumption does not allow for the distinction of risk associated with larger losses. Therefore, we need to introduce additional risk criteria to emphasize consequences of more pessimistic scenarios [14].

For a discrete random variable  $Z$  represented by its realizations  $z_s$ , the worst realization

$$M(Z) = \min_{s \in S} z_s \quad (14)$$

is a well-appealing extreme scenario performance measure. Maximization of the worst realization corresponds to a common approach to multiple deterministic outcomes. The so-called maximin (or minimax) approaches are crucial solution concepts in multiple criteria optimization [45]. Recently, the measure  $M(z)$  was applied to portfolio optimization [52]. The worst realization although expressing the notion of risk is not a typical (dispersion type) risk measure as its larger values are preferred. The maximum (downside) semideviation  $\Delta(Z) = \bar{z} - M(Z) = \max_{s \in S} (\bar{z} - z_s)$  may be considered the corresponding (dispersion) risk measure. The worst realization can easily be introduced to our decision model as additional risk criterion  $\underline{z} = M(Z)$  defined by the LP computable formula:

$$\underline{z} = \max y \quad \text{s.t.} \quad y \leq z_s, \quad \text{for } s \in S, \quad (15)$$

where  $y$  is an auxiliary (unbounded) variable.

A natural generalization of the measure  $M(Z)$  is the worst conditional expectation defined as the mean of the specified size (quantile) of worst realizations. For the simplest case of equally probable scenarios ( $p_s = 1/|S|$ ), one may define the worst conditional expectation  $M_{k/|S|}(Z)$  as the mean return under the  $k$  worst scenarios. In general, for any tolerance level  $0 < \beta \leq 1$  (replacing the quotient  $k/|S|$ ) the worst conditional expectation is defined as

$$M_\beta(Z) = \frac{1}{\beta} \int_0^\beta F_Z^{(-1)}(\alpha) d\alpha \quad \text{for } 0 < \beta \leq 1, \quad (16)$$

where  $F_Z^{(-1)}(p) = \inf\{\eta: F_Z(\eta) \geq p\}$  is the left-continuous inverse of the cumulative distribution function  $F_Z$ . Note that  $M_1(Z) = \mathbb{E}\{Z\}$  and  $M_\beta(Z)$  tends to  $M(Z)$  for  $\beta$  approaching 0.

By the theory of convex conjugent (dual) functions, the worst conditional expectation may be defined by optimization [32]:

$$M_\beta(Z) = \max_{\eta \in \mathbb{R}} \left( \eta - \frac{1}{\beta} F_Z^{(2)}(\eta) \right) = \max_{\eta \in \mathbb{R}} \left( \eta - \frac{1}{\beta} \mathbb{E}\{\max\{\eta - Z, 0\}\} \right), \quad (17)$$

where  $\eta$  is a real variable taking the value of  $\beta$ -quantile  $Q_\beta(Z) = F_Z^{(-1)}(\beta)$  at the optimum. For any  $0 < \beta \leq 1$  the conditional worst realization  $M_\beta(Z)$  is an SSD consistent measure. Actually, the conditional worst expectations provide an alternative characterization of the SSD relation [32]. Similar to the worst realization, the worst conditional expectation although expressing the notion of risk is not a typical (dispersion type) risk measure as its larger values are preferred. The conditional (downside) semideviation  $\Delta_\beta(Z) = \bar{z} - M_\beta(Z)$  may be considered the corresponding (dispersion type) risk measure.

While the value of  $\beta$ -quantile  $Q_\beta(Z) = F_Z^{(-1)}(\beta)$  is commonly called Value-at-Risk (VaR) measure, the worst conditional expectation is closely related to the so-called Conditional Value-at-Risk (CVaR) or Expected Shortfall which may be expressed as  $CVaR_\beta(Z) = \mathbb{E}\{Z|Z \leq Q_\beta(Z)\}$ . Exactly,  $M_\beta(Z) = CVaR_\beta(Z)$  in the case of continuous distributions of returns, while they can take different values for discrete distributions. For instance, with returns  $Z$  given as

$$\mathbb{P}\{Z = \xi\} = \begin{cases} 0.03, & \xi = -10, \\ 0.05, & \xi = -4, \\ 0.90, & \xi = 10, \\ 0.02, & \xi = 25, \\ 0, & \text{otherwise,} \end{cases}$$

for tolerance level  $\beta = 0.05$  we get  $M_{0.05}(Z) = (-10 \cdot 0.03 - 4 \cdot 0.02)/0.05 = -7.6$  while  $Q_{0.05}(Z) = -4$  and  $CVaR_{0.05}(Z) = (-10 \cdot 0.03 - 4 \cdot 0.05)/0.08 = -6.26$ . Nevertheless, recently considered models for portfolio optimization [2] use formula (17) for the worst conditional expectation as a computational approximation to CVaR for continuous distributions. Therefore, the maximization of the worst conditional expectation we will refer to as the CVaR criterion optimization.

For a discrete random variable represented by its realizations  $z_s$ , problem (17) becomes an LP. This allows us to extend the decision model with the CVaR risk criterion  $\underline{z}^\beta = M_\beta(Z)$  defined by the LP computable formula:

$$\underline{z}^\beta = \max\left(y^\beta - \frac{1}{\beta} \sum_{s \in S} z_s^\beta p_s\right) \quad \text{s.t.} \quad y^\beta - z_s^\beta \leq z_s, \quad z_s^\beta \geq 0 \quad \text{for } s \in S, \quad (18)$$

where  $y^\beta$  is an auxiliary (unbounded) variable.

Although well defined for any  $0 < \beta \leq 1$ , the CVaR criterion with a relatively small value of the tolerance level  $\beta$  is interesting as a potential measure of extreme risk. In our system we use the tolerance level 0.05 to define the CVaR criterion. Certainly one may consider a different value or to introduce a few CVaR criteria related to several tolerance levels.

Finally, for our stochastic power generation decision problem we are able to formulate a deterministic equivalent which is based on multiple criteria optimization of the following performance measures:

- the mean return to be maximized;
- the mean loss to be minimized;
- the mean semideviation below the mean return to be minimized;
- the worst return realization to be maximized;
- the CVaR criterion to be maximized.

While the first criterion is focused on the expected return maximization, all other four criteria are risk related. The risk criteria represent quite different risk measures and, therefore, they allow us to model various risk attitudes of a DM.

In order to keep the multiple criteria model consistent we unify all the criteria to be maximized. Instead of the mean loss  $\bar{\delta}_0(Z)$  minimization we will rather maximize its complement criterion

$$z^- = 0 - \bar{\delta}_0(Z) = \mathbb{E}\{\min\{Z, 0\}\} \quad (19)$$

expressing the mean loss-effected underachievement. In the scenario analysis approach the latter is LP computable as

$$z^- = \max\left(-\sum_{s \in S} z_s^- p_s\right) \quad \text{subject to} \quad z_s^- \geq -z_s, \quad z_s^- \geq 0 \quad \text{for } s \in S. \quad (20)$$

Similarly, we replace the mean semideviation  $\bar{\delta}(Z)$  minimization with maximization of its complement criterion

$$\bar{z}^u = \bar{z} - \bar{\delta}(Z) = \mathbb{E}\{\min\{Z, \bar{z}\}\} \quad (21)$$

expressing the mean below-mean underachievement. The latter is LP computable with respect to the realizations  $z_s$  as:

$$\bar{z}^u = \max \sum_{s \in S} z_s^u p_s \quad \text{subject to} \quad z_s^u \leq z_s, \quad z_s^u \leq \bar{z} \quad \text{for } s \in S. \quad (22)$$

With these modified criteria we get the multiple criteria maximization model

$$\max \{\mathbf{q} = (q_1, q_2, \dots, q_m) \mid \mathbf{q} \in \mathcal{Q}\}, \quad (23)$$

where  $q_1 = \bar{z}$ ,  $q_2, \dots, q_m$  represent the risk criteria, and  $\mathcal{Q}$  represents the set of attainable values when taking into account the generation constraints and the price scenarios. We essentially focus on the case of  $m = 5$  and  $q_2 = z^-$ ,  $q_3 = \bar{z}^u$ ,  $q_4 = \underline{z}$ ,  $q_5 = \underline{z}^\beta$ . However, some risk criteria may be not used in the model or one may consider more criteria, like a few CVaR criteria for various tolerance levels. Due to the use multiple risk criteria, model (23) provides tools for modeling various risk attitudes in connection with the overall return maximization. It is important to notice that the model preserves risk-averse preferences since it is consistent with the SSD relation. Exactly, from the properties of the risk criteria used in the model [32,33], the multiple criteria model (23) is SSD consistent in the sense that  $Z' \succeq_{\text{SSD}} Z''$  implies  $q'_i \geq q''_i$  for all  $i = 1, \dots, m$ .

Note that all the criteria used in model (23) are LP computable with respect the return realizations  $z_s$ . Hence, the multiple criteria model maintains the original structure of the deterministic planning model. Exactly, due to LP computable formulas (9), (20), (22), (15) and (18), the set  $\mathcal{Q}$  of attainable outcomes in model (23) can be expressed in the following form:

$$\begin{aligned} q_1 &= \sum_{s \in S} z_s p_s, \\ q_2 &\leq -\sum_{s \in S} z_s^- p_s, \end{aligned}$$

$$\begin{aligned}
 & z_s^- \geq -z_s, \quad z_s^- \geq 0, && \text{for } s \in S, \\
 q_3 & \leq \sum_{s \in S} z_s^u p_s, \\
 & z_s^u \leq z_s, \quad z_s^u \leq q_1, && \text{for } s \in S, \\
 q_4 & \leq z_s, && \text{for } s \in S, \\
 q_5 & \leq y^\beta - \frac{1}{\beta} \sum_{s \in S} z_s^\beta p_s, \\
 & y^\beta - z_s^\beta \leq z_s, \quad z_s^\beta \geq 0, && \text{for } s \in S, \\
 z_s & = \sum_h \sum_j (c_h^s P_{jh} - K_{jh} - b_j r_{jh}) - K_0, && \text{for } s \in S, \\
 & \text{and generation constraints (1)–(5).}
 \end{aligned}$$

### 3. Interactive multiple criteria analysis

In the previous section we formulated our decision problem under uncertainty as multiple criteria optimization model (23). In the model, an outcome vector  $\mathbf{q} = (q_1, q_2, \dots, q_m)$  evaluates a corresponding generation scheme with respect to the specified criteria of mean return and four risk measures. It is clear that an outcome vector is better than another if all of its individual outcomes are better or at least one individual outcome is better whereas no other one is worse. Such a relation is called domination of outcome vectors. Unfortunately, there usually does not exist an outcome vector that dominates all others with respect to all the criteria. Thus in terms of strict mathematical relations we cannot distinguish the best outcome vector. The nondominated vectors are incomparable on the basis of the specified set of criteria. The decisions that generate nondominated outcome vectors are called efficient or Pareto-optimal solutions to the multiple criteria problem.

In theory, one may consider a multiple criteria optimization as a problem depending on identification of the entire set of efficient solutions. We are interested, however, in an operational use of multiple criteria analysis as a DSS module to help the decision maker to select one efficient solution for implementation. Certainly, the original criteria do not allow one to select any efficient solution as better than any other one. Therefore, the decision support process must depend on additional preference information gained from the DM. This can be achieved with the so-called quasi-satisficing approach to multiple criteria decision problems. The best formalization of the quasi-satisficing approach to multiple criteria optimization was proposed and developed mainly by Wierzbicki [49] as the reference point method. The reference point method was later extended and, eventually, led to efficient implementations of the so-called aspiration/reservation based decision support (ARBDS) approach with many successful applications [25,50].

The ARBDS approach is an interactive technique. The basic concept of the interactive scheme is as follows. The DM specifies requirements in terms of aspiration and reservation levels, i.e., by introducing acceptable and required values for several crite-

ria. Depending on the specified aspiration and reservation levels, a special scalarizing achievement function is built which may be directly interpreted as expressing utility to be maximized. Maximization of the scalarizing achievement function generates an efficient solution to the multiple criteria problem. The computed efficient solution is presented to the DM as the current solution in a form that allows comparison with the previous ones and modification of the aspiration and reservation levels if necessary.

While building the scalarizing achievement function the following properties of the preference model are assumed. First of all, for any individual outcome  $q_i$  more is preferred to less (maximization). To meet this requirement the function must be strictly increasing with respect to each outcome. Second, a solution with all individual outcomes  $q_i$  satisfying the corresponding reservation levels is preferred to any solution with at least one individual outcome worse (smaller) than its reservation level. Next, provided that all the reservation levels are satisfied, a solution with all individual outcomes  $q_i$  equal to the corresponding aspiration levels is preferred to any solution with at least one individual outcome worse (smaller) than its aspiration level. That means, the scalarizing achievement function maximization must enforce reaching the reservation levels prior to further improving of criteria. In other words, the reservation levels represent some soft lower bounds on the maximized criteria. When all these lower bounds are satisfied, then the optimization process attempts to reach the aspiration levels. Thus, similar to the goal programming approaches [7], the aspiration levels are then treated as the targets but following the quasi-satisficing approach they are interpreted consistently with basic concepts of efficiency in the sense that the optimization is continued even when the target point has been reached already.

The generic scalarizing achievement function takes the following form [49]:

$$a(\mathbf{q}) = \min_{1 \leq i \leq m} \{a_i(q_i, q_i^a, q_i^r)\} + \varepsilon \sum_{i=1}^m a_i(q_i, q_i^a, q_i^r), \quad (24)$$

where  $\varepsilon$  is an arbitrary small positive number and  $a_i : \mathbb{R}^3 \rightarrow \mathbb{R}$ , for  $i = 1, 2, \dots, m$ , are the partial achievement functions measuring actual achievement of the individual outcome  $q_i$ , with respect to the corresponding aspiration and reservation levels ( $q_i^a$  and  $q_i^r$ , respectively). Thus the scalarizing achievement function is, essentially, defined by the worst partial (individual) achievement but additionally regularized with the sum of all partial achievements. The regularization term is introduced only to guarantee the solution efficiency in the case when the maximization of the main term (the worst partial achievement) results in a non-unique optimal solution.

The partial achievement function  $a_i$ , can be interpreted as a measure of the DM's satisfaction with the current value of outcome of the  $i$ th criterion. It is a strictly increasing function of outcome  $q_i$  with value  $a_i = 1$  if  $q_i = q_i^a$ , and  $a_i = 0$  for  $q_i = q_i^r$ . Thus the partial achievement functions map the outcomes values onto a normalized scale of the DM's satisfaction. Various functions can be built meeting those requirements [50].

We use the piecewise linear partial achievement function introduced in [34]. It is given by

$$a_i(q_i, q_i^a, q_i^r) = \begin{cases} \frac{\gamma(q_i - q_i^r)}{q_i^a - q_i^r}, & \text{for } q_i \leq q_i^r, \\ \frac{q_i - q_i^r}{q_i^a - q_i^r}, & \text{for } q_i^r < q_i < q_i^a, \\ \frac{\alpha(q_i - q_i^a)}{q_i^a - q_i^r} + 1, & \text{for } q_i \geq q_i^a, \end{cases} \quad (25)$$

where  $\alpha$  and  $\gamma$  are arbitrarily defined parameters satisfying  $0 < \alpha < 1 < \gamma$ . Parameter  $\alpha$  represents additional increase of the DM's satisfaction over level 1 when a criterion generates outcomes better than the corresponding aspiration level. On the other hand, parameter  $\gamma > 1$  represents dissatisfaction connected with outcomes worse than the reservation level.

For outcomes between the reservation and the aspiration levels, the partial achievement function  $a_i$ , can be interpreted as a membership function  $\mu_i$  for a fuzzy target [54]. However, such a membership function remains constant with value 1 for all outcomes greater than the corresponding aspiration level, and with value 0 for all outcomes below the reservation level (figure 1). Hence, the fuzzy membership function is neither strictly monotonic nor concave thus not representing typical utility for a maximized outcome. The partial achievement function (25) can be viewed as an extension of the fuzzy membership function to a strictly monotonic and concave utility. One may also notice that the aggregation scheme used to build the scalarizing achievement function (24) from the partial ones may also be interpreted as some fuzzy aggregation operator [50]. In other words, maximization of the scalarizing achievement function (24) is consistent with the fuzzy methodology in the case of not attainable aspiration levels and satisfiable all reservation levels while modeling a reasonable utility for any values of aspiration and reservation levels.

Under the assumption that the parameters  $\alpha$  and  $\gamma$  satisfy inequalities  $0 < \alpha < 1 < \gamma$ , partial achievement function (25) is strictly increasing and concave. Hence, it

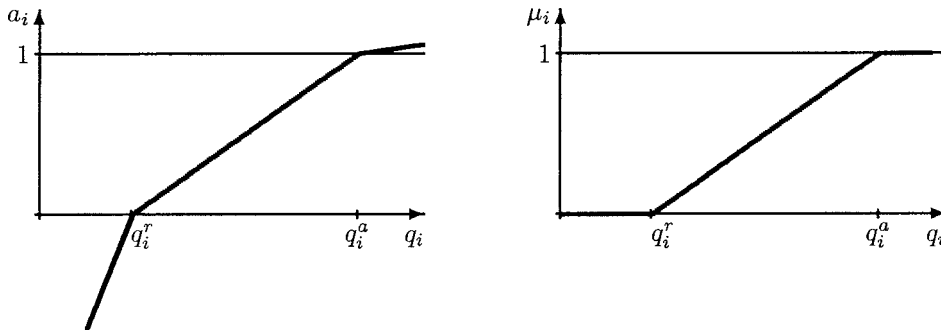


Figure 1. Partial achievement function  $a_i$  and corresponding fuzzy membership function  $\mu_i$ .

can be expressed in the form:

$$a_i(q_i, q_i^a, q_i^r) = \min \left\{ \gamma \frac{q_i - q_i^r}{q_i^a - q_i^r}, \frac{q_i^a - q_i^r}{q_i^a - q_i^r}, \alpha \frac{q_i - q_i^a}{q_i^a - q_i^r} + 1 \right\}$$

which guarantees LP computability with respect to outcomes  $q_i$ . Finally, maximization of the entire scalarizing achievement function (24) can be implemented by the following auxiliary LP constraints:

$$\begin{aligned} \max \quad & \underline{a} + \varepsilon \sum_{i=1}^m a_i \\ \text{s.t.} \quad & a_i \geq \underline{a}, & \text{for } i = 1, \dots, m, \\ & a_i \leq \gamma \frac{q_i - q_i^r}{q_i^a - q_i^r}, & \text{for } i = 1, \dots, m, \\ & a_i \leq \frac{q_i - q_i^r}{q_i^a - q_i^r}, & \text{for } i = 1, \dots, m, \\ & a_i \leq \alpha \frac{q_i - q_i^a}{q_i^a - q_i^r} + 1, & \text{for } i = 1, \dots, m, \end{aligned}$$

where  $a_i$  for  $i = 1, \dots, m$  and  $\underline{a}$  are unbounded variables introduced to represent values of several partial achievement functions and their minimum, respectively.

#### 4. Illustrative example

In order to show an outline of the interactive multiple criteria analysis performed with the ARBDS methodology, we consider a small sample problem. The data for short-term planning model considered in this example are based on real-life characteristics of a generator owning four coal-fired 180 MW units. The variable cost function of each unit is considered to be convex and it is approximated by 3 linear pieces. The mean variable cost is about 10.75 USD/MWh and start-up cost is 875 USD (for simplicity no shut-down cost and constant start-up cost is considered). The units have quite elastic characteristics (minimum power output is 60 MW), and their minimum up time is 5 hours. At the beginning of the planning horizon all units have been off, but they could be committed immediately at hour 0.

The planning horizon lasts for 48 hours which corresponds to two cycles of the market auction. It is assumed that energy spot prices forecast over planning horizon consists of 100 equally probable different scenarios. The scenarios have been generated under the assumption that prices fluctuate over a day within specified probability distribution. The price forecast distribution profiles are selected that the minimum price is at least 8 USD (at night hours), the maximum price is at most 40 USD (at peak hours), and the daily average price over all the scenarios is 19 USD. Hourly values of prices are drawn at random from those distributions thus they may be considered to be correlated as they are in historical data. Besides, a few distinctive scenarios are modeled. Hence,



Table 1  
The pay-off table with the utopia and the nadir vectors.

Optimized criteria	Efficient solutions					
	$\bar{z}$	$z^-$	$\underline{z}$	$\underline{z}^\beta$	$\bar{z}^u$	
Mean return	$\bar{z}$	239 757	5 038	-88 630	-72 358	170 319
Mean loss	$z^-$	124 704	0	0	5 630	100 580
Worst return	$\underline{z}$	71 315	0	13 238	15 115	57 193
CVaR	$\underline{z}^\beta$	77 558	0	12 371	15 901	62 233
Underachievement	$\bar{z}^u$	233 406	3 147	-64 717	-50 436	174 410
Utopia		239 757	0	13 238	15 901	174 410
Nadir		71 315	5 038	-88 630	-72 358	57 193

we manage to limit strongly the set of scenarios while preserving its representative properties.

The generator is considered a DM who wants to schedule the units over the planning horizon, so that the energy can be sold to electricity market with the maximal return taking into account uncertain spot prices. We emphasize that the spot prices are uncertain at this moment since the auction process is still not completed. For this purpose, the multiple criteria discussed in section 2 are introduced. Actually, the mean return is maximized, the mean loss is minimized while the worst return realization, the CVaR (with tolerance level  $\beta = 5\%$ ) and the mean (below-mean) underachievement are maximized. The example allows us to show the interactive process of the multiple criteria analysis to identify the efficient solution, that meets the generator’s preferences the best.

Having defined all the generation units data and the spot prices scenarios, as the first step of multiple criteria analysis the DM may examine the so-called pay-off table (see table 1). It is computed by optimization of each criterion separately. Note that mean loss is the only criterion to be minimized while all the other are maximized. Exactly, the regularized criteria  $q_i + \varepsilon \sum_{j=1}^n q_j$  are optimized to guarantee efficiency of the computed solution as well as proper reporting of unoptimized criteria values (which were introduced with inequalities). The results form the pay-off table containing values of all the criteria (columns) obtained while optimizing several individual criteria (rows). The pay-off table is known to help to understand the conflicts between several criteria. It also provides the DM with two reference vectors: the utopia and the nadir. The utopia vector represents a collection of the best values of each criterion considered separately, and the nadir vector expresses the worst values of each outcome noticed during optimization of the other criteria. Coefficients of the utopia vector represent the best values of the corresponding criteria over the entire efficient set, and the utopia vector itself is usually not attainable. Coefficients of the nadir vector cannot be considered as the worst values of the criteria over the efficient set but they usually may serve as first estimates of those bounds.

While analyzing table 1 we find out that the outcome values vary significantly depending on selected optimization. One may get up to 239 757 USD mean return (when this criterion is maximized) but it is achieved with a very risky generation schedule which may result in losses of 88 630 USD under the worst scenario while the mean loss

Table 2  
An interactive search for a satisfying efficient solution.

Iteration		Mean return $\bar{z}$	Mean loss $z^-$	Worst return $\underline{z}$	CVaR $\underline{z}^\beta$	Below-mean underach. $\bar{z}^u$
1	Reservation level	150 000	2 500	-37 500	-25 000	112 500
	Aspiration level	250 000	0	13 250	16 250	175 000
	<b>Solution</b>	<b>181 689</b>	<b>596</b>	<b>-21 418</b>	<b>-11 928</b>	<b>142 009</b>
2	Reservation level	150 000	0	-2 500	0	112 500
	Aspiration level	250 000	-250	13 250	16 250	175 000
	<b>Solution</b>	<b>133 250</b>	<b>42</b>	<b>-2 680</b>	<b>3 330</b>	<b>107 438</b>
3	Reservation level	150 000	0	-2 500	0	125 000
	Aspiration level	250 000	250	13 250	16 250	175 000
	<b>Solution</b>	<b>137 375</b>	<b>70</b>	<b>-4 233</b>	<b>2 096</b>	<b>111 080</b>
4	Reservation level	100 000	0	750	7 500	87 500
	Aspiration level	137 500	-7 500	25 000	25 000	150 000
	<b>Solution</b>	<b>118 238</b>	<b>0</b>	<b>1 744</b>	<b>7 500</b>	<b>94 819</b>
5	Reservation level	112 500	0	500	4 250	100 000
	Aspiration level	137 500	-5 000	25 000	25 000	150 000
	<b>Solution</b>	<b>123 403</b>	<b>0</b>	<b>368</b>	<b>5 775</b>	<b>99 731</b>
6	Reservation level	125 000	0	250	2 000	112 500
	Aspiration level	200 000	-1 250	17 500	20 000	200 000
	<b>Solution</b>	<b>129 192</b>	<b>17</b>	<b>-1 380</b>	<b>4 442</b>	<b>104 229</b>

is 5 038 USD. On the other hand, while maximizing the worst return, the generator may get the guaranteed minimum 13 238 USD return but with the expected value of only 71 315 USD. One may easily notice that the mean return and the worst return realization are the most conflicting criteria. Actually, the mean return maximization remains in a conflict with all three extreme events risk criteria (the worst realization, the CVaR and the mean loss) while it turns out to be quite a consistent with the mean underachievement (representing some average events risk measure). Hence, we can distinguish two groups of criteria: the first consisting of the mean return and the mean underachievement, the second built of the worst realization, the CVaR and the mean loss. While a strong conflict characterizes criteria from different groups, the criteria within the same group seem to be quite consistent.

Having known the utopia vector one can start the interactive search for a satisfying efficient solution. The DM uses aspiration and reservation levels to control the interactive analysis. The analysis is reported in table 2. At the beginning of the interactive analysis the DM specifies the requirements starting from, in some sense reasonable neutral requirements, which means that the aspiration levels for several criteria are close to their utopia values and the reservation levels are set in the middle of attainable range (see table 2, iteration 1). The efficient solution obtained in this step is better than the specified reservation levels, with respect to all the criteria. The DM finds the solution as too risky, especially in terms of the extreme events risk criteria. Therefore, stronger reservation levels for those risk criteria are used in the next step. In the case of the mean

loss criterion the reservation level is set at the corresponding utopia level (0) while the aspiration level is advanced to an unattainable value ( $-250$ ). This results in an efficient solution (iteration 2) which is much more satisfying in terms of risk criteria but it is more than 20% worse with respect to the mean return. Actually, both the mean return and the mean underachievement are then below their reservation levels. Next, the DM makes an attempt to improve a solution by focusing more on the average events risk criterion. For this purpose, the reservation level for the mean underachievement is slightly increased. Solution obtained in the third iteration brings a few percent improvement on the mean return and the mean underachievement but significant worsening of the other criteria.

Further, the DM wants to examine consequences of assuring a safe schedule which can guarantee no losses to be suffered under any scenario. For this purpose, the aspiration and reservation levels of the extreme events risk criteria are much more strengthened while the ones corresponding to the mean return and the mean underachievement are relaxed. Note that in order to stress the importance of some criteria the DM may set levels which are not attainable. Iterations 4 and 5 results in solutions with the worst return equal to 1 744 USD and 368 USD, respectively. However, the mean return in both solutions is below 125 000 USD which makes the schedules not acceptable for our generator. Nevertheless, a strongly risk avert generator would probably select the solution from iteration 4 as a very safe schedule for implementation.

The DM makes a further attempt to find a solution assuring higher mean return, while not causing significant losses if more pessimistic scenario would occur. For this purpose, the parameters from iteration 5 are modified by strengthening the requirements on the mean return and the mean underachievement and by simultaneous slight relaxing of the requirements on the extreme events risk criteria (iteration 6). Now, again there are some losses that may occur, but the mean loss is only 17 USD. All the extreme events risk criteria take much better values than those in iterations 2 and 3 while the mean return and the mean underachievement are only about 5% worse. As the mean return exceeds a reasonably high level of 125 000 USD with relatively good risk measures, the generator finally accepts this result and concludes the interactive analysis.

The final selection of an efficient solution depends on the DM's preferences. Nevertheless, the example illustrates how the interactive analysis allows the DM to learn the decision problem and to search effectively for a satisfying solution. For the presented example covering 4 generation units scheduled over 48 hours under uncertainty described with 100 scenarios, the average computing time of each iteration is about 100 seconds on a Pentium III processor using the CPLEX optimization package [18]. While analyzing larger problems related to scheduling 12 generation units over a horizon of one week (168 hours), we have noticed the computing times approaching 10 minutes. It should be emphasized that the size of the problem depends mostly on the number of generation units and hours within considered horizon. The number of price scenarios has a relatively less impact on the resulting MILP problem size. Thus, the multiple criteria model, we consider, can be effectively used as an on-line interactive analysis tool within a DSS for the energy market participant.

## 5. The DSS concept

In a competitive electricity market there are the physical, financial and risk-management operations. On the physical site, the electricity is produced by generators and delivered from generators to customers through transmission and distribution facilities. Moreover, ancillary services are needed to assure reliable operation of the system. Such services include installed capability, operable capability, spinning and nonspinning reserves, automatic generation control (AGC), transmission services, etc.

On the financial site, the contracts and payments specify planned and actual generation and consumption, terms for payment for electrical energy, and financial issues of contracted services. The risk-management arrangements (such as futures markets and long-term contracts) are used to manage risk, and they may or may not have any relation to the physical delivery of electricity.

The deregulated power system may be operated and managed as an interactive network of the following different types of energy markets:

- forward energy market (Power Exchange), which may be in the form of a day-ahead energy auction (DAA);
- planned production market (PPM), a day-ahead (or hour-ahead) market for power generation plan which should meet the demand forecasts, systems resource constraints and system-wide performance objectives;
- real-time production market (RTM), the market for real-time power generation which meets the real demand for energy and assures safety delivery and others system-wide performance objectives;
- ancillary services markets (ASM), which provide appropriate level of systems services (installed and operable capabilities, AGC, spinning reserve, fast nonspinning reserve, slow reserve, transmission capacities).

The stochastic short-term planning model and its multiple criteria interactive analysis tool is used as a key analytic module of the I-enviser decision support system [19]. The system is developed for companies that operate within power production, physical power transactions and distribution in several energy market segments. The decision process supported by I-enviser may be divided into three main stages (figure 2):

1. *Data preparation.* In this initial stage all the necessary information and data are specified. Input data are gained from outer sources, prepared by the user, as well as acquired from other applications. The user needs to specify all technical parameters of the power production and network units but this customization is usually made once and rarely modified. The main responsibility of the user at this stage is to define what elements and in which way need to be included. In particular, it is important to select the outside forecasting software (and appropriate parameters) to generate representative scenarios and assign them appropriate probabilities.
2. *Strategic decision support.* In this stage decisions in the long-term or medium-term time horizon are supported. The following main activities are handled:

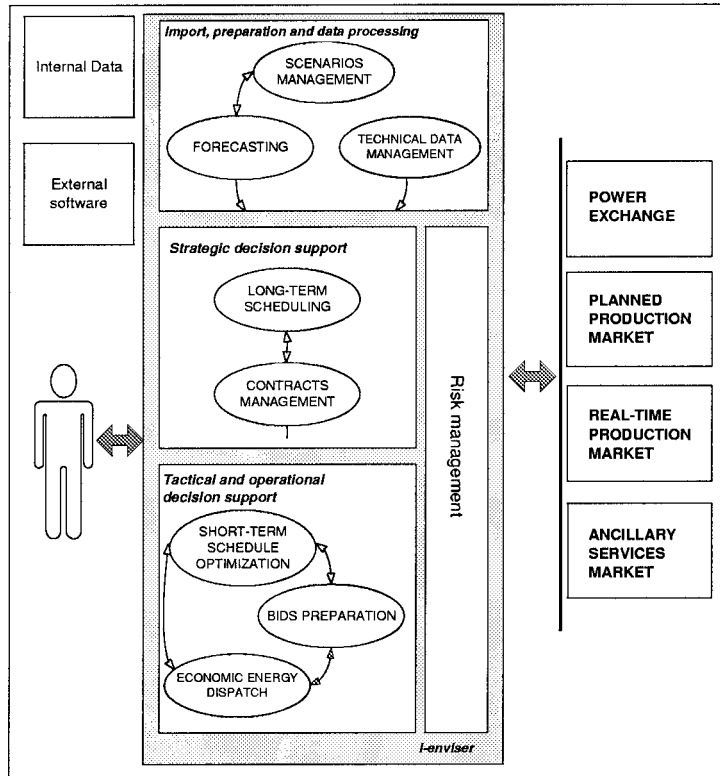


Figure 2. Decision process scheme.

- Contracts planning. Most long-term or medium-term contracts have some flexibility in time or/and load realization over the specified horizon. The system supports the disaggregation of long-term contracts into shorter periods, leading to detailed realization schedules if required.
  - Long-term resources planning. The availability of all the assets is determined over the horizon with respect to planned repairs as well as financial aspects.
3. *Tactical and operational decision support.* Operational tasks such as spot market production planning, trading planning specify an implementation of the long-term strategy. They are supported in a short time resolution and may be defined in the following areas:
- Short-term contracts planning. The contracts realization schedules obtained in strategic decision stage are further dispatched among generating units within a short-term horizon.
  - Production planning. Taking into account the prices forecast and the DM's preferences, an optimal production plan satisfying all the operating constraints is determined.

- Bidding. Depending on the optimal production plan, production costs and the DM risk attitudes, optimal bids are built to be submitted on several energy market segments.
- Final dispatching. Typical deterministic unit commitment problem is solved to find the optimal production schedule after all the market clearing processes have been finished.

Short-term planning module is a key element of the I-enviser system. The entire tactical and operational decision support is achieved by repeatedly use of the module. The module itself implements the stochastic short-term planning model and the multiple criteria interactive methodology described in this paper. Directly, the stochastic model with multiple risk criteria is used for optimization of contract positions and generation plans for several generation units, thus allowing to support the short-term contracts planning and the production planning decisions, respectively. The risk criteria are well appealing and the ARBDS interactive analysis tool enables to find an efficient solution (production plan) well adjusted to the generator preferences (risk attitude).

The criteria express the expected return and several risk measures including the extreme events measures. Hence, the multiple criteria model well depicts the uncertainty of market prices, thus allowing the optimal generation plan to be used as a background for various operational decisions. In particular, it can be used in the bids preparation process enabling an active participation in the market segments based on the clearing process mechanisms. A bid to be submitted for a particular market segment must meet specific technical requirements. Therefore, I-enviser is equipped with a special bids preparation module with predefined technical requirements. The bids themselves are built for each hour (auction time unit) of the horizon, according to the production cost caused by the implementation of the optimal production plan.

When the final results of the auction process are already known, the generation dispatch module is used to define the final production schedule for all the units. The short-term planning model based on maximization of the overall return as a function of the generation decisions. As all the model parameters, including the energy prices, are known one needs to maximize the deterministic return (6) under the typical unit commitment constraints (1)–(5) and (7).

The strategic decisions are typically related to the long-term resource planning and long-term contracts management. The decisions considered for a long horizon are hard to formal modeling and effective optimization for the sake of their mostly qualitative attributes as well as a strong uncertainty (risk) factor. The stochastic programming model can be built for the long-term resource planning [3]. Hence, there is a possibility to extend our short-term planning methodology based on the multiple risk criteria analysis to be applied to the resource planning. This is our goal, but currently I-enviser depends on possible use of external software to support resource planning decisions.

The long-term contracts management is currently supported only by verification and evaluation of solution submitted by the DM. The solutions are evaluated with simple criteria, such as schedule feasibility or the expected return. Moreover, I-enviser

provides various decision-aid tools from suitable data presentation and visualization to simple heuristics. Nevertheless, an analytical decision support for dis-aggregation of the long-term contracts obligations into shorter periods remains an important goal for our future developments. We intend to exploit the concepts of equitable optimization [22] to formalize dis-aggregation decisions.

## 6. Concluding remarks

As deregulation in the power sector is advancing, the electricity markets are moving toward greater reliance on competition. In most markets the competition is limited to include only the wholesale energy market. Even such a limited deregulation causes that the traditional methods of operating in the energy market continue to change and reformulate which makes the energy suppliers facing a necessity of new management decisions. In a regulated market only technical aspects of generation are considered by the generator (electric power producer), as the minimization of the production costs assures the maximum profit. A new competitive environment relates the profit to a success in bidding and clearing process, and the generator becomes an active market participant. This enforces the generator to serve as a decision maker (DM) dealing with new goals and decision processes as well as new types of necessary information. There is a strong need for a decision support system (DSS) dedicated to such electricity market participants.

In this paper it has been shown that the stochastic short-term planning model can be effectively used as a key analytical tool within the DSS for active participants of the electricity markets. The technical constraints result in a mixed-integer linear program while the market price uncertainty leads to the stochastic objective function. The uncertainty is modeled by a set of possible scenarios with assigned probabilities. Several risk measures have been introduced to be used as multiple criteria for modeling the generator's risk attitude. Apart from typical dispersion measures some extreme events risk measures are available. The aspiration/reservation based interactive analysis is applied to the multiple criteria problem thus allowing to find an efficient solution (generation schedule) well adjusted to the generator's preferences (risk attitude). All the criteria are LP computable while the feasible set of generation decisions defined by the mixed-integer LP constraints. Our computational experience shows that the resulting mixed-integer LP problems can be effectively solved for the scheduling several generation units over a horizon covering up to one week (168 hours) with 100 price scenarios.

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