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The multiple criteria location problem: 1. A generalized network model and the set of efficient solutions

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Abstract. This is the first of two papers in which multiple criteria location problems (MCLPs) are discussed. In this paper the main aim is to formalize a discrete MCLP and to develop a generalized network model. A critical overview of various techniques for generating efficient solutions to multiple criteria decision problems is offered. The three most commonly used methods for tackling MCLPs, namely the weighted method, the noninferior set estimation method, and the constraint method, are discussed. The main purpose of the generating techniques is to determine an exact representation of or an approximation to the set of efficient solutions among which one can choose the best or most preferred solution (location plan). To identify the best solution some information about the decisionmaker's preferences or a decision rule is needed. Consequently, in paper 2 we focus on preference-based approaches to multiple criteria decisionmaking and relate them to the concept of interactive decision support. Specifically, optimizing decision rules (utility-function-based approaches) and satisficing decision rules (goal programming methods) are discussed. Advantages and disadvantages of these two approaches to solving the MCLP are highlighted. It is suggested that the utility-maximizing and satisficing decision rules are not mutually exclusive. Accordingly, a quasi-satisficing approach that merges these two decision rules is proposed. Also, a framework for an interactive decision support system (DSS) for tackling MCLPs is presented. The system incorporates the generalized network model into a quasi-satisficing approach. It is argued that the DSS data and analytical components can be effectively integrated by means of the interactive decision support concept which allows for exploring the problem and the alternative solutions both in decision space and in criterion outcome space.

1 Introduction

Locational decisionmaking can be considered as a process of searching for the best location or pattern of locations. This process can be divided into three stages: intelligence, design, and choice (Simon, 1960). The aim in the first phase is to identify the problem environment. It requires searching the environment for conditions which call for locational decisions. For example, an analysis of the geographical distribution of public service facilities in relation to the projected distribution of population may suggest the need for a spatial shift in the supply of public services and, consequently, for locational decisions. This type of activity requires an analysis of comprehensive spatially referenced data (Armstrong et al, 1992). To this end, the most valuable support for locational decisionmaking is offered by a geographic information system (GIS): a system that performs the functions of storage, manipulation, and display of geographically referenced data, and enables the analyst (decisionmaker) to analyze large data sets (see Densham and Goodchild, 1989; Laurini and Thompson, 1992).

The design phase involves inventing, developing, and analyzing possible locational strategies, and results in the identification of a set of locational alternatives (plans).

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Typically, a formal model is used to support a decisionmaker in determining the set of alternative locational patterns. Design activities can be performed more easily, efficiently, and effectively when a substantive model of the decisionmaking situation (for example, a location-allocation model) is used to identify locational alternatives. This locational modeling involves an integration of four major elements. First, the evaluation criteria are established on the basis of problem analysis. Second, there are the various constraints—physical, economic, social, and political—associated with the specific decision situation. Third, there are the techniques for solving multiple criteria decision problems and generating alternative decisions. Fourth, there are the techniques for graphic presentation of the problem and the alternative solutions (Allard and Hodgson, 1987; Armstrong et al, 1992). These four elements should be integrated in such a way that the decisionmaking process is more effective and efficient than when an unplanned approach is used.

The third stage, choice, involves selecting an alternative. The choice depends on the decisionmaker's preferences. Given the substantive model and a decisionmaker's preferences, a choice can be made with the aid of a decision support system: a system that enables interaction between a decisionmaker (analyst) and a computer-based system, which consists of a GIS and a model-base management system. This interactive process is typically performed in a sequence of steps with inputs from and feedback to the decisionmaker at each step. The decisionmaker provides the computer-based system with information about his or her preferences, and the system generates solutions and provides feedback by means of visual displays in the form of maps, graphs, and tables (Maclaren, 1988).

In these papers, we are primarily concerned with the stage of design and choice in decisionmaking. In paper 1 a generalized network model is presented, which can be used to design a substantive model of a locational decisionmaking problem and to generate a set of nondominated locational alternatives (efficient solutions). In paper 2 the choice phase is considered. Specifically, preference-based interactive approaches to locational decisionmaking are discussed (details on the structure of these papers are given in section 1.3).

1.1 The need for multicriteria analysis

The nature of a locational decisionmaking process depends very much upon the type of economic or social activities to be located and the character and type of organization within which the decision is made. To this end, a common distinction is made between private sector and public sector organizations (ReVelle et al, 1981).

1.1.1 *Private sector*

According to the precepts of neoclassical location theory, the best locational pattern of economic activities is one that minimizes production costs or maximizes revenues, or maximizes the excess of revenue over costs (Beckmann, 1968). Put simply, the best pattern of locations maximizes profits or producer's surplus. The premise underlying profit-maximization theory is that all relevant factors involved in locational decisionmaking, as well as the process itself, can be incorporated into and adequately represented by a single-criterion function and a set of constraints imposed on the decision variables. Thus, the profit-maximizing approach to locational decisionmaking reduces the multiple criteria nature of locational problems to a single-criterion function which measures the profit associated with alternative location-distribution patterns. In essence, all classical location models such as the Weber problem, the transportation problem, the transshipment problem, and the plant location problem are based on this concept (for example, see Isard, 1969).

In general, it can be argued that the profit-maximizing criterion provides the basic rationale for the existence of private enterprise. It is not, however, a universal principle for locational decisionmaking in the private sector. There are quite distinctive factors involved in locational decisionmaking of different types of manufacturing and private services. Some analysts suggest that the profit-maximizing philosophy is almost entirely limited to traditional firms operated by owners–entrepreneurs (Czamanski, 1981), whereas managerial firms, in which direction is largely divorced from ownership (corporate organizations), are usually characterized by a complex hierarchical and multidimensional process of locational decisionmaking. Corporations are motivated in their locational decisions more by consideration of growth of the firm, control of the market, diversification of interest, entrepreneurial satisfaction, and self-preservation, rather than by profit maximization (Hamilton, 1974).

Locational decisions made by large corporations are similar, in many respects, to those hypothesized in behavioral theories of the firm (Simon, 1960; Pred, 1967). To this end, it should be emphasized that the search for a new location (a transfer or branch moves) is not a continuing corporate activity. The process of locational search is activated by the corporate recognition of an unavoidable pressure on existing plant facilities (for example, as a result of a changing pattern of demand for goods or services supplied by the firm). This process involves a search among a limited, usually small, number of alternative sites and directed towards the identification of a satisfactory location that meets a range of physical, socioeconomic, environmental, and personal requirements. Strictly financial evaluation often takes place after the satisficing site has been identified and is only infrequently used in making a selection between alternative sites (Hamilton, 1974). A number of empirical studies and surveys of industrial firms' locational decisions support the view that behavioral factors (such as the desire to avoid uncertainty) and structural factors (for example, governmental policy) are of major importance in the search for a location (Keeble, 1976; Schmenner, 1982).

1.1.2 *Public sector*

Public goods and services are typically provided and managed by governments in response to perceived and expressed need. The spatial distribution of public goods and services is strictly related to facility location decisions. Typically, these decisions involve two fundamental considerations: geographical equity and efficiency in service provision (Morrill and Symons, 1977; Mayhew and Leonardi, 1982). Most classical location–allocation studies focus on some aspects of these two factors. For example, p-median problems are primarily concerned with spatial efficiency, whereas p-center problems primarily address the equity issues. In general, location–allocation analysis has been mainly concerned with developing single-criterion models that optimize spatial demand–supply relationships and model the spatial behavior of consumers in the context of facility location. A variety of spatial-interaction-based models has been developed to incorporate the conflicting preferences of users (customers) and providers of public services (see Hodgson, 1978; Wilson et al, 1981). In the most general case, the concept of consumers' surplus (or total net benefits) is used to measure the benefits associated with alternative location–allocation patterns. Variations in benefits associated with different arrangements of public facilities are measured as the difference between customer costs (accessibility) and facility establishment costs.

Insistence that rigorous, optimizing techniques will yield the best solutions to locational choice problems is predicated on the assumption that location decisions are well structured and that solutions will be acceptable to the decisionmakers and to the public (Massam and Malczewski, 1990; Massam, 1993). If the analyst works closely with the decisionmakers and the representatives of interest groups, and they are persuaded of the credibility of the analyst's tools, then the results generated by a formal location–allocation technique may, in fact, be the ones accepted for implementation (for example, see Rushton, 1984). However, many locational problems in the public sector, especially of the noxious variety, are ill structured because of the variety of interest groups and the difficulty of measuring, assessing, and evaluating the quality and quantity of impacts associated with alternative locational patterns. A further complication is that interest groups and decisionmakers usually evaluate locational options with the use of multiple and often conflicting criteria.

The conflicting and multiple criteria nature of locational decisions in the public sector is related to the 'impurity' of public goods (Lea, 1981). There are three major 'locational sources' of impurity in public goods: the tapering effect, jurisdictional partitioning, and externalities (for example, see Pinch, 1985). Tapering results from the 'point-specific' locations of public goods and services. Such facilities as schools, libraries, health centers, post offices, and police, ambulance, and fire stations must be located at particular points and, consequently, the benefits from consumption of services provided by these facilities will diminish with the distance that users have to travel to the place of supply or the distance from the facility supply to the point of consumption, even if all other conditions of service provision are equal for all users. Therefore, criteria including minimization of aggregate or average distance, minimization of maximum distance, maximization of population covered within a given distance, etc, are of crucial importance in locating public facilities (Rushton, 1984; 1987; ReVelle, 1987).

The geographical space of a country (region) is usually subdivided into local jurisdictions. These jurisdictional units vary in terms of their economic, technological, and social development, as well as geophysical characteristics. These differences across geographic space result in variability of the quantity and quality of goods and services provided by different jurisdictional units. Because the costs and benefits associated with the provision of public goods and services should, in principle, be equally distributed, there is a political conflict over public facility location. Therefore, a variety of criteria addressing the distributional equity issues should be considered.

Locational decisions can generate positive or negative externalities. The former occur when locational decisions result in uncompensated benefits, the latter are associated with decisions that generate uncompensated costs. For example, salubrious facilities (such as parks, libraries, schools, hospitals) produce positive externalities or benefits for people who live near them. On the other hand, there are noxious facilities (for example, airports, power stations, waste dumps, hazardous waste management facilities) that produce negative externalities. These facilities may be indispensable for the regional or national economy but, at the same time, they are considered objectionable by residents who are located near them. This leads to locational conflict because different interest groups may have different perceptions of the costs and benefits associated with locational alternatives.

From the discussion above, we conclude that the search for the best locations for public facilities is a problem of collective choice (Massam, 1993). Consequently, locational decisionmaking should be seen as a process of search for consensus and a compromise solution. To this end, responsible decisionmaking requires that those in authority, who make locational decisions, should be accountable and that the selection of any formal methods (models) should contribute to this accountability by allowing the analysis to be scrutinized by the public. Although formal methods can provide specific solutions to well-posed problems, the complexity of locational decisionmaking requires that an interactive framework be found which links information and analysis to responsible, authoritative decisionmaking. This leads us to the concept of an interactive decision support system.

1.2 The need for interactive decision support

Krarup and Pruzan (1990) argue that a variety of normative approaches to locational problems developed over the last thirty years or so "together with the increasing interest in multiple criteria analysis will lead to much greater emphasis upon interactive search procedures" (page 47). More specifically, the authors "refer to systems allowing for interaction among the decisionmaker, the analyst, and the computer, and which not only employ various notions of multiple criteria analysis but also, directly or indirectly, reflect the interfaces between locational decisions and other strategic decisions" (page 47). We subscribe to this view and suggest that a complex location problem can be tackled more efficiently and effectively within an interactive, computer-based decision support framework (see also Hultz et al, 1981; Malczewski and Ogryczak, 1990; Malczewski, 1992).

A decision support system (DSS) can be defined as an interactive computer-based system designed to support a manager (decisionmaker) in achieving a higher effectiveness of decisionmaking while solving a semistructured problem (Keen and Scott-Morton, 1978). There are three terms—semistructured problem, effectiveness, and support—that capture the essence of the DSS concept.

First, semistructured problems occur when managers are not able to specify the planning problem and their objectives fully and coherently. Most location problems fall into this category. In this case, the structured (programmed) part of the problem may be amenable to automated solution by the use of a computer, whereas unstructured (nonprogrammed) aspects are tackled by managers. They can provide judgmental information in the form of preferences about the significance of impacts which cannot be expressed a priori in a formal language. Thus, judgments are not represented in the structure part of the problem but rather they are usually incorporated into the analysis as desired and acceptable levels of achievements (Massam and Malczewski, 1990). Second, although an application of a DSS for solving a decisionmaking problem may increase the efficiency of the information-processing operation, it is not the most important objective of the system. The main aim of a DSS is to improve the effectiveness of decisionmaking by incorporating managerial judgments (preferences) and computer-based programs into the decisionmaking process. Consequently, the third important feature of any DSS is that it does not replace managerial judgments. The purpose of such a system is to support a manager in achieving 'better' decisions. By better, we suggest that decisions could be reached with the use of extensive data sets and within a framework which allows sensitivity tests (Massam and Malczewski, 1991). The essence is to avoid the 'black box' style of plan and policy evaluation and selection. In order to improve the effectiveness of decisionmaking, one should incorporate into the planning process both the participation of decisionmakers (representatives of interest groups) and the substantive model of the decisionmaking situation. It is argued that these two elements are integral parts of any locational decisionmaking process. Although the use of formal, analytical procedures can be relied upon for the design and evaluation of alternative locational patterns (plans), it is most important that those who use the results of such work believe that the procedures offer credible outputs (Densham and Rushton, 1987; Massam and Malczewski, 1990).

1.3 The structure of these papers

A wide range of formal methods and procedures are available for handling multiple criteria decisionmaking, and a number of taxonomies have been proposed for classifying these techniques (see Cohon, 1978; Hwang and Masud, 1979; Rietveld, 1980; Nijkamp and Rietveld, 1986; Steuer, 1986). In these papers we will follow Cohon (1978) who has classified multiple criteria approaches into two broad categories: techniques for generating efficient or noninferior solutions and preference-based methods. The preference-based approaches can be further subdivided into two categories: explicit utility-maximization methods and interactive, implicit preference techniques. These papers are organized around this classification. Figure 1 shows the structure of these papers in relation to multiple criteria techniques.

In paper 1 a framework is provided for the modeling of multiple criteria location problems with emphasis on a generalized network model (sections 2 to 5). A critical overview of the techniques for generating efficient solutions is also given (section 6). The main purpose of the generating techniques is to determine an exact representation of, or an approximation to, the set of efficient solutions; the decisionmaker can choose the best locational scheme from among the set of efficient solutions.

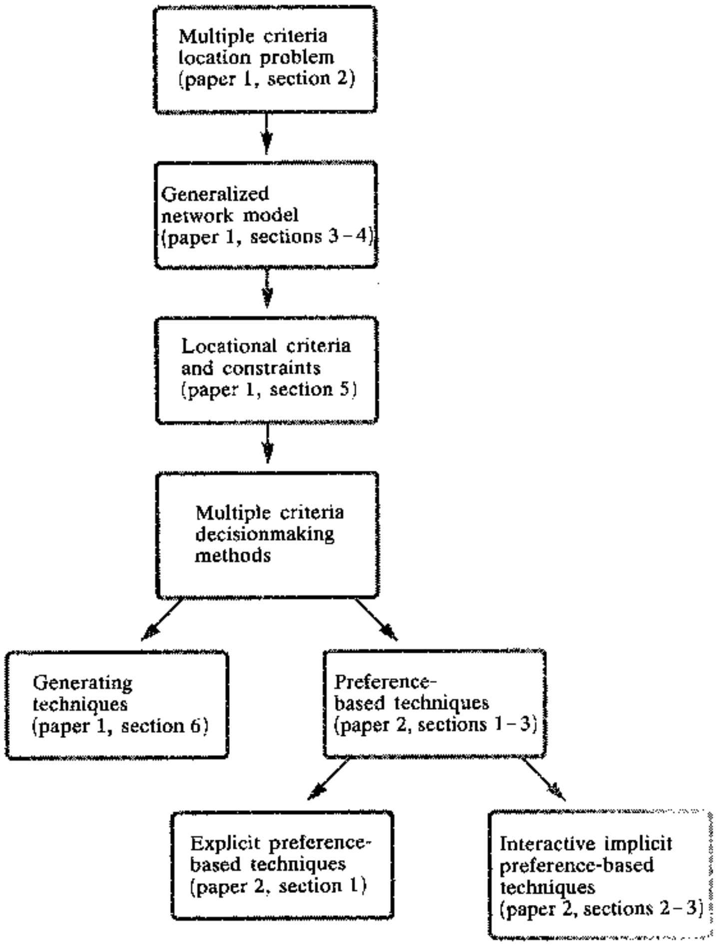

Figure 1. A framework for multiple criteria location analysis.

Decisionmakers do not have to articulate their preferences explicitly because the preferences are implicitly considered during the choice of the most preferred alternative. The major disadvantage of the generating techniques is that the size of the set of efficient solutions is usually very large for a real-life location problem. Therefore, the process of identifying the best alternative from among a set of efficient solutions requires an explicit or implicit treatment of the decisionmaker's preferences and, hence, it involves value judgments which can be incorporated into an interactive DSS.

In paper 2 we focus on preference-based approaches and an interactive DSS for locational decisionmaking. Fundamental, theoretical, and methodological differences between the preference-based approaches lie in the assumptions about decision rules that guide the selection of the best (most preferred) alternative from among the set of efficient alternatives. In practice, decision rules fall into two categories: the optimizing or utility-maximizing rules and the satisficing decision rules. In section 1 of paper 2 we provide a critical overview of these two categories of decision rules in the context of multiple criteria locational problems. The discussion is focused on the advantages and disadvantages of these two approaches. A comparative analysis of the utility-maximization and satisficing decision rules is also offered. The major disadvantage of the utility-maximizing approaches is that in many real-life situations it is very difficult or even impossible to obtain a mathematical representation of the decisionmaker's preference (utility) function. There are also some difficulties with the satisficing decision rules, which are usually operationalized with the aid of goal programming. The main weakness of this approach is that goal-programming methods may generate dominated solutions (Cohon, 1978; Chankong and Haimes, 1983).

The interactive approach presented in this paper can be considered as an extension and generalization of goal programming methods (paper 2, section 2). It is based on the quasi-satisficing rationality hypothesis. This hypothesis has been formalized in terms of the reference-point method (Wierzbicki, 1982). The quasi-satisficing decision framework is especially meaningful if it is considered in the computer-based decision support context (paper 2, section 3). In the quasi-satisficing decision support process the decisionmaker is explicitly involved in the problem-solving activities. This approach enables users to specify their requirements in terms of aspired or required criterion outcomes, and allows for the controlled generation and selection of alternatives. The levels of aspiration and reservation are used to explore the set of efficient solutions. The main idea behind the aspiration–reservation-based DSS concept is to involve a decisionmaker in a sequential process of search for a satisficing solution. In this process decisionmakers can change their preference through the process of learning and the acquisition of more information about the decisionmaking problem. The aim of such a system is to help the user to achieve a higher level of effectiveness during locational decisionmaking (see Hultz et al, 1981; Malczewski and Ogryczak, 1990; Massam and Malczewski, 1990).

2 MCLP structure

2.1 Decision space

Most real-life location problems involve choice among a discrete set of alternatives. Therefore, there are only two possible decisions associated with each of the alternative locations. These decisions are: whether to locate a facility at a site or not; no intermediate possibility exists. Let us define L, $L = \{l_j, j = 1, 2, ..., n\}$, as a finite set of all such individual locational decisions. Any locational alternative can then be expressed as a binary (logical) vector, $\boldsymbol{x} = (x_1, ..., x_n)$, where a decision variable, x_j, is defined as follows:

$$x_j = \begin{cases} 1, \text{ if the facility is located at the site } j, \\ 0, \text{ otherwise.} \end{cases}$$

Theoretically there exist 2^n different vectors (x) and corresponding alternatives in the decision space, X. However, the number of alternatives is usually less because some additional constraints define the feasible set of alternatives to be some subset of X.

For certain classes of location problems, the decision space and the feasible sets have more complex structures attributable to the consideration of allocation decisions. Such decisions are usually modeled with additional decision variables. Specifically, there is a vector of allocation variables associated with each locational alternative, j. Depending upon the nature of allocation of a quantity from a location i ($i = 1, 2, ..., m$) to site j, the allocation decision can be expressed in terms of a binary variable given by

$$x_{ij}^* = \begin{cases} 1, \text{ if location } i \text{ is to be allocated to the site } j, \\ 0, \text{ otherwise,} \end{cases}$$

or in the case where the allocated quantity can be split among two or more sites an integer or continuous variable is used, that is, x_{ij}^{**} is a portion of quantity allocated from location i to site j, ($x_{ij}^{**} \geqslant 0$). Thus, the decision vector x (and thereby the decision space X) is an aggregate of two types of decision variables:

$$x = (x', x''),$$

where x' is a binary vector of locational decisions, and

$$x'' = x^*, \quad \text{or} \quad x'' = x^{**},$$

where x'' is an integer (a $0-1$ or general integer) or continuous vector of allocation decisions. Owing to some constraints the set of feasible alternatives, A, is usually limited to some subset of the decision space, X, where $A \subset X$.

2.2 Criterion outcome space

A locational actor (decisionmaker) evaluates each alternative locational pattern with respect to a set of k criteria. Thus, we can define a criterion outcome space, Y, and a mapping (or function in case of a single criterion), F: $A \rightarrow Y$, which describes the numerical consequences of each locational alternative. Accordingly, each alternative yields a point, y, in a k-dimensional space, $\mathbb{R}^k$, that consists of all k criteria outcomes. The set of outcomes for all the feasible alternatives defines an attainable outcome set, $Y_a = \mathrm{F}(A)$. The decision problem then depends on the selection of the best attainable outcome, and identification of the decision alternative yielding this outcome.

2.3 Efficiency principle

For the sake of simplicity of the formal presentation we can assume, without a loss of generality, that all the criteria are to be minimized and that the locational problem can then be formulated as the following multiple criteria optimization model:

$$\text{minimize } \mathrm{F}(x), \tag{1}$$

subject to

$$x \in A, \tag{2}$$

where $\mathrm{F} = (\mathrm{F}_1, ..., \mathrm{F}_k)$ represents a vector of k criteria.

Central to a theory of multiple criteria decisionmaking is the efficiency principle (known also as nondominance, noninferiority, or the Pareto-optimality principle). In order to formalize this concept, let us define an achievement vector:

$$\boldsymbol{q} = \mathrm{F}(\boldsymbol{x}) \,,$$

which measures outcomes of several alternatives, $\boldsymbol{x}$, with respect to the specified set of k criteria, $\mathrm{F}_1, ..., \mathrm{F}_k$. In the case of a single criterion, we have a scalar achievement, q, and it is easy to define the best achievement simply as the minimal one within the set Y_a. When dealing with multiple criteria we face a much more complex problem. It is clear that an achievement vector is better than another one provided that all its individual outcomes are better or at least one individual outcome is better and all the others are not worse. Such a relation is called domination of achievement vectors and it is mathematically formalized (in the case for minimization problems) as follows:

$$\textit{if}\ \boldsymbol{q}' \neq \boldsymbol{q}''\ \textit{and}\ q_\nu' \leqslant q_\nu'' \,, \qquad \textit{for all} \quad \nu = 1, ..., k \,,$$

$$\textit{then}\ \boldsymbol{q}'\ \textit{dominates}\ \boldsymbol{q}'',\ \textit{and}\ \boldsymbol{q}''\ \textit{is dominated by}\ \boldsymbol{q}' \,.$$

Unfortunately, there usually does not exist any achievement vector that dominates all of the others with respect to all of the criteria, that is,

$$\textit{there does not exist}\ y \in Y\ \textit{such that for any}\ \boldsymbol{q} \in Y,$$

$$y_\nu \leqslant q_\nu \,, \qquad \textit{for all} \quad \nu = 1, ..., k \,.$$

Thus, from the point of view of strict mathematical relations, we cannot distinguish the best achievement vector. Instead, we can classify each achievement vector, $\boldsymbol{q}$, as a dominated one—such that there exists another vector, $y \in Y_a$, dominating $\boldsymbol{q}$—or as a nondominated one—such that there does not exist any vector $y \in Y_a$ dominating $\boldsymbol{q}$. The dominated achievement vectors represent nonoptimal locational alternatives. On the other hand, all the nondominated achievement vectors represent alternatives for which we cannot improve any individual achievement without worsening another one; thus, these alternatives can be considered as optimal by virtue of being mathematically efficient. Accordingly, a locational alternative is said to be efficient (sometimes also called nondominated or noninferior) if it is feasible and no other feasible location exists which can improve performance on one criterion outcome without reducing the performance on another. It implies that all efficient locations (nondominated achievement vectors) are noncomparable to each other on the basis of the specified set of criterion functions.

2.4 The decision rules

A set of assumptions that allows us to order alternatives is referred to as a decision rule (Chankong and Haimes, 1983). Decision rules provide an explicit way of selecting one or more alternatives from a set of alternatives available to the decision-maker. Note that the efficiency principle does not allow for an ordering of alternative decision outcomes. It can be used only to classify the set of feasible solutions into two categories: the set of efficient and the set of nonefficient decisions. A further decision rule is required, therefore, to choose the best alternative from among the set of efficient alternatives. In general, the decision rule can be simply stated as: choose an efficient decision with an outcome that is most preferred by the decision-maker. The complexity of the choice based on this rule stems from the fact that in a real-life location problem (especially when the problem involves allocation decisions) the set of efficient decision alternatives and, consequently, the number of location–allocation patterns are very large. Therefore, there arises a need for further analysis, or rather decision support, to help the decisionmaker to choose one efficient solution for future implementation. As the efficient solutions (alternative efficient locations) are noncomparable on the basis of the specified set of criteria, the analysis has to involve some additional information about the decisionmaker's preferences. The best alternative depends on the underlying preference structure, which, in turn, determines the decision rule used to make a locational choice. A number of decision rules are available for making locational decisions (see Isard, 1969). In essence, these decision rules fall into two fundamental categories—the optimizing rules and the satisficing rules (see paper 2).

3 Network structure

To formalize a locational decisionmaking problem in terms of the model (1)–(2), it is convenient to have some standard terminology and notions associated with location problems on a network.

3.1 Fixed nodes

A network is a collection of nodes and arcs (or links). These two elements are defined as point entities and line entities, respectively (Laurini and Thompson, 1992). The nodes can be subdivided into two sets: a set of fixed nodes and a set of potential nodes. Fixed nodes represent point entities, the location of which is known and fixed in a network. These nodes are characterized by some specified amount of attribute or quantity, b_i. The value of b_i can be defined in terms of the number of people, amount of goods, commodities, information, capital, or demand at node i. Parameters b_i can take positive values corresponding to the supply of some goods or services, negative values to represent needs or demands for the attribute, or they can equal zero.

3.2 Potential nodes

Potential nodes or candidate locations represent a set of potential sites for various facilities (production, supply points, or service centers). All nodes in a network may be candidates, that is, the potential and fixed nodes may have the same locations in a network. For example, in figure 2, fixed (demand) node D_1 and potential (supply) node P_1 have the same location. The potential nodes are represented, however, as quite independent entities in our abstract network. Each potential node is characterized by its capacity, h_j. It is assumed that a potential node can supply an amount of goods, service, information, etc., defined by its capacity or it can be considered to be a transshipment point.

3.3 Selections

In certain applications the potential nodes have to be differentiated according to their geographical location and/or type of facilities (for example, a limited number of facilities are to be located in a given region and/or the potential nodes are differentiated according to the size of facilities that can be established at the same site). For these reasons, the potential nodes can be subdivided into specific groups. These groups are referred to as selections. Each selection S_r, S_r $(r = 1, 2, ..., z)$, defines a group of potential nodes (members) and the lower and upper limit on the number of members to be selected (located). In particular, if a selection represents a few variants of the same object, lower and upper limits equal to 0 and 1, respectively, will be used. Some selections can overlay each other with respect to their members.

3.4 Arcs
The nodes are connected by a set of arcs or links. A link is a path of feasible direct transportation between two nodes, with no intervening nodes. Where more than one feasible link exists between two nodes, the shortest one is used. An arc (i, j) from node i to node j is characterized by capacity h_{ij}. The capacity is the maximum amount of flow that an arc (i, j) can carry. In general, arcs are oriented and nonsymmetric, that is, arc (i, j) differs from arc (j, i), and they can be associated with different parameters. However, in many location-allocation models they can be defined as symmetric.

3.5 Example 1
Consider a central facility location problem in a given region (see figure 2). There exist two centers, S_1 and S_2, which can serve 9000 and 8000 consumers, respectively. The demand for the service is projected to be 24000. Consequently, it is planned that two new centers will be built in the future. Six potential sites (P_1, P_2, ..., P_6) for locating the two new facilities are considered. It is assumed that the capacity of each new center should not be greater than 5000 for sites P_1 and P_4, and not be greater than 6000 for the remaining sites. Furthermore, the region is subdivided into two administrative units (subregions) and the population has exclusive use of the facility within its areal units. Therefore the potential sites are divided into two subsets associated with the corresponding two subregions: North = (P_1, P_2, P_3) and South = (P_4, P_5, P_6), and the sites located in a given subregion are considered as exclusive alternatives.

To identify the spatial distribution of demand for the services offered by the facilities, the region is subdivided into twelve spatial units (residential areas). The size of the population is used as a surrogate for demand. To measure the distance involved in traveling from residential areas to the service locations, it is assumed that the population in a spatial unit is concentrated in its center. Thus, there are twelve demand points, from D_1 and D_{12} (see figure 2).

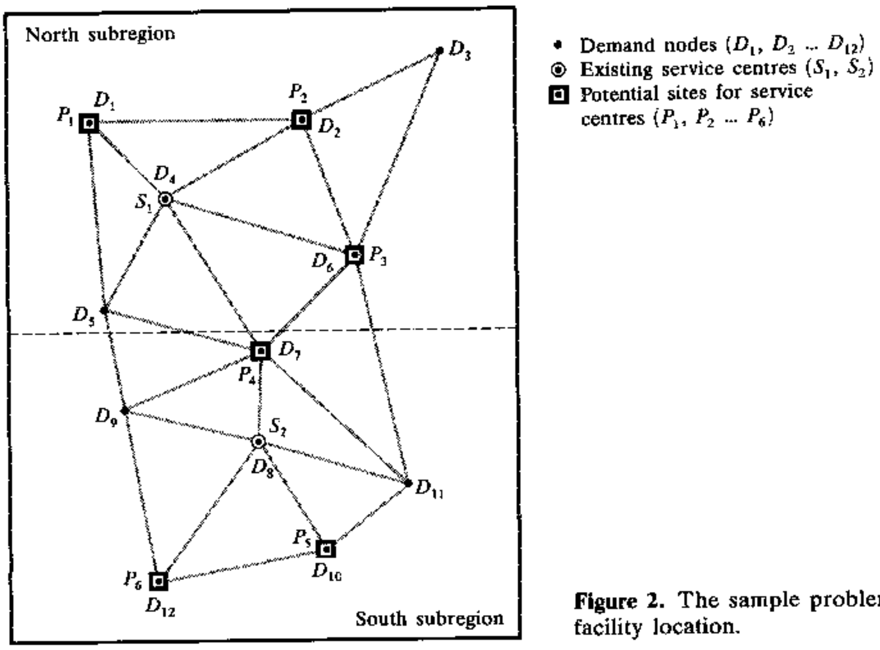

Figure 2. The sample problem: central facility location.

The problem is to find the best pattern of locations and the size of new facilities in each subregion, and allocate the demand to the existing and new facilities (this problem will be solved and analyzed in sections 4 and 6 as example 2 and example 3).

The above problem can be easily described in network terminology. Because the locations of the demand points (D_1, D_2, ..., D_{12}) and existing centers (S_1 and S_2) are known and fixed in the network, they are referred to as fixed nodes. Similarly, all the potential sites for new centers are potential nodes (P_1, P_2, ..., P_6). Arcs represent all the possible assignments of individuals to the centers. A flow along the arc from a center, c, to an area, a, indicates the number of consumers in the area a serviced by the center c (variable x_{ca}^{**}). The capacity of each of the existing and potential centers is then represented as supply in the corresponding nodes. A scheme of the network is shown in figure 3.

As we have mentioned, the locations belonging to the same subregion are considered as exclusive alternatives; that is, no more than one location from the subregion can be used. Therefore we introduce into the network model selections that represent this requirement. In our model there are two selections associated with two subregions: North and South. Both the selections have the lower numbers equal to 0 and the upper numbers equal to 1. This guarantees that at most one potential node in each selection is active.

The arcs connecting the supply nodes with the demand nodes have essentially unlimited capacities. However, in practice, flows along these arcs are also bounded by capacities of the corresponding supply centers and one can use them as arc capacities (figure 3).

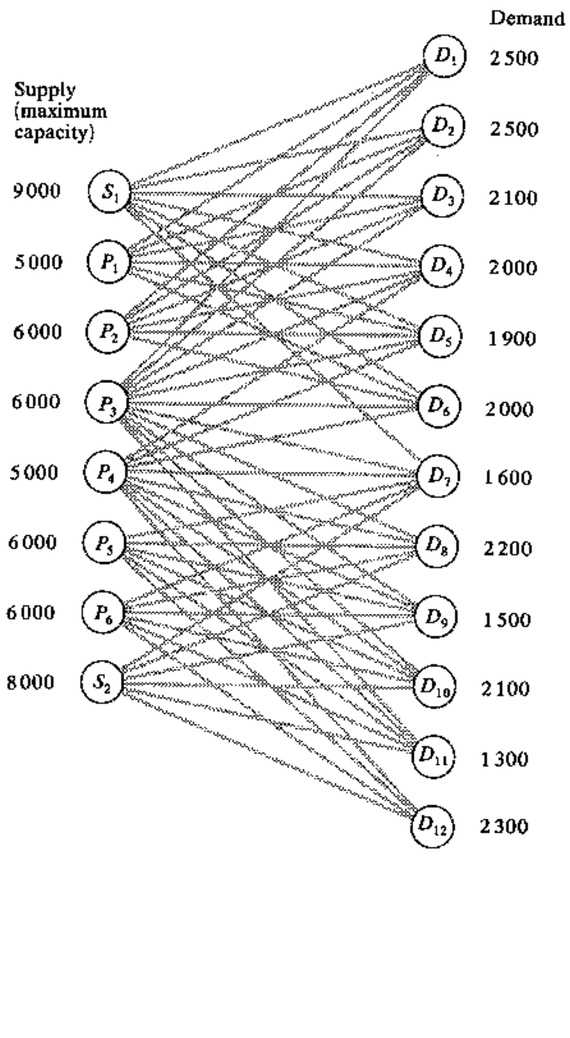

Figure 3. Network representation of the sample problem.

4 Generalized network model for multiple criteria location problems

The network model of the location-allocation problems discussed in the previous section has some weaknesses. For instance, as one can notice from example 1, to allocate entire areas without splitting them (usage of $x'' = x^*$) requires quite a different definition of network attributes (supply, demands, and capacities) compared with the model allowing for splitting. Moreover, two types of nodes disturb simplicity and soundness of the network model. In order to avoid the weaknesses of the standard network model we formulated a generalized network model (GNM). It simplifies remarkably the network structure and allows easy implementation of additional constraints or requirements.

What distinguishes the potential nodes and simultaneously disturbs the network homogeneity is their potentiality. Note that, in fact, in each network model one has a high level of potentiality associated with arcs. A flow along an arc may or may not occur. This suggests that the network model of location-allocation problems can be simplified by moving the entire potentiality included in the model to the arcs. This transfer can be accomplished by replacing each potential node with a pair of fixed nodes connected by an artificial arc. All the attributes associated with potentiality can then be assigned to the arc. Consequently, the network can be represented as a structure with all nodes being fixed and each arc representing a potential flow.

All arcs are characterized by their capacities. For artificial (potential) arcs the corresponding capacity is, however, variable instead of constant. Flows along arcs are modeled with two variables: a 0-1 variable representing the existence of the arc (locational decision—x') and a continuous or integer variable representing the amount of flow (allocation decision—x''). To be more specific, a flow along the arc (i, j) is bounded by the quantity $h_{ij}x'_{ij}$, where x'_{ij} denotes the existence of the arc (1 it exists, 0 it does not exist), and h_{ij} is the capacity of the arc. For the arcs associated with locational decisions (artificial ones), x'_{ij} is a binary decision variable. For the others, it is a parameter fixed as equal to 1.

Given this simplified network structure, the generalized model can be written as follows:

There is a given set of nodes, N, $N = \{1, 2, ..., m\}$. The nodes are characterized by some specific amount of attribute b_i. These quantities can be positive (supply), negative (demand), or equal zero (intermediate points). The nodes are connected by a set of arcs, E, $E = \{(i, j): i, j \in N\}$, characterized by the capacities, h_{ij}. The status of the model is described by two types of variables associated with arcs:

x'_{ij} denotes existence (1 it exists, 0 it does not exist) of the arc (i, j),

x''_{ij} denotes amount of the flow along the arc (i, j).

At each node the variables have to satisfy balanced equations given by

$$\sum_{j \in N} x''_{ij} - \sum_{j \in N} x''_{ji} = b_i , \qquad \text{for} \quad i \in N , \tag{3}$$

and capacity restrictions for each arc, given by

$$x''_{ij} \leqslant h_{ij}x'_{ij} , \qquad \text{for} \quad (i, j) \in E , \tag{4}$$

$$x''_{ij} \geqslant 0 , \qquad x'_{ij} = 0 \text{ or } 1 , \qquad \text{for} \quad (i, j) \in E . \tag{5}$$

Further, as in the basic network model, there are selections, S_r, defining multiple choice requirements on some groups of x' variables,

$$p_r^{\min} \leqslant \sum_{(i,j) \in S_r} x'_{ij} \leqslant p_r^{\max}, \qquad \text{for} \quad r = 1, 2, ..., z, \tag{6}$$

where $p_r^{\min}$ and $p_r^{\max}$ are lower and upper limits for the rth selection, respectively.

Note that the constraints (4) do not destroy the original network structure of the model. They are the so-called variable upper bounds (see Schrage, 1975; Todd, 1982; Ogryczak, 1992) and, similar to the standard simple upper bounds (SUB), they can be handled outside the main structure of the model. Likewise, the constraints (6) can be regarded as some additional discrete mechanisms defining feasible variables x' (special ordered sets). Thus a GNM may be effectively solved with mixed integer programming packages armed with the network solvers, such as CPLEX (1993).

The generalized network model is very flexible as it allows one to keep the same network structure while adapting additional characteristics to meet modeling requirements. For instance, if one wishes to introduce a requirement that the entire quantity from node i has to be assigned to exactly one node from the set S, it can be implemented simply by adding a new selection built on all the arcs from node i to nodes of the set S,

$$1 \leqslant \sum_{j \in S} x'_{ij} \leqslant 1 \ .$$

4.1 Example 2

Consider again the problem from example 1 (subsection 3.5). Let us number nodes as follows: demand points D_1 through D_{12} as nodes 1 to 12, existing centers S_1 and S_2 as nodes 13 and 20, potential centers P_1 through P_6 as nodes 14 to 19, respectively (see figures 3 and 4). In order to transform the model into the GNM, one can add an additional node to each potential node. However, in most cases (including the one under consideration) we are able to create all the necessary artificial arcs with only one additional node. Namely we add node 0, with supply of services equal to the total of all the demands ($b_0 = 24000$) and arcs, to all the existing and potential centers. These arcs represent the centers and therefore their capacities are defined as equal to the corresponding center capacities ($h_{0,13} = 9000$, $h_{0,14} = 5000$, $h_{0,15} = 6000$, etc.). A scheme of the network is shown in figure 4.

The algebraic description of the model is as follows. The balanced equation for node 0 takes the form

$$x''_{0,13} + x''_{0,14} + x''_{0,15} + ... + x''_{0,20} = 24\,000 \ .$$

Balance equations run as

$$x''_{i,1} + x''_{i,2} + ... + x''_{i,12} - x''_{0,i} = 0 \ , \qquad i = 13, 14, ..., 20 \ ,$$

for service centers, and as

$$-x''_{13,j} - x''_{14,j} - x''_{15,j} - ... - x''_{20,j} = b_j \ , \qquad j = 1, 2, ..., 12 \ ,$$

for demand points (note that b_j, for $j = 1, 2, ..., 12$, are negative because they represent demands).

Nodes 13 and 20 represent existing centers. Therefore the corresponding binary variables, $x'_{0,13}$ and $x'_{0,20}$, are fixed at level 1 and the corresponding capacity constraints take the form of simple upper bounds,

$$x''_{0,13} \leqslant 9000 \ , \qquad \text{and} \quad x''_{0,20} = 8000 \ .$$

Nodes 14 – 19 represent potential facilities, so the capacities of the corresponding arcs are characterized by variable upper bounds,

$$x''_{0j} \leqslant h_{0j} x'_{0j}, \qquad \text{for} \quad j = 14, 15, \ldots, 19,$$

where h_{0j} denote capacities of the corresponding facilities.

Flows along arcs from the centers (nodes 13 – 20) to the subareas (nodes 1 – 12) are formally unbounded and there is no need to define capacity constraints for them. In fact, they are limited by flows through the corresponding center nodes.

Furthermore, there are two selections, $S_1 = \{(0, 14), (0, 15), (0, 16)\}$ and $S_2 = \{(0, 17), (0, 18), (0, 19)\}$, representing subregions North and South, respectively. They generate the inequalities

$$0 \leqslant x'_{0,14} + x'_{0,15} + x'_{0,16} \leqslant 1,$$

$$0 \leqslant x'_{0,17} + x'_{0,18} + x'_{0,19} \leqslant 1.$$

If one wants to implement a requirement that each demand point is served by only one center (there is no split among two or more centers), additional selections are necessary. They can be algebraically expressed by the following inequalities:

$$0 \leqslant x'_{13,j} + x'_{14,j} + x'_{15,j} + \ldots + x'_{20,j} \leqslant 1, \qquad j = 1, 2, \ldots, 12.$$

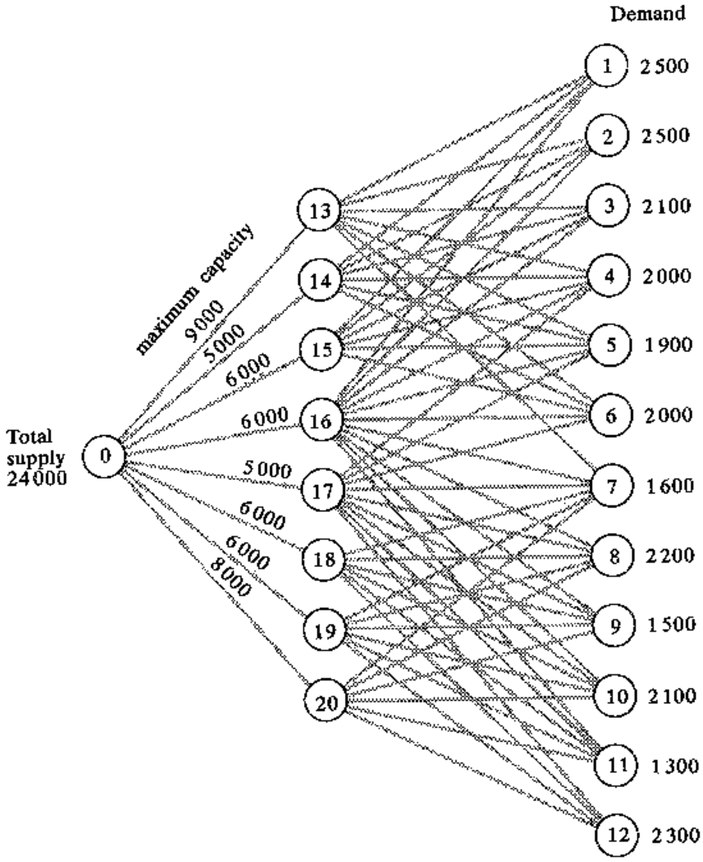

Figure 4. Network model for the sample problem of central facility location.

5 Locational criteria and constraints

Although the GNM may at first seem somewhat restrictive for a locational modeler, it is possible to express all typical location-allocation building blocks in terms of the GNM. The structure of the model (1)-(2) can take a variety of forms depending on the decisionmaking situation. Locational analysts often make a distinction between private and public sector facilities by pointing to the special features of each. Despite this distinction, a wide range of locational decisionmaking problems in the public and private sectors can be structured by means of an appropriate combination of generic criterion functions and constraints.

5.1 Criterion functions

In GNMs there are two types of variables (both associated with arcs): binary variables (x'_{ij}) denoting the existence of the corresponding arcs, and continuous or integer variables (x''_{ij}) denoting the amount of flow along the corresponding arcs. Therefore any criterion function has to be defined via coefficients assigned to those variables. Hence, there are the following three general cases of linear functions:

$$\mathrm{F}(x) = \sum_{(i,j) \in E} c'_{ij} x'_{ij} , \tag{7}$$

$$\mathrm{F}(x) = \sum_{(i,j) \in E} c''_{ij} x''_{ij} , \tag{8}$$

$$\mathrm{F}(x) = \sum_{(i,j) \in E} (c'_{ij} x'_{ij} + c''_{ij} x''_{ij}) . \tag{9}$$

Function (7) is associated with locational decisions. The corresponding criterion function minimizes (or maximizes) the amount of an attribute at the sites chosen for establishing facilities. The amount of an attribute (characteristic) associated with an alternative location is expressed by coefficient c'_{ij}. An interpolation of this coefficient depends on the specific decisionmaking situation. Typical examples of the coefficient c'_{ij} include economic, environmental, and social attributes, such as the total acquisition and development costs (investment costs), physical and social suitability of the site for facility location, environmental pollution associated with siting the facility at location j, etc. In more general terms, function (7) can be used to structure the site selection problem—that is, a problem involving choice of one or more sites from among a finite and exhaustive set of locational alternatives on the basis of a set of attributes (evaluation criteria). In this case, coefficient c'_{ij} represents an outcome of a decision to locate a facility at site j with respect to the ith attribute.

Function (8) is explicitly associated with allocation decisions. Although because of the constraints (4) it is, in fact, a function both of location and of allocation decisions. The real-world interpretation of this type of criterion function depends on a definition of the coefficients c''_{ij}. Typically, in location-allocation models, coefficients c''_{ij} are defined as some functions of d_{ij} (the shortest distance between locations i and j). If $c''_{ij} = d_{ij}$, then function (8) expresses the total transportation effort (to be minimized). Using $c''_{ij} = d_{ij}/b$, where b is the total demand, one gets the function expressing the average distance (to be minimized). Furthermore, the effect of distance (spatial separation) on the intensity of flow between a pair of nodes can be defined by $c''_{ij} = 1/\mathrm{f}(d_{ij})$. For example, $\mathrm{f}(d_{ij})$ can take the form of a negative power function, $(d_{ij})^{\alpha}$, or a negative exponential function, $\exp(-\alpha d_{ij})$, where α is a measure of the frictional effects of distance on the intensity of flows.

Hillsman (1984) has shown that function (8) can be modified by editing information in the matrix of coefficients, c_{ij}, to yield a wide variety of location-allocation criteria. According to Hillsman's approach, many location-allocation problems are structurally equivalent to the classical p-median problem and they can be cast within a unified linear model structure (see also Goodchild and Noronha, 1983; Densham and Rushton, 1992).

Function (9) is a sum of functions (7) and (8), and therefore it explicitly depends on both types of decisions. It is usually referred to as the total cost or budget criterion. It can be used, for example, to express the total investment and/or operating costs involved in establishing and running all facilities in an area. Investment and operating costs can be divided into fixed costs, c'_{ij}, and variable costs, c''_{ij}. In the classical approaches the total cost was frequently introduced into the model as a budget constraint. However, multiple criteria analysis allows us to deal with soft (fuzzy) budget constraints and examine relations between the cost and other outcomes. Note that function (9) can also be used in the context of noxious facility location to express the total pollution emission by facilities to be built because the corresponding coefficients can be split into fixed and variable.

Ross and Soland (1980) have demonstrated that function (9) can be considered as a generalization of a wide variety of criterion functions, which can be formulated by an appropriate definition of the c'_{ij} and c''_{ij} constants. Also, function (9)—along with a set of constraints imposed on the decision variables, x'_{ij} and x''_{ij}—can be recognized as a generalized assignment problem (Ross and Soland, 1977). Furthermore, Ross and Soland (1977; 1980) have shown that many classical location-allocation problems, such as the p-median and plant location problem, can be structured in terms of a generalized assignment problem.

For some applications the maximum operator can be used in criterion functions (7), (8), or (9) instead of the sum operator. For instance, if the decision situation requires minimization (or maximization) of maximal (or minimal) distance instead of the total or average distance, then function (7) can be replaced with the following expression:

$$\mathrm{F}(\boldsymbol{x}) = \operatorname*{maximum}_{(i,j) \in E} c'_{ij} x'_{ij} , \quad \text{or} \quad \mathrm{F}(\boldsymbol{x}) = \operatorname*{minimum}_{(i,j) \in E} c'_{ij} x'_{ij} , \tag{10}$$

to be minimized or maximized, respectively. Thus function (10) can be considered as a minimax or maximin criterion function. The former is of particular importance for locating emergency facilities, whereas the latter is frequently used as a criterion in noxious facility locational decisionmaking. To this end, coefficient c'_{ij} usually captures the various notions of maximum distance from facility to consumer locations. One can consider minimization (maximization) of the maximum (minimum) distance or the maximum (minimum) weighted distance, that is, $c'_{ij} = d_{ij}$ or $c'_{ij} = b_i d_{ij}$, respectively.

It is well known that a minimax linear programming problem can be transformed into a standard one by introducing additional inequalities,

$$c'_{ij} x'_{ij} \leqslant z , \qquad \text{for} \quad (i,j) \in E ,$$

where z is an additional state variable to be minimized. Such a transformation not only increases the size of the problem but also introduces coefficients c'_{ij} into the problem matrix, which can destroy the special network structure of the original matrix.

Ogryczak et al (1992) proposed a primal simplex algorithm which handles implicitly the additional inequalities. The algorithm is based on the linear programming basis partitions within the main steps of the cycle of the simplex method—that is, the inequalities are treated as a special kind of constraint and handled outside the linear programming basis like variable upper bounds in the respective algorithm (Schrage, 1975; Todd, 1982; Ogryczak, 1992). It leads to more complex formulas for the simplex steps but, on the other hand, limits the explicit basis representation to the size of the original problem and thus allows us to take advantage of the special basis structure. Thus the minimax criterion function (10) does not destroy the network structure of the problem, as the standard network simplex algorithm (Grigoriadis, 1986) can be adapted for this criterion.

5.2 Constraints

A set of feasible location and allocation decisions can be restricted by some additional requirements. Very often restrictions are imposed on the number of facilities, p, to be established. Let $p^{\min}$ and $p^{\max}$ be the minimum and maximum number of facilities to be built. In general, $0 \leqslant p^{\min} < p^{\max}$, but it is quite permissible to have $p^{\min} = p^{\max}$ to ensure that exactly p facilities are to be located (as in the case of the p-median problem). Furthermore, a decision situation may require a limit to the number of facilities in some regions (areas, jurisdictions) or prohibit an allocation of a demand in one areal unit to a supply facility in a different area. All these requirements are easily implementable by means of the GNM with selections. One simply needs to define a selection, S_r, as a set of potential nodes (represented in a GNM by arcs) and then the constraint (6) can be used to define the corresponding requirement. This means that the number of facilities in region r ($r = 1, 2, ..., z$) lies within a given minimum ($p_r^{\min}$) and maximum ($p_r^{\max}$) value.

In many approaches to location-allocation problems there is a requirement that the entire quantity from a node be assigned to exactly one facility (for example, if an area is serviced by only one center). Thus, an all-or-nothing rule is applied to the allocation of a quantity. This requirement can be implemented in a GNM with the selection mechanism. Let node i be allocated according to the all-or-nothing rule. One can set a selection S_1, defined as $S_i = \{(i, j): j \in N\}$, or $S_i = \{(j, i): j \in N\}$, depending on the orientation of the allocation and $p_i^{\min} = p_i^{\max} = 1$. The requirement is then guaranteed by the standard selection constraint (6).

The flow of quantities along an arc (i, j) can be required to stay within specified minimum and maximum limits (that is, $h_{ij}^{\min}$ and $h_{ij}^{\max}$, respectively). It can be implemented in a GNM by minor modification of the capacity constraints. Namely, inequalities (4) need to be replaced with the following:

$$h_{ij}^{\min} x'_{ij} \leqslant x''_{ij} \leqslant h_{ij}^{\max} x'_{ij}, \qquad \text{for} \quad (i, j) \in E. \tag{11}$$

In order to ensure operating efficiency for an individual facility, a lower bound can be imposed. Similarly, an upper bound can be placed on the size of the facility to ensure effective utilization. If the potential facility is considered to be a transshipment node, then one may want to ensure that the quantity shipped to and from the facility lies within a specified minimum and maximum value. This means that storage capacity can be incorporated as an upper bound on the allocation variable. On the other hand, a minimum flow is required to ensure economic efficiency. As the facilities are represented in a GNM by arcs, this requirement imposes some lower and upper bounds on flows along the arcs, and these are implemented with constraints (11).

In our network model it was assumed that nodes are characterized by some specified amount of attribute or quantity, b_i (supply or demand). In a real-life problem, instead of exact amounts some minimum ($b_i^{\min}$) and maximum ($b_i^{\max}$) amount of an attribute at node i may be specified. To implement this extension in a GNM, one needs a minor modification of the balance constraints, which is that inequalities (3) need to be replaced with the following:

$$b_i^{\min} \leqslant \sum_{j \in N} x''_{ij} - \sum_{j \in N} x''_{ji} \leqslant b_i^{\max}, \qquad \text{for} \quad i \in N. \tag{12}$$

Some requirements may be introduced to define the desirable structure of the location–allocation system. For example, $c_{ij}^{\min}$ can be used as the minimum distance between an obnoxious facility and a population center and $c_{ij}^{\max}$ can be considered as a travel-time standard or any other limit that the decisionmaker may wish to impose on locating an emergency facility. However, just as with the budget constraints, having allowed an interactive multiple criteria analysis (Hultz et al, 1981), one can easily deal with these characteristics as criterion functions [that is, similar to function (10)]. Such an approach allows the decisionmaker to learn all the relations between these and other characteristics of the solution.

The set of criterion functions (7)–(10) and the constraints can be considered as the basic building blocks of a substantive model of locational decisionmaking. Indeed, with the use of an appropriate set of criterion functions and constraints, it is possible to formulate almost any type of single-criterion location problem. The well-known elementary location problems (such as p-median, p-center, plant location, and transshipment problems) can be structured by means of GNMs. It should be emphasized that these basic models can be easily extended and modified to describe fairly complex locational decisionmaking situations (see Ross and Soland, 1977; Handler and Mirchandani, 1979; Hillsman, 1984; Colorni, 1987). Most importantly, however, an equivalent formulation of the criterion functions and constraints can be expressed in the form of a GNM for the multiple criteria location problem.

6 Generating the set of efficient solutions

6.1 Techniques for generating efficient solutions

The first step in searching for the best or most preferred decision outcome is to generate a set of efficient solutions (locational patterns with nondominated outcomes). Several techniques for generating efficient solutions are available (for an overview see, for example, Chankong and Haimes, 1983; Steuer, 1986). Here we are concerned with the three most commonly used methods for tackling MCLPs. These techniques include: the weighting method, the noninferior set estimation (NISE) method, and the constraint method (see Cohon 1978). A common feature of these techniques is that they transform the multiple criteria model (1)–(2) into a single-criterion form to generate one efficient soloution and then, by parametric variation of the single-criterion problem, the complete set or a subset of efficient solutions can be generated. The basic difference among the three methods lies in how they make the transformation from a multiple to a single-criterion problem.

The weighting method involves assigning a weight, w_v ($v = 1, 2, ..., k$), to each of the criterion functions, F_v. The multiple criteria function (1) can then be converted into a single-criterion form through the linear combination of the criteria together with the corresponding weights. Thus, the problem (1)–(2) can be transformed into the following form:

$$\text{minimize } w_1 \mathrm{F}_1(\boldsymbol{x}) + w_2 \mathrm{F}_2(\boldsymbol{x}) + ... + w_k \mathrm{F}_k(\boldsymbol{x}),$$

subject to

$$\boldsymbol{x} \in A, \quad \text{and} \quad w_v \geqslant 0, \qquad \text{for} \quad v = 1, 2, ..., k;$$

and consequently, the problem can be solved by means of standard linear programming methods. The set of efficient solutions to the original problem (1)–(2) is generated by parametric variation of the weights.

The NISE method is an extension of the weighting technique. The essential difference between these two approaches is that the NISE method allows for a control of the accuracy of the efficient set approximation. The method operates by finding a number of efficient solutions, via the weighting technique, and evaluating the properties of the line segment between them (for a detailed discussion of the NISE method for bicriterion problems see Cohon, 1978; and for three-criterion problems see Balachandran and Gero, 1985).

Another approach that can be used to generate the set of efficient solutions is the constraint method. Like the weighting and NISE methods, it is based on the idea of converting the multiple criteria optimization problem to a single-criterion one. This can be done by maximizing only one of the criterion functions whereas all the others are converted into inequality constraints. Thus, the multiple criteria problem (1)-(2) can be transformed to the following single-criterion problem:

$$\text{minimize } \mathrm{F}_l(x)$$

subject to

$$x \in A, \quad \text{and} \quad \mathrm{F}_p(x) \geqslant \varepsilon_p, \qquad \text{for} \quad p = 1, 2, ..., l-1, l+1, ..., k,$$

where ε_p is the minimum allowable level for the pth criterion function. The set of efficient solutions can be generated by solving the single-criterion problem with parametric variation of the ε_p.

A comparative analysis of the weighting method, NISE, and the constraint method can be found in Balachandran and Gero (1984). Current et al (1990) provide a comprehensive review of papers concerned with the application of generating techniques to multiple criteria analysis of facility location decisions. This review includes forty-five articles dealing with public and private facility location decisions. The following discussion provides a selective and critical overview of studies on generating the set of efficient solutions for MCLPs to complement the survey by Current et al (1990).

6.2 Advantages and disadvantages

The main advantage of the generating techniques is that they require a very limited amount of information to be provided by the decisionmaker in order to solve a MCLP. In essence, an assumption that 'more is better' or 'less is better' is all that is needed to solve a MCLP by means of generating techniques. It is argued that decisionmakers involved in searching for the best decision outcome are not able or are reluctant to articulate explicitly their preferences. For this reason, some location analysts strongly advocate this approach to MCLPs (for example, ReVelle et al, 1981). It is suggested that decisionmakers are more comfortable with articulating their preferences once the possible decision outcomes and the trade-offs involved are presented to them and clearly understood.

One of the most significant shortcomings of the generating techniques is that they are of limited applicability for large-sized problems. The generating techniques are very intensive, computationally. They can be tedious and expensive. For example, the computational requirements for the weighting and constraint methods depend on the number of criterion functions and the number of weights or constraints imposed for each criterion outcome (Cohon, 1978; Balachandran and Gero, 1984). To be more specific, there is an exponential relationship between the number of criterion functions and the computational burden. One should point out that the weighting and constraint methods do not guarantee an exploration of all 'important' segments of the efficient set. It happens especially for discrete decision problems, that is also for MCLPs. As the resulting subset of efficient solutions depends on the particular weights or constraints applied, the techniques will not necessarily generate a good representation of the entire efficient set. One possible way of handling this problem is to reduce the scale of weights or the intervals of the constraints.

However, this will increase computational burden. Also, the practical question remains of how to vary the weights or constraint intervals so that a representative subset of efficient space can be generated (Chankong and Haimes, 1983).

To some extent the NISE method avoids this drawback. It allows a quick and good approximation of the set of efficient solutions (Cohon, 1978). Although the NISE technique guarantees a representative coverage of the efficient set for multiple criteria linear problems, it is not possible to explore nonconvex portions of the efficient set with this method. Also, the weighting method cannot provide information about nonconvex segments of the efficient space. It is particularly true when the weighting or NISE methods are used to solve integer or mixed-integer linear programs (this is also typical of MCLPs). Although the NISE method can be used to solve problems involving more than two criteria (Balachandran and Gero, 1985), it is, essentially, applicable to bicriterion linear problems. Note that the efficient set for a bicriterion problem can be easily identified with the standard parametric procedures (compare Prasad and Karwan, 1992). Solanki (1991) has developed an algorithm to approximate the bicriterion integer noninferior set (ABIN), which avoids this weakness of the weighting and NISE methods. ABIN can generate a representative subset of the efficient set even if the set is nonconvex. This is achieved by applying an augmented weighted Chebyshev metric for measuring the distance between an ideal outcome and efficient criterion outcomes (see also Steuer, 1986). It should be emphasized that both the NISE and the ABIN methods are designed primarily for handling bicriterion decision problems (for applications of these methods to bicriterion location–allocation problems see Schilling, 1980; Storbeck and Vohra, 1988; Church et al, 1991; 1992).

The major practical disadvantage of the generating techniques is that the size of the approximate efficient set is usually large for a real-life MCLP. The following hypothetical problem illustrates this point.

6.3 Example 3

Consider the central facility location problem we have discussed in previous sections (examples 1 and 2 in sections 3 and 4). Further, let us assume that the decisionmaking problem involves an optimization of three criterion functions. The first criterion is to maximize the level of user satisfaction for a location pattern of central facilities. In order to measure the level of user satisfaction, let us assume that the space discount parameter, α, has been determined empirically by the calibration of a spatial interaction model. Given a value of α equal to 0.05, an exponential distance decay function can be used to measure the level of user satisfaction for alternative locational patterns, and the criterion function is to be maximized and can be written in terms of the GNM as follows:

$$\mathrm{F}_1(\boldsymbol{x}) = \sum_{(i,j)\in E} \exp(-\alpha d_{ij}) x''_{ij} . \tag{13}$$

The second criterion involves minimization of fixed (c'_{ij}) and variable (c''_{ij}) costs. It is assumed that the fixed costs do not change with the amount of services offered (for example, the size of facility), but they vary geographically. The variable costs vary locally and increase proportionally along with the size of facility. Thus, the criterion can be formalized in terms of a GNM as follows:

$$\mathrm{F}_2(\boldsymbol{x}) = \sum_{(i,j)\in E} (c'_{ij} x'_{ij} + c''_{ij} x''_{ij}) . \tag{14}$$

The third criterion is to minimize maximum distance between facility and demand locations. In this case the criterion function (10) of a GNM can be employed. Thus, the criterion is to minimize the following function:

$$F_3(\boldsymbol{x}) = \underset{(i,j) \in E}{\text{maximum}}\, d_{ij} x'_{ij} . \tag{15}$$

An optimization of these three criterion functions is subject to the following constraints:

$$\sum_{j \in N} x''_{ij} - \sum_{j \in N} x''_{ji} = b_i , \qquad \text{for} \quad i \in N , \tag{16}$$

$$x''_{ij} \leqslant h^{\max}_{ij} x'_{ij} , \qquad \text{for} \quad (i,j) \in E , \tag{17}$$

$$\sum_{(i,j) \in S_r} x'_{ij} \leqslant p_r , \qquad \text{for} \quad r = 1, 2 , \tag{18}$$

$$x''_{ij} \geqslant 0 , \qquad x'_{ij} = 0 \text{ or } 1 , \qquad \text{for} \quad (i,j) \in E . \tag{19}$$

The set of constraints (16) guarantees that the demand of every consumer is satisfied. The constraints (17) ensure that the capacity of any facility will not be exceeded and consumers will be supplied only from an open facility. Constraints (18) require that no more than p facilities will be located in a subregion r ($p_1 = p_2 = 2$) and prevent demand nodes in S_1 being allocated to facilities located in S_2, and nodes situated in S_2 being assigned to facilities in S_1. The data for the problem (13)–(19) are given in table 1.

Before we proceed to the solution of this problem by means of a generating technique, it is useful first to discuss briefly the concept of the payoff matrix in multiple criteria analysis. The matrix can be obtained by optimizing each criterion function separately. Specifically, the following single-criterion programs are solved:

$$\text{optimize } \{F_v(\boldsymbol{x}) \colon (\boldsymbol{x}) \in A\} , \qquad v = 1, 2, \ldots, k .$$

As a result, a square-matrix $\boldsymbol{R} = (q_{vt})$ ($v = 1, 2, \ldots, k$; $t = 1, 2, \ldots, k$) is obtained. This matrix allows for identification of the individual maximum and individual minimum of each criterion function under a given set of constraints—that is, ideal (utopia) and nadir vectors can be defined.

The vector with elements q_{pp}, that is, the diagonal of $\boldsymbol{R}$, defines the ideal point. This point, denoted further by q^b, is usually not attainable but it can be presented to the decisionmaker as a limit to the best numerical values of the criteria. To be more precise, it provides the decisionmaker with lower limits for minimized criterion functions and upper limits for the functions to be maximized.

Table 1. Data for the sample central facility problem.

Facility site S_j or P_j	Distance (km) from demand node D_i												Maximum capacity $h_j^{\max}$	Costs	
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}		fixed c'_{ij} (1000 \$)	variable c'_{ij} (\$ per unit)
S_1	24	30	62	0	30	36	∞	∞	∞	∞	∞	∞	9000	350	30
P_1	0	45	77	14	44	50	∞	∞	∞	∞	∞	∞	5000	1000	35
P_2	45	0	32	30	60	31	∞	∞	∞	∞	∞	∞	6000	1250	25
P_3	50	31	45	36	65	0	∞	∞	∞	∞	∞	∞	6000	1200	36
S_2	∞	∞	∞	∞	∞	∞	16	0	30	33	30	31	8000	300	32
P_4	∞	∞	∞	∞	∞	∞	0	16	30	52	40	48	5000	900	29
P_5	∞	∞	∞	∞	∞	∞	49	33	63	0	25	37	6000	1350	37
P_6	∞	∞	∞	∞	∞	∞	48	32	62	38	57	0	6000	1250	36
Demand[a]	25	25	21	20	19	20	16	22	15	21	13	23			

Note: S_j existing sites; P_j potential sites.

[a] Figures expressed in hundreds of tons.

The νth column of the matrix $\boldsymbol{R}$ represents values of the criterion function F_ν obtained during several optimizations. Let q_ν^{w} be the worst value; that is,

$$q_\nu^{w} = \max_{1 \leqslant p \leqslant k} q_{p\nu} \quad \text{or} \quad q_\nu^{w} = \min_{1 \leqslant p \leqslant k} q_{p\nu} ,$$

if the criterion function F_ν is to be minimized or maximized, respectively. Vector $\boldsymbol{q}^{w}$ is called the nadir vector.

It should be emphasized that the nadir vector represents an estimate of the worst criterion values over the efficient set, that is, the payoff matrix may provide overestimation or underestimation of the actual worst criterion outcomes. Generally, the use of these estimates does not lead to computational problems if the nadir vectors are provided purely for information. It is also possible that a payoff matrix contains a dominated solution. For this reason the matrix should be used with caution during the computational process (Steuer, 1986).

Table 2. The payoff matrix.

Optimized criterion function	Criterion value		
	$F_1(\boldsymbol{x})$	$F_2(\boldsymbol{x})$ (\$)	$F_3(\boldsymbol{x})$ (km)
$F_1(\boldsymbol{x})$	12775	33229000	38
$F_2(\boldsymbol{x})$	10979	26247000	77
$F_3(\boldsymbol{x})$	11759	28702700	36
Ideal vector ($\boldsymbol{q}^{b}$)	12775	26247000	36
Nadir vector ($\boldsymbol{q}^{w}$)	10979	33229000	77

The payoff matrix for the problem (13)–(19) is given in table 2. The information about the range of possible outcomes (the ideal and nadir vectors) can be used to generate the set of efficient solutions to the problem (13)–(19). Specifically, we have applied the constraint method (for real-life application of this method to location problems see Cohon et al, 1980; Sewell, 1990). In order to use this method, the multiple criteria model (13)–(19) is adjusted to the following single-criterion form:

maximize function (13), subject to: constraints (16)–(19),

and

$$\sum_{(i,j) \in E} (c_{ij}' x_{ij}' + c_{ij}'' x_{ij}'') \leqslant C , \tag{20}$$

$$d_{ij} x_{ij}' \leqslant d , \tag{21}$$

where C is the maximum allowable amount of money for establishing and operating the central facility system and d is the maximum allowable distance that may separate node i from its nearest facility, j.

An approximation of the set of efficient solutions to the MCLP (13)–(19) can be generated by solving the single-criterion model for a range of values C and d. Specifically, the problem has been solved for various combinations of these two parameters with the use of the LINDO package (Schrage, 1991). It was decided to set thirty combinations of constraints C and d resulting in nine efficient solutions. The results are shown in table 3.

The efficient solutions are well distributed over the entire efficient set. The results provide important information about the shape of the set of efficient solutions, the range of possible decision outcomes, and the trade-offs involved. In spite of the fact that this information is very useful in searching for the best decision outcomes and corresponding location–allocation pattern, a decisionmaker is likely to find it difficult to choose the best solution even for this very small location–allocation problem. To this end, it is suggested that graphic presentation techniques can be used to support the decisionmaker in analyzing alternative solutions and in arriving at a preferred decision.

Table 3. The set of efficient solutions for the sample central facility location problem.

Solution	Criterion outcome		
	$F_1(x)$	$F_2(x)$ (\$)[a]	$F_3(x)$ (km)
1	12775	33229	38
2	12691	32224	38
3	12165	30809	77
4	12156	31779	45
5	12081	29769	77
6	11759	28703	36
7	11224	28258	45
8	11149	26253	77
9	10979	26247	77

[a] Figures expressed in thousands.

6.4 Graphic presentation techniques

An important distinction must be made between graphic techniques for presenting information about alternative solutions in decision space and criterion space (Schilling et al, 1982; Church et al, 1992). Decision space consists of two types of decision variables: a set of 0–1 locational variables and a set of 0–1, or integer or continuous allocation variables (see section 2). A combination of these two types of variables defines a location–allocation pattern or spatial pattern. Criterion outcome space (criterion space) represents the performance of a particular spatial pattern in terms of several criteria. Given this distinction, different graphic presentation techniques are used to display information about alternative solutions to MCLPs in decision and criterion space.

Cartographic techniques are typically used to represent alternative solutions in decision space (Allard and Hodgson, 1987). Armstrong et al (1992) provide a review of a wide variety of cartographic displays for locational decisionmaking. In particular, monoplan displays and delta displays can be used to visualize solutions to MCLPs in decision space. The monoplan display techniques are designed to show a single location–allocation pattern. They include center-border displays, center-region displays, nodalchromatic maps, and spider diagrams. Center-delta and allocation-delta displays are two techniques that can be used to compare two solutions in decision space. In general, the spider displays are more effective in conveying the information about solutions when a small number of fixed nodes (demand points) are allocated to each potential node (supply point), whereas other displays may be more effective in visualizing spatial patterns that involve a large number of location and allocation variables (Armstrong et al, 1992).

There are a number of graphic techniques for visualizing alternative solutions to multiple criteria decision problems in criterion space (Schilling et al, 1983;

Maclaren, 1988; Klimberg, 1992). Most of these techniques were originally developed for the statistical analysis of multivariate data (du Toit et al, 1986). Such multivariate data displays as the bar charts, scatterplots, profiles, spider-web charts, glyphs, Chernoff faces, and Andrews's curves can be applied to visualize alternative solutions to MCLPs. Value path displays are probably the most effective methods for visualizing in criterion space (Schilling et al, 1983). The most serious shortcoming of these techniques is their limited applicability to problems involving either a large number of criteria (for example, glyphs) or a large number of alternative solutions (such as Andrews's curves, value paths).

The purpose of visual representation is to provide the decisionmaker with insights into solutions to multiple criteria decision problems not readily obtained by nonvisual methods (for example, tabular display). However, one should point to the possibility of bias in the perception of alternative solutions in decision and criterion space. Aspects of the information about alternative solutions that might be missed when visualizing in decision space might become apparent when viewing in criterion space (Steuer, 1986). This issue is of particular importance in multiple criteria location–allocation analysis. It can be argued that the spatial patterns that seem to be 'insignificantly' different when viewing in decision space, might vary 'significantly' in criterion space and vice versa. Example 4 illustrates this point.

6.4.1 *Example 4*

Consider the solutions 8 and 9 (table 3) to the central facility location problem (examples 1, 2, and 3). We have also generated another solution (solution 10) that is characterized by the following criterion outcomes: $F_1(\boldsymbol{x}) = 11032$, $F_2(\boldsymbol{x}) = \$26273000$, and $F_3(\boldsymbol{x}) = 77$ km.

These three alternative solutions are visualized in criterion space by value paths (figure 5). Each criterion outcome is represented as a percentage deviation from the ideal (or nadir) value. At first sight it can be argued that the three solutions are insignificantly different. Their performance is the same with respect to the maximum distance criterion $F_3(\boldsymbol{x})$. The differences are negligible for the total cost criterion $F_2(\boldsymbol{x})$. The deviations from the nadir value with respect to the accessibility criterion $F_1(\boldsymbol{x})$ are 0%, 3%, and 9% for solutions 9, 10, and 8, respectively. A closer inspection of the value paths indicates that solution 10 is dominated by solution 8. The latter performs slightly better than the former with respect to accessibility and cost criteria, and the alternatives are the same on the maximum distance criterion.

Most importantly, however, these three alternatives might be considered significantly different when viewed in decision space. Figure 5 shows the alternative solutions by means of spider displays. Although the spatial patterns are the same in the North subregion, the allocation patterns in the South subregion vary considerably from one solution to the other. Given the analysis of the alternative solutions in decision and criterion space, one can argue that the choice of the most preferred solution will depend on the decisionmaker's perception of the location–allocation patterns and on his or her preferences with respect to the evaluation criteria.

Clearly, it is important to represent and analyze alternative spatial patterns and associated criterion outcomes within the context both of decision and of criterion space. The quality of the graphic presentation and the way of conveying the information to the decisionmaker might significantly affect the decisionmaking process (Schilling et al, 1983; Klimberg, 1992). To this end, the visual display techniques should be considered a part of interactive decision support approaches that allow decisionmakers to analyze alternative spatial patterns and associated criterion outcomes with respect to their preferences and priorities.

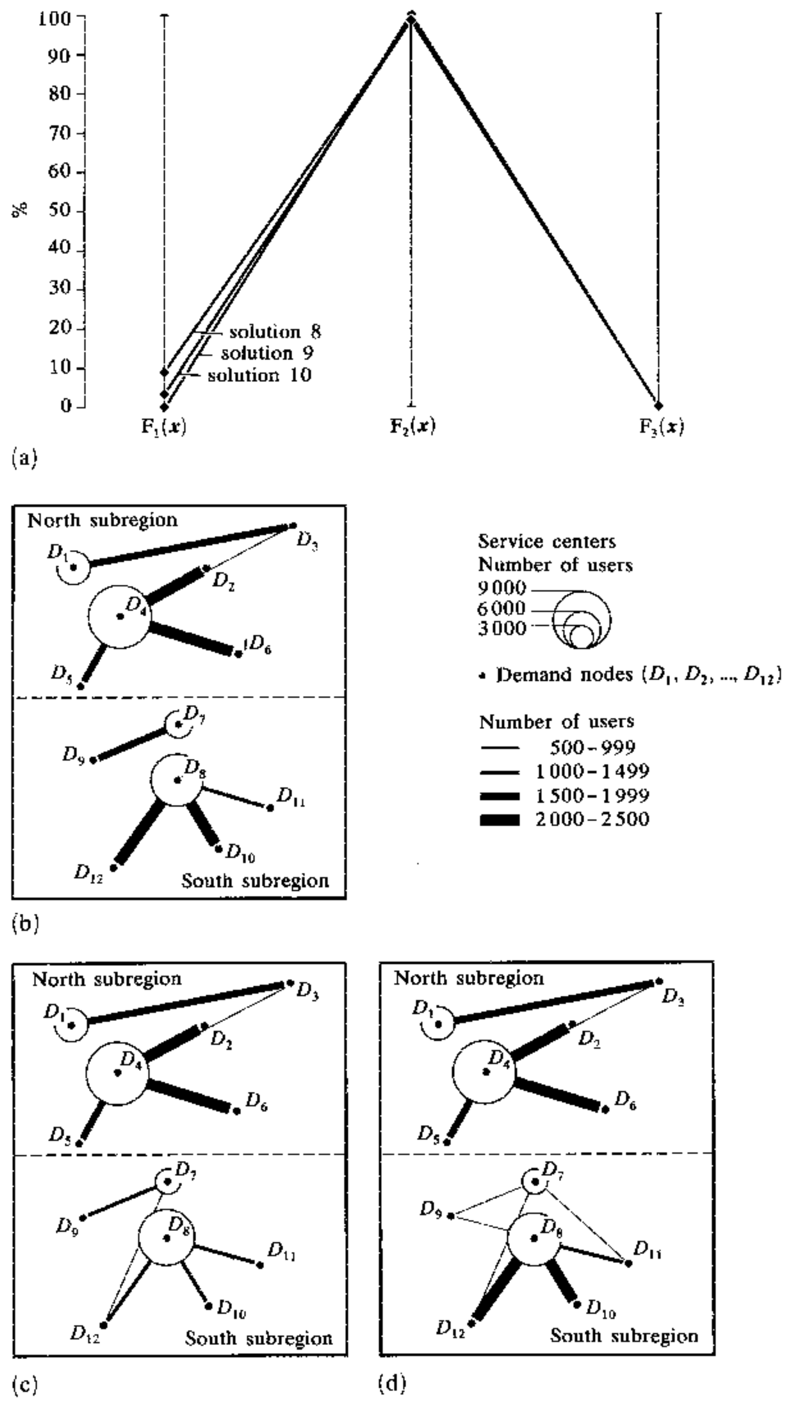

Figure 5. Alternative solutions to the sample problem of central facility location: (a) value paths, (b) location-allocation pattern for solution 8, (c) location-allocation pattern for solution 9, (d) location-allocation pattern for solution 10. Note: $F_1(x)$ is the users' satisfaction criterion, $F_2(x)$ is the total costs criterion, $F_3(x)$ is the maximum distance criterion. The criterion outcomes are represented as percentage deviations from the nadir value ($q_v^w = 0$) and ideal value ($q_v^b = 100$).

6.5 Generating techniques and interactive decision support

As location–allocation problems typically involve hundreds of decision variables and constraints, and several criterion functions, it can be argued that the generating techniques provide limited support for the decisionmaker in the choice phase of the decisionmaking process. For this reason some analysts suggest that the generating techniques are best suited to integration with the subsequent interactive approach to multiple criteria decisionmaking (Chankong and Haimes, 1983).

This point has been supported by empirical research on information processing in decisionmaking. Payne (1976) pointed to an important relationship between the amount of information provided to decisionmakers and the proportion of this information used by them (see also Kok, 1986). The general principle suggests that the percentage of information used by an individual decreases with an increase in the amount of information available. For example, it is possible to define the percentage of information used by a decisionmaker if he or she were provided with the information on the set of efficient solutions for the location–allocation problem discussed earlier (example 4, table 3). According to Payne's (1976) experiment, the decisionmaker would use approximately 60% of the information on the set of efficient solutions.

Perhaps the most important feature of the generating techniques is that they can be used to classify the set of feasible solutions into two categories: efficient and nonefficient solutions, and consequently all nonefficient decisions can be discarded from further consideration. Even so, there still remains the need for a DSS to help the decisionmaker in choosing the best solution from among the set of efficient decisions. In order to choose the best solution some information about the decisionmaker's preference structure must be obtained. The decisionmaker, working interactively with the DSS, has to specify his or her current preferences in terms of some control parameters and the DSS provides the decisionmaker with an efficient solution that is the best according to the specified control parameters. For such an analysis, however, there is no need to identify the entire set of efficient solutions prior to the analysis. Contemporary optimization software is powerful enough to be used on-line for direct computation of the best (in terms of the specified control parameters) efficient solution at each interactive step. Thus the DSS can generate at each interactive step only one efficient solution that meets the current preferences. Such a DSS can be applied for analysis of decision problems with small, large, and even with infinite (which may occur in the case of continuous decision variables) sets of efficient solutions. This leads us to the preference-based methods and techniques. In paper 2 we will focus on the preference-based approaches to MCLPs.

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The multiple criteria location problem: 2. Preference-based techniques and interactive decision support†

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Abstract. This is the second of two papers in which multiple criteria location problems (MCLPs) are discussed. In this paper two major approaches to locational decisionmaking are overviewed: optimizing decision rules (utility-function-based methods) and satisficing decision rules (goal-programming methods). Their advantages and disadvantages are discussed. From these two concepts a quasi-satisficing decision rule is developed and operationalized through a reference point method. A framework for an interactive decision support system (DSS) for tackling MCLPs is proposed. The system integrates a network model with the quasi-satisficing approach. It is argued that the DSS data and analytical components can be effectively integrated by means of the interactive decision support concept that involves a feedback exchange of information between a decisionmaker and a computer-based support system. This concept allows for the exploration of the locational decision problem and the alternative solutions both in decision space and in criterion outcome space.

1 Optimizing and satisficing decision rules

In paper 1 an overview of various techniques for generating efficient solutions to multiple criteria location problems (MCLPs) was presented (see Malczewski and Ogryczak, 1995). The main purpose of the generating techniques is to determine an exact representation of or an approximation to the set of efficient solutions. The techniques can only be used to classify the set of feasible solutions into two categories: the sets of efficient and nonefficient decisions. A further decision rule is required, therefore, to identify the best alternative from among the set of efficient alternatives according to the decisionmaker's preferences. As defined in paper 1, a set of assumptions that allows for a complete ordering of alternatives is referred to as the decision rule. A number of decision rules are available for making locational decisions (see Isard, 1969). They can be classified into two fundamental categories: the optimizing and satisficing decision rules. In this paper we suggest that these two approaches are not mutually exclusive and it is possible to merge them into a quasi-satisficing framework for an interactive decision support (Wierzbicki, 1982; 1983).

1.1 Optimizing decision rule

Classical location theories are organized around the optimizing decision rule. The fundamental assumption underlying optimal decisions is that the locational player is an economic human (*homo economicus*) who has perfect knowledge of the relevant aspects of the decisionmaking environment, behaves rationally, and is able to analyze the alternative courses of locational strategy fully and comprehensively in order to choose an optimum location. This implies that the decisionmaker has

† Introductory section of paper 1 summarizes the structure of the two papers.

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unlimited information-processing capacity. It is also assumed that the preferences of homo economicus are well-ordered, stable, and given exogenously (see Isard, 1969).

The economic human is an optimizer who is searching for a location that will yield the maximum utility. In the most elementary sense, this process can be described as follows: first, the decisionmaker identifies all feasible locational alternatives; second, he or she orders the alternatives according to his or her preference and, finally, the individual chooses the most preferred (the best) alternative. The crucial question involved in this process is about how the decisionmaker's preference structure translates into complete ordering of the feasible alternatives (see Myers and Papageorgiou, 1991).

1.1.1 *Utility - value function approach*

As stated earlier, in a complex location problem the evaluation criteria are conflicting and noncommensurable (see paper 1). Consequently, there does not exist a point in the set of feasible alternatives A, which simultaneously maximizes all k criterion functions. Hence, in addition to the specification of a decisionmaking problem in the form of an optimization program, such as the multiple criteria location - allocation model, some procedure for identifying the best compromise solution is required. To this end, the classic interactive approaches to multiple criteria decision analysis are based on the assumption that the decisionmaker behaves according to the optimizing or utility-maximizing decision rule (Keeney and Sicherman, 1976). Usually the existence of some individual or group utility - value function (also called a preference function) is assumed (Fishburn, 1970). The utility function approach deals with the case when some probability measure (uncertainty) is incorporated into the decisionmaker's preferences, whereas the value function techniques are used in deterministic problems. In either case, the problem is solved by defining a utility - value function, $\mu[F(x)]$, with the property that if x' and x'' are feasible solutions, then x' is preferred to x'' if and only if $\mu[F(x')] > \mu[F(x'')]$.

The interactive decision support process depends on identification of the utility function (for example, Zionts and Wallenius, 1976). To be more specific, the utility-function-based interactive procedure involves four steps (Keeney and Sicherman, 1976): (a) structuring the multiple criteria decisionmaking problem; (b) quantifying the uncertainties about possible decision outcomes (if the problem involves uncertainties) or determining the decision outcomes associated with alternative locational patterns (under deterministic conditions); (c) translating the decisionmaker's preference structure into a utility - value function; and (d) evaluating the alternative locational patterns. The crucial task of an analyst is then to solve the MCLP (1) by defining the decisionmaker's utility function over the multiple criteria of the problem under consideration [see paper 1, equation (1)]. Given the decisionmaker's utility function, μ, the multiple criteria decision problem (1) can be unambiguously stated as the following utility function program, maximize $\{\mu[F(x)]: x \in A\}$, and solved by means of standard single-criterion mathematical programming techniques (Steuer, 1986).

The utility function can be specified in any mathematical form, providing that it meets a set of underlying axioms. An additive form of the utility function is the simplest and the one most frequently applied to locational choice problems. This approach uses a weighting schema to combine criteria into a single measure of utility. The standard assumptions underlying this method involve preferential independence (that is, the trade-off of pairs of criteria must be independent of the fixed value of any other criterion at hand) and utility independence (that is, the utility of an alternative on a criterion is independent of the outcomes on the other criteria).

The literature on the application of the utility-function-based approach to locational choice is voluminous (see Keeney, 1980, for an overview). This method, particularly of the additive form, has been applied to many real-life and hypothetical locational decisionmaking problems. It has been used mostly for locating major facilities such as power plants, airports, dams, refineries, and waste disposal facilities. Hobbs (1980) provides a critical overview of the application of additive models to power plant siting decisions. DiMento et al (1985) have discussed the utility-based approach to the location of hazardous waste disposal facilities. The utility function approach has been used extensively for locational analysis by Keeney and associates. They applied it to a variety of real-world decisionmaking problems, such as the location of an airport (Keeney, 1973) and the siting of a nuclear plant (Keeney and Nair, 1975; Keeney and Robilliard, 1977).

The common feature of these studies is that the utility-based approaches are used for locational choice from among a small number of alternatives in an environment of uncertainty to resolve difficult public policy problems. To this end, an interesting approach to the utility-function-based decisions has been proposed by Mirchandani and Reilly (1987). These authors incorporated a utility function into a classical p-median model and by doing so they were able to analyze a large set of alternative location–allocation patterns for spatial distribution of fire fighting units.

1.1.2 *Advantages and disadvantages*

An important advantage of the utility function approach is that the multiple criteria function $F(x)$ is reduced to a scalar-valued function. Consequently, the multiple criteria decision problem can be solved by means of single-criterion optimization techniques. This means that the vast body of algorithms, software, and experience that currently exist for single-criterion optimization can be directly applied to tackling multiple criteria problems. This is of major importance considering the extent to which single-criterion optimization has influenced the development of location theories, location–allocation modeling, and the use of computers to solve locational decisionmaking problems (see Rushton et al, 1973; Goodchild and Noronha, 1983; Densham and Rushton, 1992). For instance, the vertex-substitution algorithm, which is one of the most widely used procedures for solving location–allocation problems can be employed for tackling utility-function-based MCLPs. Indeed, Mirchandani and Reilly (1987) have employed this algorithm to solve a location–allocation decision problem. One should point out, however, that the utility function for location decisionmaking is usually nonlinear with respect to decision variables. This in turn may cause difficulties in solving MCLPs by means of standard mathematical programming methods.

A further advantage of the utility function approach is that, if the $\mu[F(x)]$ function is correctly constructed and optimized, the resulting solution is preferred at least as much as any other feasible solution. This means that a best compromise solution will also be an efficient one. This capability of the utility function approach to generate an efficient solution is of particular importance in the context of some other multiple criteria decision methods that can produce inferior solutions (for example, the goal-programming method).

The difficulties in assessing the utility function for the locational choice problem should be emphasized. Usually it is quite difficult, impractical, or even impossible to obtain a mathematical representation of the decisionmaker's preferences. There are two major reasons for this: (1) the procedure for assessing utility functions with even a moderate number of criteria can be very time consuming and tedious, and (2) it places considerable information-processing demands on the decisionmaker.

This is particularly true in the case of an MCLP. For example, Sarin et al (1979) have investigated the difficulties in assessing various preference functions in the context of a real-life study on power plant siting. Similarly, Mirchandani and Reilly (1987) have discussed the difficulties they encountered in estimating the decisionmaker's utility function for the location of a fire fighting unit. In particular, the decisionmakers found it difficult to relate an increase in expected utility to the cost of resources required to achieve that increase. It can be argued that decisionmakers are not able to or are reluctant to articulate their preferences without knowing the possible consequences associated with alternative decisions (ReVelle et al, 1981).

This issue has been discussed by Hobbs (1980) with reference to requirements underlying an additive model for locational choice. He has shown that the weights must be proportional to the relative value of unit changes in their criterion value functions. Further, he argued that many empirical location choice studies did not meet this requirement and consequently the locational choice models inadequately represented the decisionmaker's preferences. It is also likely that the preferential and utility independence axioms will not be met in most real-life location choice problems (ReVelle et al, 1981; Solomon and Haynes, 1984). ReVelle et al (1981) pointed out the fact that the utility function approach neglects the existence of spatial relationships among alternative locations. For example, in the case of the coal-based power station siting problem, the level of emission allowed for a given power plant depends on the emissions from other facilities located in a given region. Furthermore, it can be argued that location choice is a Gestalt process in which alternatives are considered holistically. This means that the value of a criterion function is greater than the sum of its elements. This point is missed in the utility-function-based approach because its components are determined separately and then combined (see French, 1984). Furthermore the utility function approach is focused on the decisionmaker's preferences in criterion space, and fails to analyze the decisionmaker's perception of alternative locational patterns in decision space. It can be argued that the decisionmaker may have significantly different preferences with respect to evaluation criteria if he or she is allowed to analyze alternative solutions in decision space (Steuer, 1986; Church et al, 1992).

Last, one should emphasize that the utility function approach is based essentially on the assumption that once the decisionmaker's preferences have been specified, he or she plays a passive role during the choice process. This implies that the decisiomaker's preferences are assumed to be stable with reference to locational alternatives and time. A number of studies that have attempted to measure utility functions have revealed numerous difficulties concerning the spatial and temporal ordering of decision outcomes as perceived by decisionmakers. Isard (1969) pointed to the fact that "learning, changes in aspiration, and other processes that take place during the actual perception of outcomes over time and [geographical] space may significantly alter the individual's preference pattern" (page 181). Many analysts and practitioners have also stressed the importance of learning during the interactive session with the decision support system, and there are numerous examples in which people systematically violate consistency and coherence of their preferences (see MacLean, 1985).

1.2 Satisficing decision rules

The hypothesis that people seldom maximize some utility function while preparing individual decisions led to approaches based on the bounded rationality or satisficing behavior concept (Simon, 1957). In these approaches, which depend on recurrent observation, it is assumed that people tend to summarize their learning of the state of the world by forming aspirations on desirable outcomes of their decisions. When the outcomes fail to satisfy their aspirations, people tend to seek ways of improving the outcomes. When their aspirations are satisfied, however, they turn their attention to other outcomes. Thus when dealing with locational decisions individuals adjust their aspiration levels to reality on the basis of information about locational alternatives and the decisionmaking environment. An aspiration level can be interpreted as a threshold reference point which is used by an individual as a criterion for evaluating the utility of alternative locations (Wolpert, 1964). It plays an instrumental role in determining whether a satisficing alternative exists among a limited number of locational alternatives at hand. In the locational decisionmaking process the threshold reference point serves as a guideline in the search for a satisficing location. The process of searching is continued until an alternative that meets the aspired levels is found, and finally this alternative is chosen as the satisficing one (Pred, 1967).

Thus, the satisficing behavior assumes that the decisionmaker's preferences are represented by a binary-valued ordinal utility function, referred to as a satisficing utility function, $s[F(\boldsymbol{x})]$. Given the attainable outcomes, Y_a, the decisionmaker differentiates the set of feasible outcomes by setting his or her aspiration levels, a_ν, for each $\nu = 1, ..., k$ and by then assigning to each possible decision outcome one of two categories—satisfactory or unsatisfactory; that is to say, the satisficing utility function $s[F(\boldsymbol{x})] = 1$ when $F_\nu(\boldsymbol{x}) \geqslant a_\nu$, for all $\nu = 1, ..., k$, and $s[F(\boldsymbol{x})] = 0$ if $F_\nu(\boldsymbol{x}) < a_\nu$, for at least one ν.

The satisficing decision rules involve a dynamic search for the best (satisficing) locational alternative (Rees, 1974). Simon (1979) argues that a decisionmaker "copes with the complexity that confronts him by highly selective serial search of the environment, guided and interrupted by the demands of his motivational system, and regulated, in particular, by dynamically adjusting multidimensional levels of aspiration" (page 4). An individual is consistently concerned with his or her environment and in the decisionmaking process he or she always relates possible decision outcomes and their consequences to the environment with its unique conditions. The complexity and uncertainty of the environment makes global rationality impossible. Consequently, decisionmakers do not optimize, they instead try to satisfice. It is argued that the decisionmaker is rational only within the limits imposed by a complex, evolving, and partially unknown environment. Therefore, the decisionmaker's preferences are unstable over time. The preferences may also change during the process of searching for a satisficing decision as a result of learning and acquisition of more information about the decisionmaking environment (for example, see Britton, 1974; Rees, 1974).

One may describe a wide range of possible decision situations and associated decision rules that fit into the satisficing behavior (Isard, 1969). Of particular interest in the locational decisionmaking context is a situation in which the levels of aspiration are not attainable. In such a case, the decisionmaker may be concerned with the discrepancy between possible outcomes and his or her aspired goals. Accordingly, the best alternative (decision outcome) is that which most nearly approximates his or her stated goals. This idea underlies the goal-programming approach.

1.2.1 *Goal-programming methods*

The satisficing behavior concept can be operationalized in terms of goal programming, so that the decisionmaker's preferences, specified in the form of a series of goals or aspiration levels, are incorporated in an operational model of search for satisficing decision outcomes.

The goal-programming method, originally proposed by Charnes and Cooper (1961), is now probably the most widely used approach to handling multiple criteria decisionmaking problems in general (White, 1990), and MCLPs in particular (for examples, see Charnes and Storbeck, 1980; Schniederjans et al, 1982; Kwak and Schniederjans, 1985; Min, 1987).

The goal-programming approach requires the decisionmaker to specify *the most wanted value* for each criterion as the aspiration level. The criteria (1) [see equation (1), paper 1] are then transformed into goals:

$$F_v(\boldsymbol{x})+d_v^- - d_v^+ = a_v, \quad \text{for} \quad v = 1, ..., k, \tag{1}$$

$$d_v^-, d_v^+ \geqslant 0, \qquad d_v^- d_v^+ = 0,$$

where a_v is the aspiration level for the vth criterion and d_v^-, d_v^+ are the negative and positive goal deviations, respectively; that is, nonnegative state variables which measure deviations of the current value of the vth criterion function from the corresponding aspiration level.

An optimal solution is then understood as the one that minimizes the deviations from the aspiration levels. Various measures of multidimensional deviations were introduced. They are expressed as the so-called achievement functions. Accordingly, a range of goal-programming forms has been proposed. Specifically, three basic approaches to goal programming can be distinguished: (a) weighted goal programming; (b) Chebyshev goal programming; and (c) lexicographic goal programming. These three formulations are also known as minisum, minimax, and preemptive priority goal programming.

Weighted goal programming The simplest form of achievement function was introduced by Charnes and Cooper (1961) as a sum of weighted deviations, that is

$$g(\boldsymbol{d}^-, \boldsymbol{d}^+) = \sum_{v=1}^{k} (w_v^- d_v^- + w_v^+ d_v^+), \tag{2}$$

where w_v^- and w_v^+ are weights corresponding to several goal deviations. The weights represent, in fact, additional information reflecting the decisionmaker's preferences with respect to the deviation variables. Therefore they must be considered as additional parameters (data) of the goal-programming model specified by the decisionmaker. It is never explicitly pointed out but, because of the goal-programming philosophy, it is understood that all the weights are nonnegative. Moreover, it is assumed that the positive and negative deviations of the criterion outcomes from aspired goals are equally undesirable; that is, the decisionmaker perceives both overachievement and underachievement of specified goals as equally undesirable outcomes. In this sense, the decisionmaker behaves according to a strictly satisficing principle. (For applications of the weighted goal-programming method to locational decisionmaking see Warczberger, 1976; Kwak and Schniederjans, 1985.)

Chebyshev goal programming This method can be considered as a specific form of the weighted goal-programming approach. In particular, the achievement function (2) can be recognized mathematically as the weighted l_1 norm. Using other l_p norms to measure multidimensional distances, one gets other reasonable achievement functions defined as follows:

$$g(\boldsymbol{d}^-, \boldsymbol{d}^+) = \left[\sum_{v=1}^{k} (w_v^- d_v^- + w_v^+ d_v^+)^p\right]^{1/p}. \tag{3}$$

In particular, for $p = 2$ one can obtain the classic least squares problem. The l_2 norm is rarely used in goal programming because in the case of linear programming problems it destroys their linear structure. In fact Charnes and Cooper (1961) proposed the weighted linear goal-programming model as an approximation to the least squares problem.

For $p = \infty$ the achievement function (3) takes the form of the weighted Chebyshev norm,

$$g(\boldsymbol{d}^-, \boldsymbol{d}^+) = \underset{\nu = 1, \ldots, k}{\text{maximum}} \, (w_\nu^- d_\nu^- + w_\nu^+ d_\nu^+) \, . \tag{4}$$

The corresponding goal-programming model is referred to as fuzzy goal programming because it reflects a fuzzy approach to mathematical programming (Ignizio, 1982; for application of this method to public and private facility location problems, see also Min, 1987; 1988; 1989). Fuzzy goal programming can be implemented via linear programming techniques, thereby allowing it to protect the linear structure of the original multiple criteria problem.

Lexicographic goal programming Lee (1972) has advanced the goal-programming method by considering a preemptive goal-preference structure. In this method some hierarchy of goals is assumed. A vector of a few achievement functions is constructed,

$$g(\boldsymbol{d}^-, \boldsymbol{d}^+) = [g_1(\boldsymbol{d}^-, \boldsymbol{d}^+), g_2(\boldsymbol{d}^-, \boldsymbol{d}^+), \ldots, g_m(\boldsymbol{d}^-, \boldsymbol{d}^+] \, , \tag{5}$$

where $g_j(\boldsymbol{d}^-, \boldsymbol{d}^+)$ are achievement functions similar to functions (2), (3), or (4), and minimized according to the lexicographic order. This means that the first achievement function is minimized, then, on the set of optimal solutions with respect to the first function, the second function is minimized and so on, until a unique solution is obtained or all the specified functions are minimized. This implies that goals of higher priority must be met before those of lower priority are considered; that is, a preference weight of positive infinity is assigned to a goal of higher priority compared with that of the goal of next lower priority. This approach has been widely used for tackling MCLPs both in the public sector and in the private sector (Lee et al, 1981; Min, 1987; Sinha and Sastry, 1987; Zografos et al, 1989).

1.2.2 *Advantages and disadvantages*

The major advantage of goal programming is its computational efficiency. When we are dealing with multiple criteria linear programs, goal-programming approaches allow us to stay within an efficient linear programming computational environment. The interactive goal-programming analysis can be supported by dual quantities, and sensitivity analysis as the duality theory was developed even for lexicographic goal programming (Ignizio, 1982; Ogryczak, 1988). For instance, Zanakis (1981) has demonstrated some of these properties of goal programming in a real-life decisionmaking situation. He efficiently solved quite a large goal-programming model (175 variables and 81 individual goals grouped into six priorities) for a public facility location problem.

There are several conceptual and technical problems with the use of goal-programming methods for tackling MCLPs. First, the standard goal-programming methods require the decisionmaker to specify fairly detailed a priori information about his or her aspiration levels, preemptive priorities, and the importance of goals in the form of weights (Nijkamp, 1979). One can expect that, in a complex location decision situation, the decisionmaker will find it difficult (or even impossible) to provide the precise information required by these methods. This is particularly true when locational choice involves multiple decisionmakers. Empirical studies showed that decisionmakers found it relatively easy to specify ordinal rankings for goals, but they were unable to derive meaningful preference weights on a cardinal scale (Hotvedt et al, 1982; Solomon and Haynes, 1984). These difficulties are further aggravated when the goals are unrelated to each other (Dykstra, 1984).

Another problem with weighted goal programming is related to the assumption of equal valuation of overachievement and underachievement of specific goals. Because locational criteria are usually either monotonically increasing or decreasing with respect to site suitability, the above assumption is unlikely to be true in most real-world location choice problems. For this reason some analysts argue that the weighted goal-programming method is inapplicable to most complex locational decisionmaking (Solomon and Haynes, 1984). On balance, Ghosh and McLafferty (1987) suggest that in the context of retail service location, this method provides a flexible tool for locational decisionmaking by facilitating a sensitivity analysis.

A serious weakness of weighted goal programming is its poor controllability of the interactive process (compare with Wierzbicki, 1986) in the case of discrete problems (Hallefjord and Jornsten, 1988). In MCLPs this may mean that some efficient locational decisions (vertices of the convex hull) are very likely to be selected for various aspiration levels and weights, whereas other decisions (in fact, compromise decisions), despite being efficient, are rarely selected except for aspiration levels defined very close to their outcomes.

The lexicographic model can simplify the problem of weight definition because the decisionmaker is only required to specify weights within the groups of goals considered with the same priority level. Albeit, just in this case, usage of weights as control parameters raises the most serious theoretical doubts. Namely, the lexicographic optimization is essentially unstable (Klepikova, 1985). Fortunately, under some reasonable assumptions lexicographic goal programming is stable with respect to the changes of aspiration levels (Ogryczak, 1988) but this is not true with respect to the changes of weights.

Thus, it can be argued that the lexicographic approach at least partially solves the problem of preference weighting. It seems, however, that this method is not in general superior to the cardinal-weight model. In the lexicographic method it is assumed that the higher priority goal is of overriding importance with respect to a goal of the next lower priority, and hence there is no substitution trade-off possible between goals (Warczberger, 1976). Consequently, an alternative locational plan, which performs best on the criterion of highest priority, will always be identified as the best irrespective of its performances on other criteria and no matter how well other alternative plans performed on the other criteria. This property of the preemptive approach seriously limits its applicability to locational decisionmaking problems, which inherently involve conflicting criteria. Therefore, some analysts suggest that the preemptive goal method should always be used along with a sensitivity analysis that can be performed by changing the ordering of the priorities (Kwak and Schniederjans, 1985; Min, 1987).

The problems associated with a priori information required by standard goal-programming methods can be overcome, at least partially, by an interactive approach. To this end, it should be noted that the aspiration levels, preemptive priorities, and weights are considered as a part of data for the goal-programming models. They have to be specified by the decisionmaker. However, they can be changed during the analysis depending on the decisionmaker's learning of the decision problem if a goal-programming model is used as a basis of some interactive decision support approach. This is particularly true in the locational decisionmaking context. There is much evidence to suggest that the decisionmakers tend to develop their preferences and goals during the decisionmaking process (Nijkamp, 1979; Malczewski and Ogryczak, 1990).

For example, Nijkamp (1979) has developed, and applied to MCLPs, an interactive multiple goal-programming method which does not require the decisionmaker to specify explicitly trade-offs or weights. An interactive approach to locational decisionmaking has also been advocated by Min (1988; 1989). In the context of the fuzzy goal-programming application to MCLPs, Min (1989) has pointed to several advantages of this method, such as computational efficiency and flexibility to incorporate imprecise or linguistic goals without a priori information on the lexicographic importance of the goals.

Finally, we should point to the fact that both the cardinal-weight and the preemptive model have a strong tendency to generate inefficient solutions. This is the most important weakness of the goal-programming approach to multiple criteria decisionmaking. For many analysts, this 'inefficiency' problem seriously limits the utility of goal-programming methods as tools for tackling multiple criteria decision problems (Cohon, 1978). The goal-programming approach does not attempt to use additional information to find an efficient solution. Having specified an attainable set of aspiration levels, analysts (decisionmakers) receive exactly what they want even if better decision outcomes are possible. Goal-programming models using achievement functions (2), (3), (4), or (5) often generate inefficient solutions even when nonattainable aspiration levels are specified. They yield only decisions that have the closest outcomes to the specified aspiration levels. This raises a question about whether or not the satisficing model should replace the optimizing one.

1.3 Optimizing versus satisficing decision rules

The fundamental distinction between the two rationality frameworks is the way in which optimal and satisficing alternatives are identified. Recall that, according to the optimizing decision rules, the decisionmaker is assumed to examine all the outcomes and alternative locations in order to choose the best (optimal) one, whereas the satisficing decision rules postulate a search for the best (satisficing) alternative from among a limited number of locational alternatives. Therefore, the utility-maximization behavior and associated optimizing decision rules can be considered as a closed decision model because they in principle disregard the contextual aspects of decisionmaking. By contrast, the satisficing behavior assumes an openness of the decisionmaking environment and stresses the importance of contextual aspects of decisionmaking. Hence, it corresponds to an open decision model (Wilson and Alexis, 1962).

In contrasting these two rationality frameworks, one needs to draw a distinction between their logical and empirical validity. Although proponents of the satisficing behavior concept essentially accept the logical underpinning of the utility-maximization hypothesis, they deny its empirical validity. It is argued that neoclassical theory lacks an explanatory power. The theory offers no explanation of the actual behavior of the decisionmaker because it concentrates on the outcome of the choice process, without indicating how an individual arrives at a decision. Assuming the consistency of the preference-choice structure, this theory stresses an instrumental view of rationality.

The satisficing model postulates procedural rationality. It concentrates on the decisionmaking process rather than on the decision outcome. Satisfying rationality does not guarantee the consistency of the preference-choice structure. As a matter of fact, the decisionmaker's preferences, specified in terms of aspired goals, can be defined intuitively or even in some sense irrationally. Furthermore the satisficing model is formally simpler than the utility-maximization one because it does not call for evaluating the utility on criterion outcomes and does not require comparability of incommensurable criteria. It does not even call for a complete exploration of the decision space. Also, one should point out that the validity of the satisficing behavior hypothesis in explaining locational behavior has been supported by empirical studies (for example, Wolpert, 1964; Walker, 1975). Moreover, there is some evidence to show that an individual does form an aspiration level as a guideline in making decisions (Tietz, 1983).

It seems, however, that the empirical superiority of the satisficing model over the optimizing one has been overemphasized (Boland, 1981). Many authors argue that it is impossible or at least extremely difficult to prove empirically whether an individual is primarily either adoptive (satisficing) or analytic (optimizing) (see Shelly and Bryan, 1964). Most importantly, the satisficing decision rule can be considered as a form of the utility-maximization model or these two concepts can even be interpreted as equivalent logical structures (see Isard, 1969, for a consideration of the satisficing behavior within an optimizing rationality framework).

Furthermore it can be argued that the satisficing model is plausible only in those situations where the process of searching for alternatives is infeasible or too costly. Otherwise, there is no reasonable justification for stopping the search when a satisficing alternative is found, instead of looking for a better alternative or even for an optimal one. This is closely related to Simon's assumptions about the limitations of memory and information-processing capabilities under which a human decisionmaker operates. In this respect, it is important to note that the satisficing rationality concept was proposed in the 1950s (Simon, 1957), and recent advances in computer technology at least partially undermine this assumption. This is especially true in the context of recent development in human being-machine interaction concepts, such as the decision support system (DSS) and expert systems (for a discussion on relevant issues in the locational planning context, see Densham and Rushton, 1987; Waters, 1989).

The above arguments lead us to the conclusion that the optimizing and satisficing behavior models of rationality are not mutually exclusive. The optimizing and satisficing decision rules are equivalent rather than opposite principles.

1.4 The quasi-satisficing decision rule

Taking into account the arguments presented in the previous section, one may develop a framework that merges the optimizing and satisficing decision rules (Wierzbicki, 1982; 1983). It can be argued that an individual has some tendency towards maximization of his or her utility even if he or she behaves according to satisficing rationality principles—that is, forms aspiration levels as a guide for locational decisionmaking. The aspired goals may or may not be attainable. If the specified aspiration levels are attainable, then a better location (alternative decision) may exist; otherwise one does not exist. In the first case, an individual may lose the tendency towards maximization of his or her utility after attaining specified goals or he or she may increase the aspiration levels in order to search for a better alternative. If the levels of aspiration are unattainable, then the decisionmakers have to adjust their behavior to constraints imposed by the decisionmaking environment, but they may still strive to optimize the decision outcomes. Such a behavior is referred to as quasi-satisficing rationality.

Accordingly, an alternative is said to be quasi-satisficing if: (1) there exist a set of criteria and corresponding aspiration levels that describe a satisficing alternative; (2) the alternative in question is efficient (it has nondominated outcomes); and (3) an outcome associated with this alternative is worse than the corresponding aspiration level then it is as close to the aspiration level as possible.

The key element in the quasi-satisficing decision framework is the relationship between the efficient set of solutions and aspired goals. If the decisionmaker behaves according to the quasi-satisficing decision rule then, irrespective of the attainability of his or her aspiration levels, he or she should identify the best (most preferred) alternative as the one which belongs to the set of efficient solutions.

2 Interactive decision support

2.1 Introduction

The distinguishing feature of interactive approaches to multiple criteria decision-making is that these methods do not require a priori information about the decisionmaker's preference structure. The existence of a utility–value function is implicitly assumed and the function is maximized by means of a formal mechanism which involves an interactive exchange of information between a computer-based system (model) and the decisionmaker.

All interactive procedures consist of two phases: a judgmental phase and a computational phase. In the judgmental stage of these procedures the decision-maker analyses and evaluates information provided by a computer-based system and articulates his or her preferences with respect to the values of the criteria. In the computational phase, a solution, or a group of solutions, that meets the decisionmaker's requirements specified in the judgmental phase is generated. This interactive exchange of information is continued until a criterion outcome is deemed acceptable to the decisionmaker. The main idea behind the interactive method is to determine the best (compromise or satisficing) decision outcome from among the set of efficient solutions by means of a progressive communication process between the decisionmaker and a computer-based system (for example, see Nijkamp, 1979; Steuer, 1986).

This general framework provides a basis for various approaches to interactive decisionmaking. An overview of these procedures can be found in Rietveld (1980), and Shin and Ravindran (1991). Applications of interactive approaches to locational decisionmaking have been reported by Nijkamp (1979), Ross and Soland (1980), Hultz et al (1981), Nijkamp and Spronk (1981), Soland (1983), Reeves et al (1988), Ogryczak et al (1989a), and Malczewski and Ogryczak (1990).

Nijkamp and associates appear to be the first to have considered an interactive approach to MCLPs. Nijkamp (1979) discusses a goal-programming-based method to interactive locational decisions. Nijkamp and Spronk (1981) presented an interactive procedure for solving a multiple criteria Weber location problem. This approach requires the decisionmaker to adjust selectively his or her aspiration levels. Ross and Soland (1980) have developed an interactive algorithm that involves the decision-makers in the solution procedure by asking them to compare two efficient solutions and to indicate the one they prefer. This procedure also requires the decision-makers to articulate their aspiration (satisfaction) levels in each judgmental phase. Hultz et al (1981) have subsequently incorporated this algorithm into an interactive computer-based system for locational decisionmaking. Ogryczak et al (1989a) and Malczewski and Ogryczak (1990) have presented an interactive approach to MCLPs that is based on the quasi-satisficing decision rule.

2.2 Aspiration – reservation based decision support

In order to operationalize quasi-satisficing behavior, let us assume that in making locational decisions an individual is supported by a computer-based system. The locational decisionmaking problems are usually ill defined or semistructured. The structured part of the problem can be expressed in the form of a substantive model, for example, a multiple criteria location model which can be structured by means of a generalized network model—GNM (see part 1), whereas the decisionmaker can concentrate on the intangible, unstructured aspects of the locational decision (Massam and Malczewski, 1990).

The best formalization of the quasi-satisficing approach to multiple criteria decisionmaking was proposed and developed mainly by Wierzbicki (1982) as the reference-point method. This is an interactive technique. The basic concept of the interactive scheme is as follows. The decisionmaker forms his or her requirements in terms of aspiration levels. Depending on the specified aspiration levels, a special scalarizing achievement function is built which, while being minimized, generates an efficient solution to the problem. The computed efficient solution is presented to the decisionmaker as the current solution in the form that allows him or her to analyze achievements of this outcome in comparison with the previous solutions and to modify the aspiration levels if necessary. The scalarizing achievement function is slightly similar to a utility function and, in fact, can be used as an approximation to a class of utility functions. It is, however, explicitly dependent on aspiration levels stated and modified by the decisionmaker and therefore it makes operational the concept of adaptive dependence of utility on learning and context. Here, completeness, computational robustness, and controllability of the interactive scheme are more important than consistency and coherence (Wierzbicki, 1986).

The reference-point method has been extended to gain additional information from decisionmakers not only about their aspiration levels, but also about reservation levels that refer to the minimum requirements and correspond to some lower limits of tolerance. Thus, the decisionmaker can specify acceptable as well as required values for given criteria. This concept has been implemented as the so-called aspiration-reservation based decision support (ARBDS) (Lewandowski and Wierzbicki, 1989).

Central to the ARBDS concept is the scalarizing achievement function, which not only guarantees efficiency of the solution, but also reflects the decisionmaker's expectation specified via aspiration and reservation levels. Namely, while building the function, we can make the following assumptions regarding the decisionmaker's expectations:

(A1) the decisionmaker prefers outcomes that satisfy all the reservation levels to any outcome that does not satisfy at least one of the reservation levels,

(A2) provided that all the reservation levels are satisfied, the decisionmaker prefers outcomes that satisfy all the aspiration levels to any outcome that does not satisfy at least one of the aspiration levels.

One of the simplest scalarizing functions can be written as follows:

$$s(\boldsymbol{q}, \boldsymbol{a}, \boldsymbol{r}) = \max_{1 \leqslant \nu \leqslant k} \mathrm{u}_\nu(q_\nu, a_\nu, r_\nu) + \frac{\rho}{k} \sum_{\nu=1}^{k} \mathrm{u}_\nu(q_\nu, a_\nu, r_\nu) , \tag{6}$$

where $\boldsymbol{q}$ is the outcome vector, $\boldsymbol{q} = \mathrm{F}(\boldsymbol{x})$; $\boldsymbol{a}$ and $\boldsymbol{r}$ denote vectors of aspiration and reservation level, respectively; ρ is an arbitrarily small positive number; and u_ν is a function which measures the deviation of results from the decisionmaker's expectations with respect to the νth criterion, depending on the given aspiration level, a_ν, and reservation level, r_ν.

The function, $\mathrm{u}_\nu(q_\nu, a_\nu, r_\nu)$, is a strictly monotonic function of q_ν with value $\mathrm{u}_\nu = 0$ if $q_\nu = a_\nu$, and $\mathrm{u}_\nu = 1$ if $q_\nu = r_\nu$. This function can be interpreted as some measure of the decisiomaker's dissatisfaction with the current value of the νth criterion function. In the case of minimization it can be defined, for instance, as a piecewise linear function as follows (Lewandowski and Wierzbicki, 1988):

$$\mathrm{u}_v(q_v, a_v, r_v) = \begin{cases} \dfrac{-\beta(q_v - a_v)}{(q_v^b - a_v)}, & \text{if } q_v \leqslant a_v, \\ \dfrac{(q_v - a_v)}{(r_v - a_v)}, & \text{if } a_v < q_v < r_v, \\ \dfrac{\gamma(q_v - r_v)}{(q_v^w - r_v) + 1}, & \text{if } q_v \geqslant r_v, \end{cases} \tag{7}$$

where q_v^b and q_v^w denote the best and the worst possible value of the vth criterion, respectively, which are assumed to be known from the predecision analysis, and β and γ are arbitrarily defined positive parameters. $\beta \geqslant 0$ represents additional satisfaction of the decisionmaker caused by achievement better than the corresponding aspiration level, whereas $\gamma > 1$ represents dissatisfaction connected with achievement worse than the reservation level.

In an implementation of the ARBDS system for the multiple criteria transshipment problem with facility location (Ogryczak et al, 1989a) an even simpler type of the function u_v has been used. It is given by

$$\mathrm{u}_v(q_v, a_v, r_v) = \begin{cases} \dfrac{-\beta(q_v - a_v)}{r_v - a_v)}, & \text{if } q_v \leqslant a_v, \\ \dfrac{(q_v - a_v)}{(r_v - a_v)}, & \text{if } a_v < q_v < r_v, \\ \dfrac{\gamma(q_v - r_v)}{(r_v - a_v) + 1}, & \text{if } q_v \geqslant r_v. \end{cases} \tag{8}$$

It is also a piecewise linear function but it does not require any estimation of the best and worst values. Under the reasonable assumption that the parameters β and γ satisfy inequalities $\beta < 1$ and $\gamma > 1$, the achievement functions (8) are convex and thereby they can be modeled via linear programming methodology. Accordingly, the entire scalarizing achievement function (6) can be modeled with linear programming methodology. So the ARBDS approach not only uses the best control parameters of the goal-programming method (aspiration levels), but also keeps its computational efficiency.

2.3 Goal programming and the ARBDS approach

The ARBDS approach to MCLPs, despite being similar to goal programming, seems to have many advantages in comparison with the latter. ARBDS uses only well-defined control parameters (aspiration and reservation levels) whereas goal programming requires that one must also specify some weights. Although ARBDS makes use of fewer control parameters it always generates an efficient solution to the MCLP, whereas goal programming does not. Therefore, it is of interest to find a reason for these advantages and to determine if they really do not apply to goal-programming models. In this section we will show how the ARBDS approach can be modeled via the goal-programming methodology.

The main difference between these two approaches is in the usage of the second reference vector (reservation levels) in the ARBDS approach. The reservation levels can, however, be introduced into the goal-programming model. The simplest way is to build two goals for each criterion function: one associated with deviations from the aspiration level and the second associated with deviations from the reservation level. However, one can avoid this increase of the problem size by using a modeling technique similar to interval goal programming (compare Ignizio, 1982; Ogryczak, 1988)—that is, by transformation of the criterion functions into the following goals (in the case of minimization):

$$F_\nu(x)+d_\nu^- - d_\nu^a - d_\nu^r = a_\nu, \quad \text{for } \nu = 1, \dots, k, \tag{9}$$
$$d_\nu^- \geqslant 0, \quad 0 \leqslant d_\nu^a \leqslant r_\nu - a_\nu, \quad d_\nu^r \geqslant 0, \quad d_\nu^- d_\nu^a = 0, \quad (r_\nu - a_\nu - d_\nu^a)d_\nu^r = 0,$$

where a_ν and r_ν denote aspiration and reservation levels, respectively, for the νth criterion; d_ν^-, d_ν^a, d_ν^r are nonnegative state variables which measure deviations of the current value of the νth criterion function from the corresponding aspiration and reservation levels:

d_ν^- is the negative deviation from the aspiration level,
d_ν^a is the positive deviation from the aspiration level within the interval between the aspiration and reservation level, and
d_ν^r is the positive deviation from the reservation level.

The goals (9) differ from the typical ones (1) only through the splitting of the positive deviation d_ν^+ into a sum of two deviations d_ν^a and d_ν^r, where the first one is limited to the interval between the aspiration and reservation levels, and the second one can be positive only if $d_\nu^a = r_\nu - a_\nu$.

The most important advantage of the ARBDS approach is in its generation of efficient solutions. The basis for this advantage is concealed in the formulas for the scalarizing achievement functions (6) and (7) or (8). Using the three types of deviations defined in equation (9), one can write formulas (7) and (8) as follows:

$$u_\nu(q_\nu, a_\nu, r_\nu) = \begin{cases} -\beta w_\nu^- d_\nu^-, & \text{if } q_\nu \leqslant a_\nu, \\ w_\nu^a d_\nu^a, & \text{if } a_\nu < q_\nu < r_\nu, \\ \gamma w_\nu^r d_\nu^r + 1, & \text{if } q_\nu \geqslant r_\nu, \end{cases} \tag{10}$$

where w_ν^-, w_ν^a, and w_ν^r are positive weights defined depending on the corresponding aspiration and reservation levels, and β and γ are arbitrarily defined positive parameters. Thus like the standard goal-programming techniques the ARBDS approach deals with deviations accompanied by weights, but these weights are now automatically calculated. Provided that $w_\nu^a = 1/(r_\nu - a_\nu)$, as in formulas (7) and (8), the function (10) can be written as

$$u_\nu(d_\nu^-, d_\nu^a, d_\nu^r) = -\beta w_\nu^- d_\nu^- + w_\nu^a d_\nu^a + \gamma w_\nu^r d_\nu^r, \tag{11}$$

which is a weighted sum of the deviations. However, there is one specificity in the function (11). Namely, there is a negative weight coefficient, $-\beta w_\nu^-$, associated with the negative deviation, d_ν^-. This is the reason why the ARBDS approach attempts to reach an efficient solution even if the aspiration levels are attainable. This small change of the coefficient represents, however, a crucial change in the goal-programming philosophy, where all weights are assumed to be nonnegative. Provided that we accept negative weight coefficients, we can consider the function (11) as a specific case of goal programming achievement functions.

Now let us analyze formula (6) defining the final scalarizing achievement function. The scalarizing function is built there as a sum of the Chebyshev norm of the individual achievements u_ν and a small regularization term (the sum of the achievements). Using lexicographic optimization, one can avoid the problem of choosing an arbitrarily small positive parameter ρ [compare equation (6)] and introduce the regularization term as an additional priority level. One can then form the scalarizing achievement function as the following lexicographic goal-programming achievement function:

$$g(\boldsymbol{d}^-, \boldsymbol{d}^a, \boldsymbol{d}^r) = [g_1(\boldsymbol{d}^-, \boldsymbol{d}^a, \boldsymbol{d}^r), g_2(\boldsymbol{d}^-, \boldsymbol{d}^a, \boldsymbol{d}^r)] , \tag{12}$$

$$g_1(\boldsymbol{d}^-, \boldsymbol{d}^a, \boldsymbol{d}^r) = \max_{1 \leqslant \nu \leqslant k} \{-\beta w_\nu^- d_\nu^- + w_\nu^a d_\nu^a + \gamma w_\nu^r d_\nu^r\} ,$$

$$g_2(\boldsymbol{d}^-, \boldsymbol{d}^a, \boldsymbol{d}^r) = \sum_{\nu=1}^{k} (-\beta w_\nu^- d_\nu^- + w_\nu^a d_\nu^a + \gamma w_\nu^r d_\nu^r) ,$$

where w_ν^-, w_ν^a, and w_ν^r are positive weights depending on the corresponding aspiration and reservation levels [for example, to satisfy formula (8) one can put $w_\nu^- = w_\nu^a = w_\nu^r = 1/(r_\nu - a_\nu)$], and β and γ are arbitrarily defined positive parameters satisfying inequalities $0 < \beta w_\nu^- < w_\nu^a < \gamma w_\nu^r$.

Ogryczak and Lahoda (1992) showed that lexicographic minimization of the above achievement function over the goals (9) always generates an efficient solution to the original multiple criteria problem and satisfies simultaneously the rules of the ARBDS approach, that is the assumptions A1 and A2. Moreover, just as in the standard goal-programming method, the nonlinear constraints on the deviations, $d_\nu^- d_\nu^a = 0$, and $(r_\nu - a_\nu - d_\nu^a)d_\nu^r = 0$, can simply be omitted as they are automatically satisfied by optimization.

2.3.1 *Example*

In order to demonstrate and compare the ARBDS approach with the goal-programming method, consider the following hypothetical plant location problem. A firm specializing in the production of a chemical product is evaluating five sites (S_1 to S_5) for locating two new plants that would supply ten markets (D_1 to D_{10}) (see figure 1). Thus, there are ten alternative locational patterns and each of them generates many allocations (flows of products) schemes. It is expected that the total annual demand for the product will be 90 000 tons. The top management of the firm decided that the maximum annual production capacity of each new plant should not exceed 50 000 tons. The firm has collected the data on costs involved in transporting the products from the new plants to markets (the costs are assumed to be proportional to distance), unit cost of establishing and running a plant in the five potential locations, and suitability of the potential sites for locating the plants. The data are summarized in table 1. The top management of the firm felt that the potential sites should be evaluated on the basis of transportation costs, the unit cost of establishing

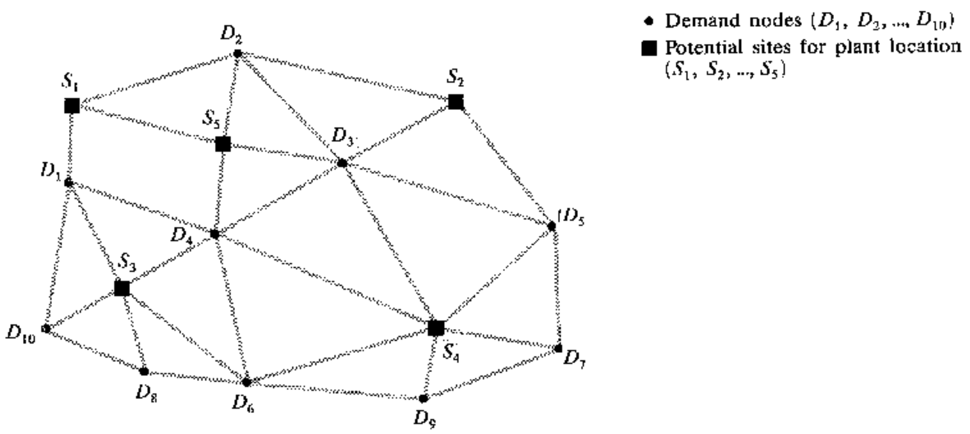

Figure 1. The sample plant location problem.

and operating new plants, and suitability of the sites. Accordingly, three criterion functions should be considered: minimization of total transportation costs or aggregated transport distance [$F_1(x)$], minimization of the fixed capital costs [$F_2(x)$], and optimization of the sites suitability [$F_3(x)$], which is to be minimized as a result of the evaluation scale (see table 1). Also, the managers set aspiration and reservation levels (a_v and r_v, respectively, where $v = 1, 2, 3$).

Table 1. Data for the plant location problem.

Plant site S_i	Distance (km) from market D_i										Fixed costs ($ per unit)	Site suitability[a]
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}		
S_1	18	38	72	52	122	81	131	64	119	60	17	4
S_2	95	50	28	61	35	96	62	104	84	103	16	5
S_3	26	63	56	23	106	37	102	20	75	19	19	3
S_4	105	68	40	53	35	40	27	62	16	85	24	1
S_5	55	20	25	20	75	55	82	63	89	62	26	2
Demand[b]	10	8	9	13	9	11	6	4	8	12		

[a] 1 represents the most suitable site, 5 represents the least suitable site.

[b] Figures expressed in thousands of tons.

Given the aspiration and reservation levels for each criterion, the problem is to find the satisficing or compromise location-distribution pattern subject to a set of constraints imposed on the number of plants to be located, their maximum production capacities, and the expected demand at markets. This location–allocation problem can be structured by means of a GNM as shown in figure 2. Note that there is an additional node 0 and that all the nodes are fixed. To avoid renumbering of nodes we use their original names (D_1, ..., D_{10} and S_1, ..., S_5) as node indices. In algebraic form we denote the set of all nodes by N and its subsets as $S = \{S_1, S_2, ..., S_5\}$ and $D = \{D_1, D_2, ..., D_{10}\}$, respectively. The GNM model can be formulated in terms of the standard goal-programming method and the ARBDS approach as follows.

The decision variables are:

$$x'_{0i} = \begin{cases} 1, \text{ if facility is located at the site } i \text{ (for } i \in S), \\ 0, \text{ otherwise;} \end{cases}$$

x''_{0i} is the annual production at plant i, for $i \in S$;

x''_{ij} is a portion of the product allocated from plant i to market j, for $i \in S$ and $j \in D$.

The state variables are:

d^a_v is the positive deviation from the aspiration level for goals 1, 2, and 3, respectively;

d^r_v is the positive deviation from the reservation level for goals 1, 2, and 3, respectively;

d^-_v is the negative deviation from the aspiration level for goals 1, 2, and 3, respectively.

The parameters are defined as:

b_j represents minus demand at market j, for $j \in D$;

b_0 is the total demand;

$b_i = 0$, for $i \in S$;

$h^{\max}_{0i}$ is the maximum production capacity at location i, for $i \in S$;

c_{0i} is the cost of establishing and operating a plant at location i, for $i \in S$;

e_{0i} is the suitability of the site i for locating a plant, for $i \in S$;

t_{ij} is the unit cost of transporting the product from i to j, for $i \in S$ and $j \in D$;

p is the number of plants to be built ($p = 2$).

The goal-programming model can then be written as:

$$\text{minimise} \sum_{v=1}^{3} d_v^a + d_v^r + d_v^- \ , \tag{13}$$

subject to

$$\sum_{i \in S} \sum_{j \in D} t_{ij} x''_{ij} + d_1^- - d_1^a - d_1^r = a_1 \ , \tag{14}$$

$$\sum_{i \in S} c_{0i} x'_{0i} + d_2^- - d_2^a - d_2^r = a_2 \ , \tag{15}$$

$$\sum_{i \in S} e_{0i} x'_{0i} + d_3^- - d_3^a - d_3^r = a_3 \ , \tag{16}$$

$$r_v - a_v \geqslant d_v^a \geqslant 0 \ , \quad \text{for } v = 1, 2, 3 \ , \tag{17}$$

$$d_v^-, d_v^r \geqslant 0 \ , \quad \text{for } v = 1, 2, 3 \ , \tag{18}$$

$$x'_{0i} \leqslant h_{0i}^{\max} x''_{0i} \ , \quad \text{for } i \in S \ , \tag{19}$$

$$\sum_{j \in N} x''_{ij} - \sum_{j \in N} x''_{ji} = b_i \ , \quad \text{for } j \in N \ , \tag{20}$$

$$\sum_{i \in S} x'_{0i} = p \ , \tag{21}$$

$$x''_{ij} \geqslant 0, \quad x'_{0i} = 0 \text{ or } 1 \ , \quad \text{for } i, j \in N \ . \tag{22}$$

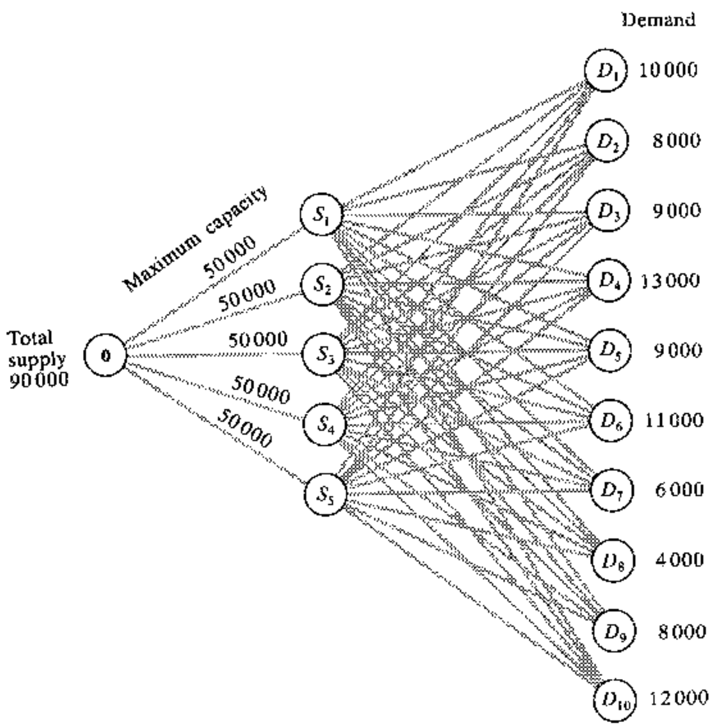

Figure 2. Network model for the plant location problem.

Table 2. Summary of the results obtained by the goal-programming method (GP) and the aspiration–reservation based decision support (ARBDS) method for given aspiration (a_v) and reservation (r_v) levels.

Run number		Reservation–aspiration level and criterion outcome			[Location decisions] Allocation decisions
		$F_1(\boldsymbol{x})$[a]	$F_2(\boldsymbol{x})$	$F_3(\boldsymbol{x})$	
1	a_v	2 761	33	3	
	r_v	4 710	50	9	
	GP	2 761	43	4	$[S_3; S_4]$ $S_3-(D_1; D_2; D_4; D_8; D_9; D_{10})$, $S_4-(D_3; D_5; D_6; D_7)$
	ARBDS	3 527	41	5	$[S_1; S_4]$ $S_1-(D_1; D_2; D_4; D_{10})$, $S_4-(D_3; D_5; D_6; D_7; D_8; D_9)$
2		3 000	33	5	
		4 710	40	9	
		3 500	45	5	$[S_3; S_5]$ $S_3-(D_1; D_2; D_4; D_8; D_9; D_{10})$, $S_5-(D_3; D_5; D_6; D_7; D_{10})$
		3 285	35	8	$[S_2; S_3]$ $S_2-(D_2; D_3; D_5; D_7; D_9)$, $S_3-(D_1; D_4; D_6; D_8; D_{10})$
3		3 800	33	5	
		4 710	40	9	
		3 800	41	5	$[S_1; S_4]$ $S_1-(D_1; D_2; D_8; D_{10})$, $S_4-(D_1; D_3; D_4; D_5; D_6; D_7; D_9; D_{10})$
		3 285	35	8	$[S_2; S_3]$ $S_2-(D_2; D_3; D_5; D_7; D_9)$, $S_3-(D_1; D_4; D_6; D_8; D_{10})$
4		4 000	40	3	
		4 710	50	5	
		4 000	50	3	$[S_4; S_5]$ $S_4-(D_1; D_3; D_7; D_9)$, $S_5-(D_1; D_2; D_4; D_6; D_8; D_9; D_{10})$
		2 761	43	4	$[S_3; S_4]$ $S_3-(D_1; D_2; D_4; D_6; D_8; D_{10})$, $S_4-(D_3; D_5; D_6; D_7; D_9)$
5		4 000	40	3	
		4 710	50	4	
		4 000	50	3	$[S_4; S_5]$ $S_4-(D_1; D_3; D_4; D_5; D_6; D_7; D_9)$, $S_5-(D_1; D_2; D_4; D_6; D_8;$ $D_9; D_{10})$
		3 256	50	3	$[S_4; S_5]$ $S_4-(D_3; D_5; D_6; D_7; D_8; D_9)$, $S_5-(D_1; D_2; D_3; D_4; D_{10})$
6		4 000	33	3	
		4 710	33	5	
		4 000	41	5	$[S_1; S_4]$ $S_1-(D_1; D_2; D_3; D_6; D_{10})$, $S_4-(D_4; D_5; D_6; D_7; D_8; D_9; D_{10})$
		4 710	33	9	$[S_1; S_2]$ $S_1-(D_1; D_2; D_4; D_6; D_8; D_{10})$, $S_2-(D_3; D_4; D_5; D_7; D_9)$
7		4 000	35	8	
		4 000	35	8	
		4 000	35	8	$[S_2; S_3]$ $S_2-(D_1; D_2; D_3; D_5; D_7; D_8)$, $S_3-(D_1; D_4; D_6; D_9; D_{10})$
		3 285	35	8	$[S_2; S_3]$ $S_2-(D_2; D_3; D_5; D_7; D_9)$, $S_3-(D_1; D_4; D_6; D_8; D_{10})$
8		3 500	40	3	
		4 000	50	5	
		3 500	45	5	$[S_3; S_5]$ $S_3-(D_1; D_2; D_4; D_8; D_9; D_{10})$, $S_5-(D_3; D_5; D_6; D_7; D_{10})$
		2 761	43	4	$[S_3; S_4]$ $S_3-(D_1; D_2; D_4; D_6; D_8; D_{10})$, $S_4-(D_3; D_5; D_6; D_7; D_9)$

Table 2 (continued)

Run number		Reservation-aspiration level and criterion outcome			[Location decisions] Allocation decisions
		$F_1(x)$[a]	$F_2(x)$	$F_3(x)$	
9	a_v	3500	33	5	
	r_v	4000	50	6	
	GP	3500	45	5	$[S_3; S_5]$ $S_3-(D_1; D_2; D_4; D_8; D_9; D_{10}), S_5-(D_3; D_5; D_6; D_7; D_{10})$
	ARBDS	3527	41	5	$[S_1; S_4]$ $S_1-(D_1; D_2; D_4; D_{10}), S_4-(D_3; D_5; D_6; D_7; D_8; D_9)$
10		3500	43	5	
		4000	43	7	
		3500	45	5	$[S_3; S_5]$ $S_3-(D_1; D_2; D_4; D_8; D_9; D_{10}), S_5-(D_3; D_5; D_6; D_7; D_{10})$
		2761	43	4	$[S_3; S_4]$ $S_3-(D_1; D_2; D_4; D_6; D_8; D_{10}), S_4-(D_3; D_5; D_6; D_7; D_9)$

[a] Figures expressed in thousands.

The ARBDS model is written as:

$$\text{lexmin } g(\boldsymbol{d}^-, \boldsymbol{d}^a, \boldsymbol{d}^r) = [g_1(\boldsymbol{d}^-, \boldsymbol{d}^a, \boldsymbol{d}^r), g_2(\boldsymbol{d}^-, \boldsymbol{d}^a, \boldsymbol{d}^r)]\,, \tag{23}$$

$$g_1(\boldsymbol{d}^-, \boldsymbol{d}^a, \boldsymbol{d}^r) = \max_{1 \leqslant v \leqslant 3} \frac{(-0.1d_v^- + d_v^a + 10d_v^r)}{(r_v - a_v)}\,, \tag{24}$$

$$g_2(\boldsymbol{d}^-, \boldsymbol{d}^a, \boldsymbol{d}^r) = \sum_{v=1}^{3} \frac{(-0.1d_v^- + d_v^a + 10d_v^r)}{(r_v - a_v)}\,, \tag{25}$$

subject to equations (14)–(22).

The standard goal-programming model was solved by means of the LINDO package (Schrage, 1991). In order to solve the ARBDS model, the DINAS package was used (Ogryczak et al, 1991; see also section 3).

First, the three criteria were considered independently to identify the utopia (ideal) and nadir vectors (see paper 1, section 6)—that is, the best and worst possible outcome for each criterion: $\boldsymbol{q}^b = [2\,761\,000;\ 33;\ 3]$, and $\boldsymbol{q}^w = [4\,710\,000;\ 50;\ 9]$.

Having defined the utopia and nadir values, we examined ten experiments (runs) for the standard goal-programming method and the ARBDS approach by changing the aspiration and reservation levels for the three criterion outcomes. The results are given in table 2 and displayed in a form of value paths for ten alternative solutions generated by the goal programming method and the ARBDS method (see figures 3 and 4).

An analysis of the results shows the conflicting nature of the problem. Specifically there is an intensive conflict between the production cost minimization and the other two criteria—transportation cost and site suitability. Most importantly, however, the results clearly show the susceptibility of the goal-programming method to generate solutions that are dominated by other feasible solutions. It can be seen that alternative solutions 2, 8, 9, and 10 generated by means of the goal-programming method are dominated by criterion outcomes obtained in the first run (see table 2 and figure 3). Note also that solution 6 is dominated by criterion outcome 3.

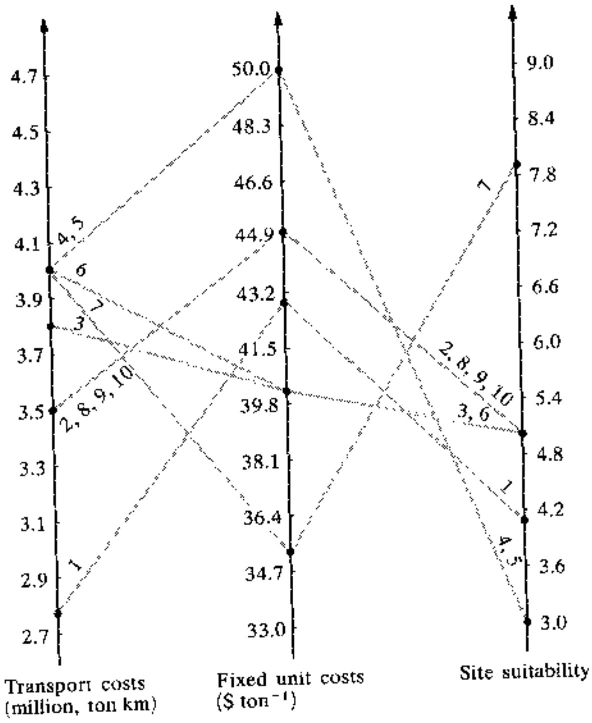

Figure 3. Value paths for the plant location problem: goal-programming solutions.

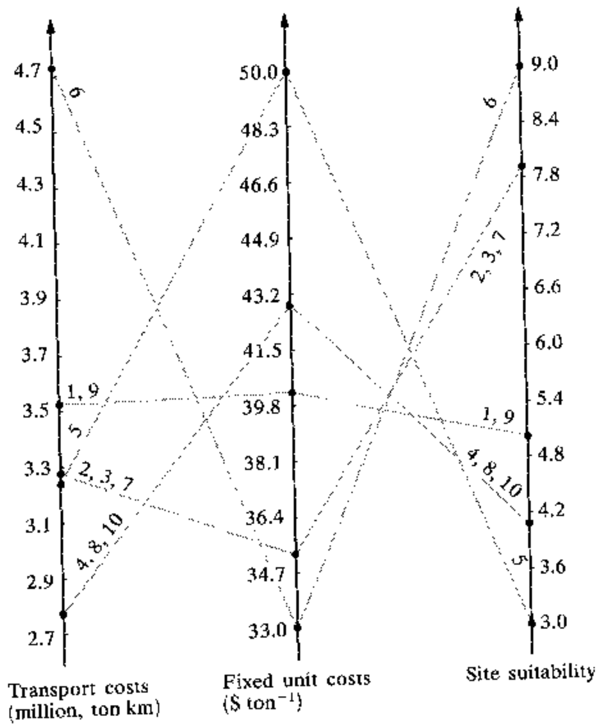

Figure 4. Value paths for the plant location problem: the ARBDS method solutions.

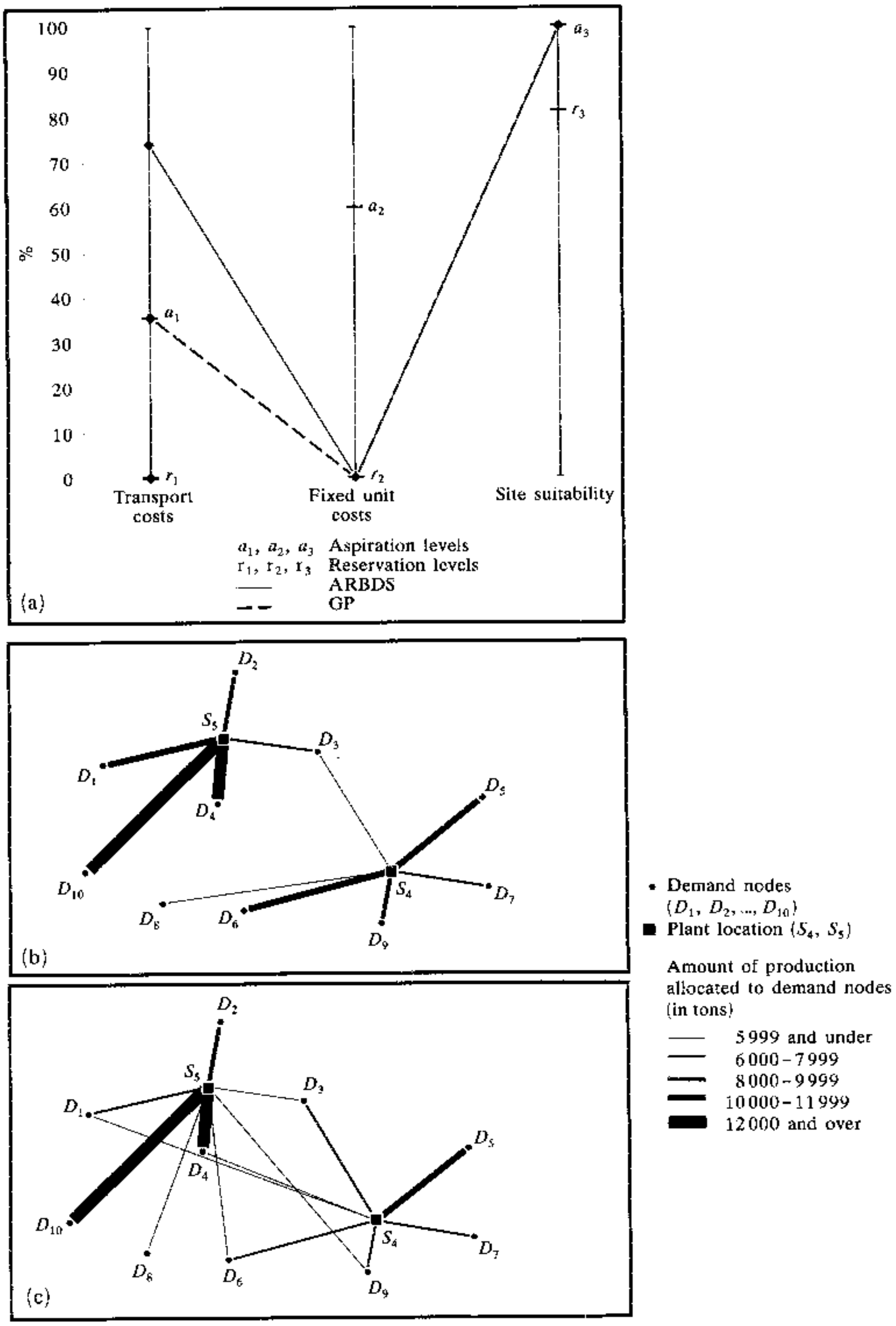

Figure 5. Alternative solutions to the sample plant location problem: (a) value paths, (b) location–allocation pattern generated by the ARBDS method, and (c) location–allocation pattern for solution generated by the goal-programming (GP) method. Note: the criterion outcomes are represented as percentage deviations from the nadir value (q_v^w = 0) and ideal value (q_v^b = 100).

As stated earlier in this paper, the ARBDS approach avoids this shortcoming of the goal-programming method. Figure 4 shows that all solutions generated by ARBDS are nondominated. It is worth noting that both methods may produce the same location patterns for given aspiration and reservation levels (for example, the solution obtained in runs 5 and 7, see table 2). However, these two methods may generate different allocation patterns for the same pattern of locations and for the same reservation and aspiration levels. This is because, in the case when aspired goals are attainable, the goal-programming method generates an allocation pattern which is characterized by total transportation costs equal to a specified aspiration level. Hence, the allocation patterns obtained by means of the ARBDS approach are more spatially efficient than those generated by the goal-programming method (compare the solution generated by these two methods in runs 5, 6, 7, 8, and 10). Graphic displays of alternative solutions in decision space and criterion space provide an effective way of illustrating this point (see paper 1, section 6). We will compare the two solutions generated in run 5 (table 2).

The criterion vectors are displayed in value path format (figure 5). The criterion function values are presented on the scale from 0 (nadir value for a given criterion function) to 100 (ideal value). As the decisionmaker's preferences (specified in the form of aspiration and reservation levels) are crucial components of the interactive approaches, the value paths are displayed in relation to these two levels. Both the aspiration and the reservation levels range from 0 (if they correspond to the nadir value) to 100 (if they correspond to the ideal value). The value paths indicate that the performance of the two solutions is the same with respect to the fixed unit costs $[F_2(\boldsymbol{x})]$ and the site suitability criterion $[F_3(\boldsymbol{x})]$. The criterion outcomes are equal to the reservation level and the aspiration level for $F_2(\boldsymbol{x})$ and $F_3(\boldsymbol{x})$, respectively. However, these solutions are considerably different with respect to $F_1(\boldsymbol{x})$ outcomes, that is, the total transportation costs. This difference becomes more apparent when viewing in the decision space. The supply–demand flow pattern generated by the ARBDS method is more efficient in terms of transportation costs. Comparing these two spatial patterns, one can easily indicate the inefficient allocations determined by the goal-programming approach.

3 Integrating the GNM and the ARBDS approaches

Ogryczak et al (1991; 1992a) have developed a computer-based system that integrates the GNM (see paper 1, section 4) and the ARBDS approaches. This system, called DINAS, has been specifically designed for tackling multiple criteria location choice and location–allocation problems. The system runs on IBM-PC XT/AT (or compatible) computers. It has been used for handling real-life location problems (Malczewski and Ogryczak, 1990; Malczewski, 1992). Ralston (1992) has presented a spatial DSS that includes the DINAS method for multiple criteria location modeling.

DINAS deals with location–allocation problems formulated according to the network methodology (see paper 1, section 3) and automatically transforms the problems into GNMs. Thus the following groups of input data define the problem: criteria; fixed nodes with their balances; potential nodes with their capacities and (fixed) cost coefficients; selections with their lower and upper limits on number of active potential nodes; and arcs with their capacities and cost coefficients.

The problem is to determine the number and location of active potential nodes and to find the flows (along arcs) so as to satisfy the balance and capacity restrictions and, simultaneously, optimize the given criterion functions. A mathematical model of the problem and its transformation into a GNM is described in detail by Ogryczak et al (1989a).

DINAS was developed as an experimental tool to test assumed methodology (GNM and ARBDS) on IBM-PC XT/AT (or compatible) computers. Therefore the basic version of the DINAS system can process only limited-size problems consisting of: up to seven criterion functions; a transportation network with up to 100 fixed nodes and 300 arcs; and up to fifteen potential locations.

For processing the network models, DINAS is equipped with a special network editor, EDINET (Ogryczak et al, 1992b). EDINET is a full-screen editor specifically designed for input and for editing the data of network problems analyzed with DINAS. It may be considered to be simplified interface to the geographical decision space, and in a commercial system this would be implemented with some standard geographic information system (GIS).

The DINAS interactive procedure works with a special file containing complete information defining the problem, and the editor enables one to prepare this file. The essential data of the problem can be divided into two groups:

(1) logical data defining the structure of a transportation network (for example, nodes, arcs, selections);

(2) numerical data describing the nodes and arcs of the network (for example, balances, capacities, coefficients of the criterion functions).

The general concept of EDINET is to edit the numerical data while defining or examining the logical structure of the network. More precisely, the essence of the editor concept is a dynamic movement from some current node to its neighbouring nodes, and vice versa, according to the network structure. The input data are inserted by a special mechanism of windows. At any time only one of the windows representing different kinds of the data is active and the corresponding piece of the data can then be edited. However, apart from the windows with local information, some special windows containing a list of nodes and a graphic scheme of the network can be activated at any moment to ease movement across the network.

DINAS utilizes aspiration and reservation levels to control the interactive analysis. The decisionmaker works with the system interactively and specifies acceptable values for several criteria as the aspiration levels, and necessary values as the reservation levels. The system searches for a satisficing efficient solution with the aid of the achievement-scalarizing function defined by formulas (6) and (8) as a criterion in single-criterion optimization. A special solver has been prepared to provide the multiple criteria analysis procedure with solutions to single-criterion problems. The solver is hidden from the user but it is the most important computational part of the DINAS system. It is the numerical kernel of the system which generates efficient solutions. The concept of the solver is based on the branch-and-bound scheme with a pioneering implementation of the simplex special ordered network (SON) algorithm proposed by Glover and Klingman (1981) with implicit representation of the simple and variable upper bounds (SUBs and VUBs) suggested by Schrage (1975). The mathematical background of the solver was given in detail by Ogryczak et al (1989b).

DINAS is a menu-driven system with very simple commands. Operations available in the DINAS interactive procedure are partitioned into three groups and corresponding three branches of the main menu: PROCESS, SOLUTION, and ANALYSIS. The PROCESS branch contains basic operations connected with processing the multiple criteria problem and the generation of several efficient solutions. There are included operations such as editing and converting the problem, computation of the payoff matrix, and, finally, a sequence of efficient solutions depending on the edited aspiration and reservation levels is generated.

The SOLUTION branch contains additional operations associated with the current solution. The decisionmaker can examine in detail the current solution using the network editor. The current solution can be visualized and analyzed both in decision space and in criterion space. Values of the criterion functions are presented in tabular form and displayed by bar charts in the aspiration–reservation scale and in the utopia–nadir scale. The bar charts show the percentage level of each criterion value with respect to the corresponding scale. The associated location–allocation patterns can be displayed in decision space in the network form. This network consists of active nodes (indicating location of facilities) and active arcs (indicating flows between a pair of nodes). The decisionmaker may also print the current solution or save it for use in further runs of the system with the same problem.

The ANALYSIS branch collects commands related to operations on the solution base. The main command, COMPARE, allows the decisionmaker to perform a comparison of all the efficient solutions from the solution base or from some subset of this base. Like the SOLUTION branch, ANALYSIS includes support for displaying the alternative solutions in decision space and criterion space. Thus, the information about alternative solutions can be simultaneously displayed in the form of bar charts and tables, and associated location–allocation patterns can be visualized by the network. Moreover, some commands allow the decisionmaker to select various efficient solutions from the solution base as the current solutions are included in this branch. There also exists an opportunity to restore some (saved earlier) efficient solution to the solution base.

As mentioned, DINAS was developed as an experimental tool to test assumed methodology based on the use of the GNM and ARBDS. The same methodology can be implemented in a commercial form for large-scale real-life problems. Note that DINAS is, essentially, made of the following three modules: a decision (geographical) space interface, a GNM solver, and an ARBDS driver. The first two modules can be easily replaced with the commercial software. A commercial GIS resolves all the problems of friendly analysis in the decision (geographical) space. Similarly, as mentioned in paper 1 (section 4), the GNM can be effectively solved with mixed integer programming systems armed with network solvers such as CPLEX (1993). Thus for an advanced implementation of the DINAS methodology one needs to prepare only a special ARBDS driver for the criterion space analysis. However, because of the simplicity of the ARBDS approach such a tool can be implemented quite easily. As shown by Korycki and Ogryczak (1995) the ARBDS driver can even be implemented within a standard spreadsheet.

4 Summary and research directions

In these two papers we have attempted to bring together works from diverse areas of multiple criteria location analysis. The approaches to MCLPs have been classified into three broad categories: the generating techniques, explicit preference-based methods, and interactive procedures. We have focused on a critical evaluation of these approaches in the context of their capabilities of supporting location decisions. A generalized network model for multiple criteria location analysis has been presented. It is suggested that this approach provides a flexible tool for modeling complex location problems. Further, the interactive implicit preference-based techniques have been advocated for use as the core of a DSS for locational planning. It is argued that an integration of the generalized network model and the interactive approach along with graphic presentation techniques provide a fairly comprehensive basis for designing a user-oriented computer-based system.

Three issues can be articulated for future research. First, an integration of multiple criteria decision approaches with GIS capabilities has recently been recognized as one of the most important areas for further research (Fedra and Reitsma, 1990; Carver, 1991; Eastman et al, 1993; Pereira and Duckstein, 1993). A GIS usually focuses on the capture, storage, manipulation, analysis, and display of geographically referenced data and only implicitly assumes a support of spatial decisionmaking through analytical modeling operations (see Densham and Goodchild, 1989). The display capabilities of the GIS typically provide the user with a number of techniques that can be used to visualize the problem and the solution in decision space. That is, once the problem has been solved by multiple criteria techniques, the results (decision variables) can subsequently be displayed with a mapping package. Most available GIS systems do not have the capabilities for addressing the solution to MCLPs in decision space and criterion space simultaneously. An application of a GIS for tackling a MCLP requires substantial user involvement to link the analytical components of the multiple criteria decision problem with the cartographic display techniques available in the GIS (Armstrong et al, 1992). Few commercially available GIS systems support multiple criteria decisionmaking techniques at present. IDRISI is a noticeable exception (Eastman, 1993; Eastman et al, 1993). ARC/INFO and TransCAD GIS systems include support for location–allocation models for site selection and analysis (see ESRI, 1987; Caliper Corporation, 1990). One can expect that an increasing number of popular GIS systems will incorporate multiple criteria decisionmaking modules (Keller, 1989; Fedra and Reitsma, 1990; Carver, 1991). It is argued that an integration of spatially referenced data with multiple criteria decision methods can provide an approach for supporting all phases of the decisionmaking process; that is, intelligence, design, and choice (see section 1, paper 1). The geographically referenced database system and multiple criteria decision model-base system can be considered as major elements of a multiple criteria spatial decision support system (MC-SDSS). Such a system has the potential of providing users with the capability of supporting a variety of decisionmaking styles in various decision situations. It allows for integrated data analysis and locational modeling, with account being taken of multiple criteria and the decisionmaker's preferences (Carver, 1991; Eastman et al, 1993).

Second, it is suggested that integration of the data analysis techniques and location models can be organized around the concept of visual interactive modeling—VIM (Hurrion, 1986). This is one of the most challenging developments in spatial decision support research (Densham and Goodchild, 1989; Armstrong et al, 1992; Monmonier, 1992; Densham, 1994). VIM is focused on the use of graphic visualization techniques as an integral part of the problem-solving process. It differs from traditional modeling approaches in that it enables the user (decisionmaker or analyst) to intervene in the problem-solving process and to observe the results of this intervention. In the context of the MCLP it is important that the user be able to interact with the data and location model via graphic techniques for visualizing alternative solutions in decision space and criterion space (Church et al, 1992). The decision space is typically represented by means of cartographic displays, whereas criterion space can be represented by a variety of graphs such as value paths, spider-web charts, bar charts, etc. (see section 6, paper 1). With VIM the user can conduct a what-if dialogue with the computer-based system interacting via graphical display techniques with decision space and criterion space. This approach can be used to visualize the impact of a change in the input data, the location–allocation patterns, and the associated criterion outcomes. This means that the user can ask questions such as "what will happen to the location–allocation pattern if the demand for services in a given spatial unit changes" or "what will happen to the location–allocation pattern if the supply of services in a given location changes". With VIM the user should be able to manipulate and change the input data by using a pointing device (a mouse) to click on the supply–demand (potential–fixed) nodes and links on a map of the network, and to see the changes in the location–allocation pattern and associated criterion outcomes. This type of VIM is focused on the analysis of alternative solutions to the MCLP in decision space. The approach can also be applied to analyze alternative solutions in criterion space. To this end, the decisionmaker can communicate to the MC-SDSS his or her preferences with respect to evaluation criteria. The preferences can be expressed by means of aspiration–reservation levels. Given the graphical display of criterion outcomes in a form of value paths on the utopia–nadir scale for each criterion, the user can modify the 'shape' of the value paths using a pointing device, and the system should be able to display the location–allocation pattern associated with the specified aspiration–reservation levels (see Kasanen et al, 1991). Thus, the underlying what-if analysis involves the following question: "what will happen to the location–allocation pattern if the decisionmaker's preferences change".

Third, more empirical research on multiple criteria decisionmaking and MC-SDS systems is needed. There have been too few actual applications of the multiple criteria approaches to real-life locational planning problems. Interaction with the decisionmaker is an integral part of procedures for structuring and solving multiple criteria decision problems, hence further research in this area is of particular importance. For example, more research is needed on the influence of the data-presentation mode on the decisionmaking process. This influence varies from one stage of decisionmaking to another (Garceau et al, 1988). It is suggested that the graphic presentation techniques are more effective tools in the intelligence and design stage of decisionmaking, whereas in the choice phase the decisionmaker should be supported by a combination of tabular and graphic presentations. There is, however, little empirical and conclusive research on this point. Furthermore, the decisionmaker's preferences are influenced by the mode of presenting the spatial components of both the problem and the alternative solutions. A solution that seems to be the most (least) preferred one in criterion space might be recognized as an inferior (superior) one when viewed in decision space (Church et al, 1992). Probably the most effective way of dealing with this problem is to present the alternative solutions in several different formats in decision space and criterion space (Steuer, 1986). To this end, the concept of graphic script for the sequenced visualization of alternative solutions can be applied (Monmonier, 1992). This concept incorporates a variety of graphic techniques useful in composing sequences of dynamic maps, graphs, tables, and text blocks. This 'new cartography' of dynamic displays should be considered as a part of VIM. Such an approach to integrating the MC-SDSS components can significantly increase the flexibility of the problem-solving process by enhancing the capabilities for exploratory analysis of the spatial components of both the problem and the alternative solutions (MacDougall, 1992). It would also make it easier to understand why a given solution is superior to other alternatives and therefore one would expect the decisionmaker to have more confidence in a decision (Maclaren, 1988; Kasanen et al, 1991).

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