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Future Challenges

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Abstract. Many important topics in multiobjective optimization and decision making have been studied in this book so far. In this chapter, we wish to discuss some new trends and challenges which the field is facing. For brevity, we here concentrate on three main issues: new problem areas in which multiobjective optimization can be of use, new procedures and algorithms to make efficient and useful applications of multiobjective optimization tools and, finally, new interesting and practically usable optimality concepts. Some research has already been started and some such topics are also mentioned here to encourage further research. Some other topics are just ideas and deserve further attention in the near future.

16.1 Introduction

Handling problems with multiple conflicting objectives has been studied for decades (as discussed, e.g., in Chapters 1 to 3); yet there still exist many interesting topics for future research. There are both theoretical questions as well as challenges set by real applications to be tackled. Some of the questions

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can be answered, for example, by hybridizing or integrating ideas from the MCDM and EMO literature.

Here we do not even pursue covering all relevant future challenges but concentrate on three major topics, mainly due to space limitation. First, we discuss new and challenging problem domains in which multiobjective optimization and decision making (or decision support) techniques can be applied. Second, we discuss some new methodologies for multiobjective optimization which allow a synergetic application of optimization and decision making or provide a more global approach to optimization. Third, we describe new and innovative definitions of optimality in multiobjective optimization, which allows one to find a subjective or preferred set of optimal solutions.

Many topics discussed in this chapter are currently under study in various research groups. We still discuss such topics here mainly from the point of view of propagating such ideas to more people. We would like to encourage readers to pursue research along these directions, but our compilation will be successful if future researchers make due acknowledgment of the cited references and this compilation. In our view, the ideas presented are important and have a long-term implication to the field of multiobjective optimization. Collaborative and focused research efforts to implement some of such ideas will be the next step towards making the field more applicable, sustainable and enjoyable.

16.2 Challenging Multiobjective Optimization Problems

Besides solving typical optimization problems having multiple objectives, multiobjective optimization methodologies can also be used in other kinds of problem solving tasks. In this section, we briefly mention some of such research directions.

16.2.1 New Problem Domains

Multiobjective optimization problems arise in many applied fields of research. Although many of these problem types are already investigated, there are also some important new problem classes which deserve to be examined in detail. In the following, we discuss a few selected problems.

Multiobjective Bilevel Optimization

In multiobjective bilevel optimization (Dempe, 2002) one considers the optimization problem

$$\begin{aligned}
&\text{minimize } \mathbf{f}(\mathbf{x},\mathbf{y})\\
&\text{subject to } \mathbf{y}\in Y \text{ and}\\
&\qquad\qquad \mathbf{x} \text{ solves } \begin{cases}\text{minimize } \mathbf{g}(\mathbf{x},\mathbf{y})\\ \text{subject to } \mathbf{x}\in X.\end{cases}
\end{aligned}$$

Here $Y \subset \mathbf{R}^n$ and $X \subset \mathbf{R}^m$ are given feasible sets possibly defined by inequalities and equalities and $\mathbf{f} : \mathbf{R}^m \times \mathbf{R}^n \to \mathbf{R}^k$ and $\mathbf{g} : \mathbf{R}^m \times \mathbf{R}^n \to \mathbf{R}^l$ are vector-valued functions. So, on the "lower level" one has to solve a multiobjective optimization problem for an arbitrary parameter $\mathbf{y} \in Y$. The problem on the "upper level" is a multiobjective optimization problem where the feasible set is defined by Y and the whole Pareto optimal set of the lower problem. Actually, we have two coupled multiobjective problems on two levels. This so-called multiobjective bilevel problem is difficult to solve because we need the complete Pareto optimal set of the problem on the lower level for every parameter $\mathbf{y} \in Y$. The use of interactive methods on the lower level is not helpful in this case. An overview on these complicated problem types in the single objective case can be found in (Dempe, 2002, 2003).

There are interesting applications for this problem class. The bilevel problem in its original form goes back to von Stackelberg (1934), who has introduced a special case of these problems. The so-called Stackelberg games are special bilevel problems. In our case the leader and the follower (in the context of Stackelberg games) have multiple objectives.

In addition to games and economical applications, there are also various applications in engineering (Bard, 1998; Dempe, 2002, 2003). For instance, certain equilibrium problems in chemical engineering can be formulated as bilevel problems (Dempe, 2002).

Semidefinite Optimization

Semidefinite Optimization is a field in optimization which has rapidly grown since the beginning of the 1990's. A multiobjective semidefinite optimization problem can be formulated as

$$\begin{aligned} &\text{minimize } \mathbf{f}(\mathbf{x}) \\ &\text{subject to } G(\mathbf{x}) \text{ is positive semidefinite and} \\ &\qquad\qquad \mathbf{x} \in \mathbb{R}^m. \end{aligned}$$

Here we assume that $\mathbf{f} : \mathbb{R}^m \to \mathbb{R}^k$ is a vector-valued function and $G : \mathbb{R}^m \to \mathcal{S}^n$ is a matrix-valued function, where $\mathcal{S}^n$ denotes the Hilbert space of symmetric (n, n)-matrices with real coefficients. Although the case that the objective function is real-valued has been studied in detail and numerical methods are available for linear and nonlinear semidefinite optimization problems, investigations of the multiobjective case and the development of numerical methods are still expected.

Many applications lead to semidefinite optimization problems (Jahn, 2007). Among others, we only mention the design of a rib in the front of the wing of the new Airbus A380. This complicated problem of material optimization has been solved by semidefinite optimization where one minimizes the weight of the structure and the compliance is treated as a constraint. Thus, the design of one rib is a solution of an ε-constraint problem (see Section 1.3.2in Chapter 1), where ε has not been varied. In this sense, this rib is a result of a special scalarization technique known from multiobjective optimization.

Set Optimization

Since the end of the 1980's, multiobjective optimization has been extended to set optimization. These are problems of the type

$$\begin{aligned} &\text{minimize } F(\mathbf{x}) \\ &\text{subject to } \mathbf{x} \in S, \end{aligned}$$

where $S \subset \mathbb{R}^n$ is a feasible set being defined by inequalities and equalities and $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^k$ is set-valued. So, for a given feasible point $\mathbf{x}$ the image $F(\mathbf{x})$ is a set of vectors in $\mathbb{R}^k$. Although there are investigations on these set problems as an extension of multiobjective optimization, we need new approaches taking into account that we have to work with partial orderings for sets (and not only for points). First steps have been taken with the KNY (Kuroiwa-Nishnianidze-Young) partial ordering (Jahn, 2004) but many significant theoretical questions are still open.

Problems of this type may occur if the objective is not clearly defined but only specified in a vague set-oriented way. If one cannot define a function value of the objective but only the range for this value, one has to solve a set problem.

An example of an industrial application is the navigation of autonomous transportation robots. Here one uses ultrasonic sensors determining the smallest distance to an obstacle in the emission cone. Since the direction of the object cannot be identified in the cone, the location of the object is set-valued. Therefore, questions of navigation may lead to problems of set optimization.

Further Problem Types

There are many other multiobjective problem types to be explored more intensively. Among others, we need more investigations in *multiobjective dynamic optimization*. Dynamic optimization is a significant field of optimization with important applications and it has been used for decades. It is essential to extend these studies to the multiobjective case and only a few studies exist to date (Bingul, 2007; Deb *et al.*, 2007a; Farina *et al.*, 2000; Palaniappan *et al.*, 2001).

Another problem class can be called *multiobjective clustering* (Delattre and Hansen, 1980). If one applies cluster analysis to a set of points in order to find out appropriate clusters, in some cases standard methods do not give the desired result. Recent investigations on multiobjective clustering show that the use of multiple objectives may result in better clusters (e.g., formulating a biobjective optimization of minimizing intra-cluster distance and maximizing inter-cluster distance and finding a set of trade-off solutions will provide solutions not accessible by methods that optimize only one of the criteria) (Handl and Knowles, 2007). This topic is certainly a future challenge as well.

16.2.2 Large-Scale Problems

By large-scale problems we understand problems with many variables, constraints or objectives. In general, an exact lower bound for the number of variables, constraints or objectives is not specified. These problems arise very naturally in concrete applications. For instance, if one discretizes a system of partial differential equations defining the constraints one gets immediately many constraints and many variables. If the considered variable of our problem is a function, the discretization of this function leads to many variables. In order to get a good approximation one has to work with many variables in this case.

Problems with many objectives may occur, for example, in engineering. For instance, the design of suspension bridges may lead to several hundred objectives being difficult to handle. Other problems in material optimization also belong to this class of large-scale problems. Standard methods of multiobjective optimization cannot be applied to these large-scale problem types without simplifying the original problem. Therefore, we need new methods being able to treat problems with many variables, constraints or objectives.

It seems to be difficult to design interactive methods for large-scale problems. There are various reasons. For the decision maker (DM) it is difficult to handle a lot of objectives (or variables). The auxiliary problems which have to be solved during every calculation phase may be so time-consuming that an interaction with the DM does not make sense. Here we have to find new concepts for interaction. Let us add that in some problems function evaluations may be very costly even though the dimensions are small. From the computational point of view, such problems can also be regarded as large-scale ones and interaction may suffer as discussed above.

Like in the case of single objective optimization, one important step for the reduction of computation time in multiobjective optimization is the parallelization of algorithms and their implementation with a distributed computing system. Some earlier studies have demonstrated the use of a distributed computing paradigm for the parallel computation of automatically allocated non-overlapping regions of the Pareto optimal set (Deb *et al.*, 2003; Branke *et al.*, 2004b). A successful treatment of large-scale problems can be reached by using computers in parallel. New approaches such as grid computing allow to use entire networks of computers as one huge parallel computer. New methods have to be designed for parallel architectures. The change from sequential structures to parallel structures will accelerate in the future. More discussions on possibilities of parallel multiobjective optimization can be found in Chapter 13.

16.2.3 Using Multiobjective Optimization to Aid in Other Problem Solving Tasks

Besides solving multiobjective optimization problems, multiobjective concepts and approaches can also be exploited to solve other optimization problems:

- *Constraint handling:* In single objective optimization problems, an additional objective of minimizing the overall constraint violation can be employed. Furthermore, in problems where the constraints form an empty feasible region, the constraints can each be converted as objective functions. This enables solving the problem by taking constraint violations as objectives to be minimized (Miettinen *et al.*, 1998).
- *Optimization with an additional requirement:* In many problems, although the goal is to minimize or maximize a single or multiple objectives, the solution should also exhibit other desirable properties. For example, in the context of evolving computer programs for performing a task using the genetic programming approach, the goal is often to execute the task as accurately as possible but with a hidden agenda of developing a strategy (program) which is as simple as possible. Bleuler *et al.* (2001) minimized the *size* of a genetic program in addition to the supplied objective functions. Since the minimization of the program size is also an important objective, the genetic programming attempts to find the optimal program without making the program unnecessarily large.
- *Improving the search landscape:* Furthermore, some recent studies (Knowles *et al.*, 2001; Neumann and Wegener, 2005) have shown that decomposing the original single objective function carefully into multiple functionally different objectives and treating the problem as a multiobjective optimization problem makes the problem easier to solve than the usual single objective optimization procedure.
- *Revisit traditional problems with multiple objectives for a better and more informative solution strategy:* Sometimes, adding extra objectives as so-called helper (or proxy) objectives allows a better handling of single objective optimization problems (Jensen, 2004). Certain problems are traditionally solved using a particular procedure. A reconsideration of such problems using a multiobjective optimization strategy can be useful in many problem solving tasks, like the multiobjective clustering problem discussed earlier.
- *Knowledge discovery from multiobjective optimization results:* A recent concept of *innovization*, innovation through optimization, makes a post-optimality analysis of obtained trade-off solutions for deciphering principles which are commonly appearing in most obtained trade-off solutions (Deb and Srinivasan, 2006). Since the solutions obtained by an EMO or an a posteriori MCDM method are close to being (or are) Pareto optimal, they are expected to have certain features which remain common to qualify these solutions to be close to the Pareto optimal set and certain features which allow them to have a trade-off among objectives. An effort to try to decipher such valuable information from a set of trade-off near-optimal solutions is a unique way of discovering salient information about "how to solve a problem in a near-optimal manner?". In many engineering design problems and game playing problems, interesting and new design principles and strategies can be unearthed by such a procedure.

The possibility of adding additional objectives to make the search more flexible or even deleting one or more objectives to restrict the search in certain directions provides flexible ways of performing various search tasks (Fliege, 2007). These possibilities certainly open up new avenues and new ways of solving problems and should be exploited more in the near future. The above and a number of other possibilities of aiding different problem solving tasks through multiobjective optimization are discussed in (Knowles *et al.*, 2008).

16.3 Challenging Methods of Finding Optimal Solutions

Having discussed new problem domains for multiobjective optimization, we now discuss new and challenging methodologies of arriving at optimal solutions to multiobjective optimization problems.

16.3.1 Hybrid Methods

As mentioned earlier in this book, in the MCDM literature, solving multiobjective optimization problems has typically been understood as a task of helping a DM in finding the most preferred solution in the presence of conflicting objectives. In this kind of a problem setting, DM's preference information plays an important role. However, until recently, EMO approaches have mostly concentrated on approximating the whole set of Pareto optimal solutions. This brings about a natural question of how MCDM and EMO approaches can complement each other. For example, EMO methodologies can be used to include preference information (Fonseca and Fleming, 1998; Parmee *et al.*, 2000) (see Chapter 6 for more studies). As an example of hybridizing ideas and methods of MCDM and EMO fields, we can mention that some reference point (see Section 2.3 in Chapter 2) based EMO methods have already been introduced (Deb *et al.*, 2006; Thiele *et al.*, 2007), but there is much more potential in preparing new hybrid methods. Other examples of augmenting interactive MCDM methods with EMO ideas include (Deb and Kumar, 2007a,b), where the reference direction approach (Korhonen, 1988) and the "light beam search" (Jaszkiewicz and Słowiński, 1999) are utilized, respectively. Overall, the goal is to analyze the strengths of different approaches and utilize and combine them.

A very simple hybridizing idea is to use continuous local search methods (with scalarizing functions used in MCDM, see Chapter 1) together with EMO. This can be useful, for example, in order to improve (or even guarantee Pareto optimality of) different solutions produced by an EMO algorithm.

Hybridizations of approximation algorithms (approximating the Pareto optimal set) and interactive MCDM methods have, for example, been given in (Klamroth and Miettinen, 2008; Miettinen *et al.*, 2003). Similar ideas can be applied with EMO and interactive MCDM methods. By first using an approximation algorithm, the DM gets a general understanding about the problem as a whole, its possibilities and limitations and it is easier for him/her to specify preference information for the interactive method used. It is, for example, easier to specify the starting point for the interactive method or to specify a reference point.

One possibility of creating hybrid methods is to apply MCDA methods developed for dealing with a discrete set of solution alternatives (Olson, 1996) to the set of solutions generated by an EMO algorithm or a subset thereof. In this way, decision support tools of MCDA could help the DM in analyzing multidimensional objective vectors and finding the most preferred solution. For an example, see (Thiele *et al.*, 2007), where using the reference direction based VIMDA method (Korhonen, 1988) is discussed. A simple implementation of an EMO-MCDA hybrid procedure is also suggested in (Deb and Chaudhuri, 2007).

Sometimes, it is difficult for DMs to move from one Pareto optimal solution to another because this necessitates giving up in some objective function values. If, for example, an EMO algorithm is used to generate an approximation of the Pareto optimal set, the solutions in the population produced are not yet necessarily Pareto optimal. This leads to an idea of a method with a natural win-win situation. Namely, populations generated during the EMO search are shown to the DM, (s)he can direct the search and get better and better solutions.

16.3.2 Global Solvers

Many multiobjective optimization problems arising in engineering are problems defined by nonconvex, nondifferentiable and multi-modal functions. These functions are highly nonlinear. In this case, we must be able to employ a global solver to find the globally optimal solutions. Often, the auxiliary single objective problems which have to be solved as subproblems in multiobjective methods do not have the necessary mathematical structure, like generalized convexity, ensuring that computed points are global solutions. In many algorithmic investigations the question whether a global solution of the auxiliary problems can be determined, is very often not discussed. But in practice this is a significant point. In single objective optimization, locally optimal solutions can still be of some use, as economists or engineers already accept a computed point if a drastic reduction of costs can be obtained by the obtained (local or global) solution. But in multiobjective optimization finding global solutions is crucial, as often the optimization task is followed by a decision making task. If the solvers used do not compute global solutions, one obtains an approximation of the set of Pareto optimal points which may be completely awkward to make decisions with (see also discussion in Chapter 1). Such an ill-functioning appears, for example, if the set of Pareto optimal points is not connected, that is, it consists of several disconnected parts. Then the gaps between these parts are difficult to identify from a numerical point of view. Evolutionary or stochastic optimization methods are better equipped in dealing with such problems and must be investigated more rigorously.

Let us point out that instead of using solvers that can guarantee only the local optimality of solutions generated, it is possible to use some global single objective solver for solving the auxiliary problems produced by MCDM methods, for example, evolutionary algorithms or a hybrid solver where a local solver is used after an evolutionary algorithm, both suggested by Miettinen and Mäkelä (2006).

In the context of single objective optimization, a recent study (Eremeev and Reeves, 2003) has suggested that after a solution is found by using an approximate solver (such as an evolutionary algorithm), a validation procedure must be used to support the result. The study suggested a sampling procedure to estimate the frequency of falling into local optima. Extensions of such studies can be made in the context of multiobjective optimization. However, there still is much to do in this field in order to find the most appropriate solvers to be used in each problem considered.

In general, because evolutionary algorithms or stochastic methods are potential global solvers, the question of interest is how to improve their algorithmic behavior with techniques using derivatives. The above-mentioned way is the simplest possibility. For instance, if one applies an evolutionary algorithm to a complicated problem with smooth functions, it certainly makes sense to combine the evolutionary algorithm with a local solver which uses information on derivatives. Such a combination may improve the evolutionary algorithm. These *memetic* algorithms are difficult to design because one has to determine when to switch from the evolutionary algorithm to the local solver and back. For example, memetic methods combining an evolutionary algorithm with the well-known sequential quadratic programming (SQP) method produce promising results. Here we need comprehensive investigations on the interface of these evolutionary methods and derivative-based methods being qualified for such a combination. These investigations should lead to modern metaheuristic approaches resulting in new global solvers. For instance, hybrid solvers involving simulated annealing and the (local) proximal bundle method are introduced in (Miettinen *et al.*, 2006).

Based on the remarks listed, there is a need for efficient global solvers (as also concluded by Aittokoski and Miettinen (2008)). We should develop hybrid methods combining standard methods with global strategies. These global hybrid solvers are very desirable and they would bring a breakthrough in finding guaranteed global Pareto optimal solutions in multiobjective optimization.

16.4 New Trends in Optimality

Finally, let us devote some thoughts to a few new trends in defining optimality in the context to multiobjective optimization: subjective preferences, different optimality concepts, and robust solutions.

16.4.1 Subjective Preferences

The MCDM literature typically places the DM at the center of the solution process of a multiobjective decision problem (as, e.g., Belton and Stewart (2001)). The DM's preferences determine which objective functions are more or less important, and how different objective values are to be rated. Consequently, aggregation across objectives has to be performed in a way which is consistent with the DM's preferences. This subjectivity is often seen as one of the characteristic features of multiobjective optimization problems, which distinguish this area from single objective optimization, where an objectively optimal solution can be found.

This subjective view is not entirely shared in the EMO literature, or more generally in multiobjective optimization. When solving multiobjective optimization problems, the aggregation across the individual objectives is often specified by model developers, with little involvement of the actual DMs. From a subjective perspective, this might seem a grave omission: an analyst who selects an aggregation mechanism across objective functions (like an additive function), and specifies weights of individual objectives to be used in this aggregation, takes away decision authority from the actual DM. Even seemingly objective concepts like dominance or Pareto optimality contain subjective elements because dominance requires at least information about the direction of preference within each objective function. But not all multiobjective optimization problems exhibit this level of subjectivity. Sometimes, even an aggregation across objective functions can be performed quite objectively, and a model developer might be even in a better position to perform such an "objective" optimization than the actual DM. However, if subjective information is not available, EMO can be used to get an idea of the Pareto optimal set, at least in the case of optimization problems with two to four objectives.

Rather than establishing a strict dichotomy between "objective" single objective optimization problems and "subjective" multiobjective problems, we can propose a taxonomy of different levels of subjectivity in multiobjective optimization problems as four cases. In this taxonomy, the classical view of multiobjective problems does not even form an endpoint, but an intermediate stage.

The proposed taxonomy consists of four cases:

i) Multiobjective optimization problems as a technical solution device.
ii) Multiobjective optimization problems as an approximation of a higher level objective.
iii) Subjective multiobjective optimization problems.
iv) Problems involving meta-criteria.

Multiobjective Optimization Problems as a Technical Solution Device

In some cases, heuristics work better on multiobjective optimization problems than on problems with a single objective function. In these cases, it might make sense to perform a "multi-objectivization" of the problem (Knowles *et al.*, 2001): to split an explicitly given criterion into several functions and solve the resulting multiobjective optimization problem, as discussed in Section 16.2.3. Of course, the aggregation procedure in this case is fully determined and has to reconstruct the original objective function.

Multiobjective Optimization Problems as an Approximation of a Higher Level Objective

In many applications of multiobjective optimization methods, the DM actually wants to maximize some higher level criterion, but this criterion can either not be directly measured, or the relationship of the decision variables to that higher level criterion is not clear. Therefore, one uses several substitute criteria and solves a multiobjective optimization problem, instead. For more details, refer to (Miettinen, 1999). A study on EMO (Handl *et al.*, 2007) called these substitute criteria 'proxy objectives'. In many MCDM applications in business, the long run profit of the firm is the ultimate goal. But the impact of many decisions on a long run profit can hardly be quantified. For example, when hiring a new executive, one cannot predict how much a particular person will contribute to profit, so substitute criteria like education and experience are used to approximate that person's productivity. Another example demonstrating the benefits of using multiobjective optimization is discussed in (Hakanen *et al.*, 2005), where estimating amortization time and interest rate for capital is avoided when balancing between investment and running costs in the case of designing a heat recovery system of a paper mill.

In these cases, neither the choice of a preference model nor the selection of parameters (e.g., weights) to be used in that model is purely subjective, but both should approximate the likely relationship of substitute criteria to the higher level criterion. A higher weight in this case does not indicate that a substitute (lower level) criterion is considered more important, but that it is considered to have a stronger influence on the higher level criterion.

Subjective Multiobjective Optimization Problems

Subjective problems are typically considered in MCDM, where the aggregation of objective functions depends solely on the subjective preferences of the DM. This type of decision problems are often illustrated by referring to personal decisions like the purchase of a car, where attributes like comfort, speed or costs need to be compared.

In this case, no objective aggregation model exists, which would be valid for all DMs. Of course, modeling can still be performed by an analyst, but only in close contact with the actual DM who has to provide the relevant preference information.

Problems Involving Meta-criteria

Many multiobjective optimization problems are related to decisions in which the interests of multiple stakeholders have to be taken into account. Such decisions occur, for example, in public policy. Even when the decision is ultimately made by one individual, for example, a politician, that individual has to consider the interests of different parties. While in the decision problems discussed so far, an improvement in any objective function could be considered to improve the overall evaluation of a decision alternative (this assumption underlies the whole concept of Pareto optimality), this is no longer true when aspects like fairness need to be taken into account. Here, further improvements of the situation of stakeholders who are already better off than the others might be considered as unfair and, thus, make a solution less preferable.

Such "meta-criteria", which evaluate the distribution of results across several criteria, occur not only in multi-person decisions. For example, when time streams of income are evaluated, income in each period could be considered as an objective. Apart from maximizing income in each period (which would correspond to the standard multiobjective formulation), DMs might prefer a constant income stream over a stream which exhibits large variations over time. This preference for particular patterns should not be confused with risk aversion; it can occur even if all payments are known in advance with certainty. To handle this type of problems, Kostreva *et al.* (2004) developed the concept of *equitable* multiobjective decision making and showed how several multiobjective optimization methods, in particular reference point methods, can be extended to handle such problems.

Further Comments on the Taxonomy

Our taxonomy of multiobjective optimization problems has several consequences for the way in which "preferences" are elicited, modelled and aggregated. The first difference concerns the person, or group of persons, from whom preference information can be obtained. In highly subjective problems, only the DM him/herself can provide information about preferences. But in problems where multiple criteria are used to approximate a higher level objective, it might be reasonable to obtain input from several experts in order to get a clearer picture of how substitute criteria will actually influence the higher level objective. In the remaining two cases no real "preference elicitation" can take place. When multiple criteria are introduced for technical reasons, their aggregation is also a technical problem. In the case of meta-criteria, the way in which individual criteria are aggregated is based on the meta-criteria involved, which can be considered as axioms an aggregation method must fulfill.

This distinction has also consequences for the likely stability of preference information. While there is some empirical evidence that individual preferences towards multiple criteria remain stable over time (Blackmond and Fischer, 1987; San Miguel *et al.*, 2002), they are still subject to more external influences than causal relationships between substitute criteria and higher level objective. Consequently, "preference" information obtained for the latter type of problems, as well as for the other two classes, needs to be elicited less often in repeated decisions than for subjective problems.

One might also view properties of solution concepts, like efficiency or independence of irrelevant alternatives, differently in the four cases of our taxonomy. In the first case, such axioms are more or less irrelevant. Aggregation has to reconstruct the original objective, regardless of whether it fulfills common axioms of decision analysis or not. In the second case, rationality (in the form of axioms) becomes more important, since in most problems, it can be expected that the true relationship between substitute criteria and the actual higher level objective also follows these principles. In the third case, acceptance of axioms is entirely up to the DM. Empirical research on bias phenomena in decision making has provided considerable evidence that subjects consciously choose to violate axioms of decision analysis, even when this violation is pointed out to them (von Winterfeldt and Edwards, 1986). Finally, in the last case, meta-criteria are themselves axioms, and their acceptance by all stakeholders is a prerequisite for acceptable solutions.

By formulating the above taxonomy, we have just started to explore the impact of different levels of subjectivity on the solution process, as well as the underlying theory of multiobjective optimization, both with MCDM and EMO methods. This could become an interesting area of future research.

16.4.2 Generalized Dominance and Redefining Optimality

Most multiobjective optimization studies use the concept of Pareto optimality for driving their search. However, there exist a number of other trends of redefining the usual Pareto optimality. Such considerations usually reduce the size of the optimal set and in some occasions make it easier for the search algorithms to handle the complexity associated with multiobjective optimization. Here we discuss a number of such trends of redefining optimality in multiobjective optimization.

In this book, the basic concept of optimality has been that of Pareto optimality, but a closely related, relaxed, concept of weak Pareto optimality is sometimes used because the latter is computationally simpler and many straightforward approaches to multiobjective optimization generate weakly Pareto optimal solutions (see, e.g., Preface and Chapter 1). However, weak Pareto optimality is not satisfactory for applications because it ignores clear possibilities of solution improvement with respect to some objectives. Actually, even the concept of Pareto optimality may be too weak for many applications. As discussed in Chapter 1, the notion of proper Pareto optimality (Geoffrion, 1968) assumes that all the trade-offs are bounded (see also Chapter 2). Sometimes, more useful for applications are solutions that are properly Pareto optimal with an a priori given bound on trade-offs.

Several dominance (and thereby efficiency or Pareto optimality) concepts can be introduced as the so-called dominance cone (Yu, 1974) as also briefly discussed in Chapter 1. The partial order of the dominance relation is implied by a convex cone D in such a sense that $\mathbf{y}'$ dominates $\mathbf{y}''$ if and only if $\mathbf{y}' - \mathbf{y}'' \in D \setminus \{\mathbf{0}\}$. The standard Pareto optimality or Pareto dominance is defined by using an orthant cone (negative orthant for minimization). A narrower cone restricts the dominance relation thus expanding the corresponding efficient set. On the other hand, a wider cone enforces more dominated outcome vectors, thus narrowing down the efficient set.

A corresponding dominance cone can be constructed by combining the orthant with the half-space (Kaliszewski, 1994; Wierzbicki, 1986). Actually, the reference point method and many other scalarizing function model such dominance by taking the sum of objective values (the half space) with a small weight to regularize the basic term of the max-min aggregation (the orthant). See also Chapters 1 and 2 as well as (Miettinen, 1999).

Most traditional MCDM approaches to multiobjective optimization seek for the best solution according to the DM's preferences while treating the dominance relation as a common principle of all rational preference models. Thus the concept of Pareto optimality is rather used as a necessary condition to establish the boundary of acceptable choices. Therefore, strengthening the dominance concept is not so crucial for the implementation of interactive MCDM procedures, although still important. On the other hand, EMO procedures use three different features: emphasis on nondominated solutions in the current population, emphasis on previously-found nondominated solutions, and emphasis on less crowded solutions in the objective space (see Chapter 3). Many studies related to different dominance relations and approaches utilizing them have been published during the years in the MCDM field. Lately, they have also attracted attention in the EMO field. For example, wider dominance cones can be used to focus an EMO search on a part of the Pareto optimal set (Branke *et al.*, 2001; Laumanns *et al.*, 2002), instead on the complete set. In particular, the cone dominance enables to formalize concepts of narrowing the Pareto optimal set related to limitations on trade-offs.

Note that the dominance cone can be changed during the solution process. Such a dominance structure appears, for instance, in the case of a given value (or utility) function maximization. The dominance structure corresponding to the comparison of the value function values is represented by the tangent cone to the isoline contours of the value function at any objective vector. For poorly characterized preferences in multiobjective problems, it is often desirable to seek (approximate) optimal solutions for a large class of value functions. Solutions corresponding to the optimal value of a large variety of linear value functions can be emphasized within the EMO procedure, thereby aiding to find *knee* objective vectors in certain problems (Branke *et al.*, 2004a). An approximate majorization relation enables the search for solutions maximizing all symmetric concave value functions (Goel and Meyerson, 2006).

There are many applications leading to problems with a large number of uniform criteria considered impartially which makes the distribution of outcomes more important than the assignment of several outcomes to the specific criteria. Such models are generally related to the evaluation and optimization of various systems which serve many users where quality of service for every individual user defines the criteria. This applies to various technical and social systems. An example arises in locating public facilities where the decisions often concern the placement of a service center or another facility in a position so that the users are allocated in an impartial way. Thus, we are interested in comparing distributions of values within the objective vectors rather than componentwise comparison of objective vectors (Ogryczak, 1999). Note that having two possible location patterns generating objective vectors $(5, 0, 5)$ and $(0, 1, 0)$, we would recognize both the location patterns as Pareto optimal in terms of (distance) minimization. However, the first location pattern generates two objectives (distances) equal to 5 and one objective equal to 0, whereas the second pattern generates one objective equal to 1 and two objectives equal to 0. Thus, in terms of the distribution of objective values, the second location pattern is clearly better.

The need to search for some optimal distribution of objective values is commonly recognized in problems which may be viewed as resource allocation models. While allocating limited resources to maximize the system efficiency they also attempt to provide a fair treatment of all the competing activities. For instance, in networking, a central issue is how to allocate bandwidth to flows efficiently and fairly (Denda *et al.*, 2000; Pióro and Medhi, 2004). Furthermore, uniform individual criteria may be associated with some events rather than physical users, like in many dynamic optimization problems where uniform individual criteria represent a similar event in various periods or in decision problems under uncertainty where uniform individual criteria represent the outcome realizations under various scenarios. Another type of model is that of approximation of discrete data by a functional form. The residuals may be viewed as objectives to be minimized, and there is no reason to treat them in any way but impartially.

In many models fair consideration of all criteria requires more than only impartiality. In order to ensure fairness in a system, all system entities have to be equally well provided with the system's services. This means that more equal objective vectors are preferred to unequal ones or, more formally, a transfer of any small amount from an objective function to any other relatively worse-off objective results in a more preferred objective vector. For instance, a solution generating all three objective values equal to 2 is considered better than any solution generating individual values 4, 2 and 0. This leads to concepts of fairness expressed by the equitable efficiency as a specific refinement of Pareto optimality taking into account impartiality and inequality minimization (Kostreva *et al.*, 2004). Thus, seeking for the optimal distribution of objective values is actually a new multiobjective problem type. However, the dominance structure for objective vectors does not represent any cone (Kostreva and Ogryczak, 1999).

Currently, some specific solution concepts are used for various application areas. Biobjective aggregations to the mean and some dispersion measure are used in the areas of decisions under risk and location analysis as well. The max-min approach additionally regularized with the lexicographic order (the so-called max-min fairness) is commonly used in resource allocation problems (Luss, 1999). Approaches exploiting the multiobjective nature of distribution optimization problems are rather rare (Ogryczak *et al.*, 2008). Actually, such problems are hard for preference modeling and identification within the interactive MCDM methods as well as for the EMO approaches. Nevertheless, they deserve to be investigated more intensively.

16.4.3 Robust Solutions

A conventional optimization approach that considers only the optimality of a decision or a design, that is, performance at decision or design condition, should work fine in a controlled environment. Real-world applications, on the other hand, inevitably involve errors and uncertainties (be it, e.g., in the design process, manufacturing process, and/or operating conditions); so that the resulting performance may be lower than expected. For instance, the aerodynamic performance of an airplane wing design is very sensitive to the wing shape and flight conditions and, thus, it may deteriorate drastically when subject to wing manufacturing errors and wind variations even if the wing design is optimized.

Several approaches have been developed to deal with uncertain or imprecise data. The approaches focused on the quality or on the variation (stability) of the solution for some data domains are considered robust. The notion of robustness applied to decision problems was first introduced almost 50 years ago by Gupta and Rosenhead (1968). Practical importance of the performance sensitivity against data uncertainty and errors has later attracted considerable attention to the search for robust solutions. Actually, as suggested by Roy (1998), the concept of robustness should be applied not only to solutions but, more generally to various assertions and recommendations generated within a decision support process. A brief comparison between conventional optimization and robust optimization is illustrated in Fig. 16.1 a). Solution A obtained by a conventional optimization is the best in terms of optimality, but disperses widely in terms of the objective function against the dispersion of design variable or environmental variable, and this dispersion may extend to an infeasible range. On the other hand, solution B obtained by a robust optimization is moderately good in terms of optimality and also good in terms of robustness, that is, dispersion of objective function is narrow against dispersion of design variable.

On the other hand, the optimal solution despite generating objective values dispersed quite widely may be clearly better than a solution not dispersed at all. As depicted in Fig. 16.1 b), solution A though characterized by dispersed results remains under all conditions better than the stable solution B. Hence, solution B is obviously dominated and it cannot be considered a robust optimal solution.

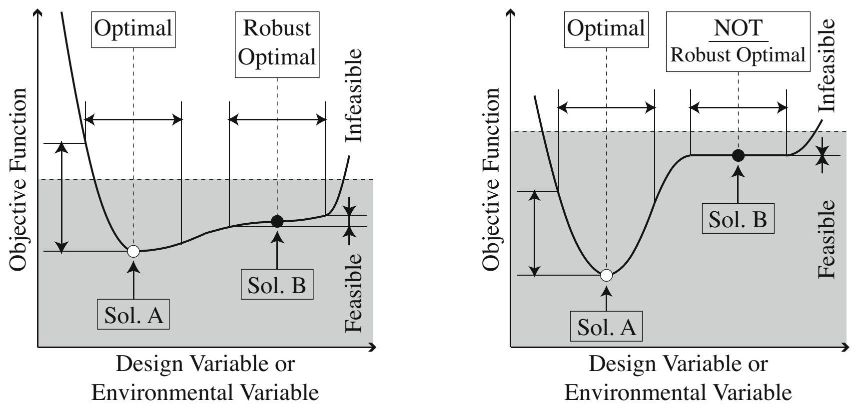

Fig. 16.1. Comparison between conventional optimization and robust optimization (for a minimization problem): a) conventional optimal solution A vs. robust optimal solution B; b) stable but not robust optimal solution B.

The precise concept of robustness depends on the way the uncertain data domains and the quality or stability characteristics are introduced. Typically, in robust analysis one does not attribute any probability distribution to represent uncertainties. Data uncertainty is rather represented by non-attributed scenarios, which means there is no specific rule to determine the data uncertainty characteristics. Since one wishes to optimize results under each scenario, robust optimization might be in some sense viewed as a multiobjective optimization problem where objectives correspond to the scenarios. However, despite of many similarities of such robust optimization concepts to multiobjective models, there are also some significant differences (Hites *et al.*, 2006). Actually, robust optimization is a problem of optimal distribution of objective values under several scenarios (c.f. Section 16.4.2) rather than a standard multiobjective optimization model.

A conservative notion of robustness focusing on worst case scenario results is widely accepted and the min-max optimization is commonly used to seek robust solutions. The worst case scenario analysis can be applied either to the absolute values of objectives (the absolute robustness) or to the regret values (the deviational robustness) (Kouvelis and Yu, 1997). The latter, when considered from the multiobjective perspective, represents a simplified reference point approach with the utopian (ideal) objective values for all the scenario used as aspiration levels. Recently, a more advanced concept of ordered weighted averaging was introduced into robust optimization (Perny *et al.*, 2006), thus, allowing to optimize combined performances under the worst case scenario together with the performances under the second worst scenario, the third worst and so on. Such an approach exploits better the entire distribution of objective vectors in search for robust solutions and, more importantly, it introduces some tools for modeling robust preferences. Actually, while more sophisticated concepts of robust optimization are considered within the area of discrete programming models, only the absolute robustness is usually applied to the majority of decision and design problems.

Taking into account the current computational capabilities of both EMO and MCDM techniques, one may expect development of new robust optimization approaches in many areas. Here, we do not make any attempt to discuss all such existing implementations.

Dealing with Risk

When an (objective or subjective) probability distribution is specified to characterize the data uncertainty, robust optimization becomes a problem of decision under risk. In this context, robustness is represented by the notion of risk aversion, and typically by a "strong" risk aversion. There exists a well-developed methodology for decisions under risk and it can be directly applied to robust optimization. In particular, the mean-risk (MR) approach (Markowitz model) quantifies the problem in a lucid form of only two objectives: the mean (expected) outcome μ and the risk ϱ, a scalar measure of the variability (dispersion) of outcomes. The latter may be equally interpreted as a robustness measure of solutions, thus allowing the MR model to be read as mean-robustness in an appropriate setting.

The MR approach allows to formalize robust optimization with two separate criteria: optimality (μ) and robustness (ϱ). Indeed, in many real-life problems improvements in optimality and robustness are competing while the MR model allows to formalize it and to analyze the trade-off between these two criteria. The classical Markowitz model uses the standard deviation σ (or variance σ^2) as the risk measure. Similarly, the biobjective model $\min\{\mu, \sigma\}$ is applied for robust optimization, although frequently in the scalarized form $\min\{\mu + \alpha\sigma\}$ with the trade-off parameter $\alpha < 0$. Unfortunately, while the mean-variance model is well suited for normal distributions, it may lead to inferior conclusions in general. Referring to the case depicted in Fig. 16.1 b), one may notice that obviously a worse solution B is characterized by $\sigma = 0$, thus, in terms of the biobjective MR model it is not dominated by solution A with a positive measure of dispersion, despite the fact that the latter is clearly better under all scenarios. This flaw of MR models may be overcome by the use of asymmetric dispersion measures focused only on disturbances negative to the optimization and combining them with the mean values.

For instance, the biobjective model $\min\{\mu, \mu+\bar{\sigma}\}$ with $\bar{\sigma}$ representing the upper side standard deviation will generate only solutions with nondominated distributions of results (Ogryczak and Ruszczyński, 1999), namely, solutions which cannot be improved under all scenarios simultaneously. One may notice that, while considering the maximum upper deviation (from the mean) Δ as a probability independent dispersion measure, one gets the criterion $\mu + \Delta$ expressing the worst case scenario result, that is, the classical conservative notion of robustness. Multiobjective approaches to decisions under risk (Ogryczak, 2002) allow to model various robust solution concepts.

Shimoyama *et al.* (2005) have proposed a multiobjective robust optimization approach called design for multiobjective six sigma (DFMOSS). The DFMOSS builds on the ideas of design for six sigma (DFSS) (Engineous Software, Inc., 2002), coupled with an EMO algorithm (Deb, 2001), for an enhanced capability to reveal trade-off information considering both optimality and robustness of design. Jin and Sendhoff (2003) have also discussed the trade-off between optimality and robustness in the context of multiobjective optimization. The DFSS is based on the "six sigma" concept, which was originally established as a measure of excellence for business processes. The aim is to achieve a process with such a small dispersion that the range of $\pm 6\sigma$ (where σ is standard deviation) around the mean value μ is included in an acceptable range for the performance parameter. The level of dispersion can be defined as "sigma level n" satisfying the following constraints:

$$\mu - n\sigma \geq \mathrm{LSL} \quad \text{and} \quad \mu + n\sigma \leq \mathrm{USL}, \tag{16.1}$$

where LSL and USL are lower and upper specification limits, respectively. A larger sigma level indicates smaller dispersion. In the context of robust design optimization, smaller dispersion translates to a more robust characteristic.

For a general single objective optimization problem where an objective function $f(\mathbf{x})$ of design variable $\mathbf{x}$ must be minimized, the DFMOSS deals with the biobjective optimization problem where the mean value (μ_f) and the standard deviation (σ_f) of $f(\mathbf{x})$ must be minimized when $\mathbf{x}$ disperses around the design condition due to errors and uncertainties. During the optimization process itself, multiple solutions (individuals) are dealt with simultaneously using EMO. For each individual, μ_f and σ_f are evaluated as two separate objective functions from $f(\mathbf{x})$ at the sample points around $\mathbf{x}$. From them, better solutions are selected based on the Pareto optimality concept between μ_f and σ_f. New solutions for the next step are reproduced by crossover and mutation from the selected solutions. This optimization process is iterated until the trade-off relation between μ_f and σ_f has converged, and multiple robust optimal solutions have been obtained. After the optimization, the sigma level n satisfying (16.1) is post-evaluated for the obtained optimal solutions. This allow one to select a robust solution with the highest sigma level (preferably with the level 6).

Note that some optimization problems do not have robust solutions that satisfy six sigma. In such cases, it is preferable to find a solution with a sigma level n as high as possible, even if it is less than six sigma. In addition, (16.1) can still be considered during the μ_f–σ_f optimization; it is better to do this when n to be satisfied is strongly determined by a certain design requirement. Let us also mention that Deb and Gupta (2005) have suggested two types of robustness in the context of multiobjective optimization. Certainly, many other variations are possible.

Uncertainty in Presence of Constraints

Uncertainty in problem parameters may affect not only the objective functions but also the feasible set, thus, threatening the feasibility of solutions. Solving such problems is frequently referred to as reliability-based optimization, where one seeks the best solution among those remaining feasible for various data perturbations. Again, the precise concept of solution depends on the way the uncertain data domains are introduced. When uncertainty is represented by non-attributed scenarios, the worst case approach can be applied. When probability distribution is specified (either objective or subjective) to characterize the data uncertainty, one gets a typical stochastic programming problem. Fig. 16.2 shows a hypothetical problem with two inequality constraints. Typically, the optimal solution lies on a constraint boundary or at the intersection of more than one constraints, as shown in the figure. In the event of uncertainties in design variables (as shown in the figure with a probability distribution around the optimal solution) in many instances such a solution will be infeasible. In order to find a solution which is more reliable (meaning that there is a very small probability of instances producing an infeasible solution), the true optimal solution must be sacrificed and a solution interior to the feasible

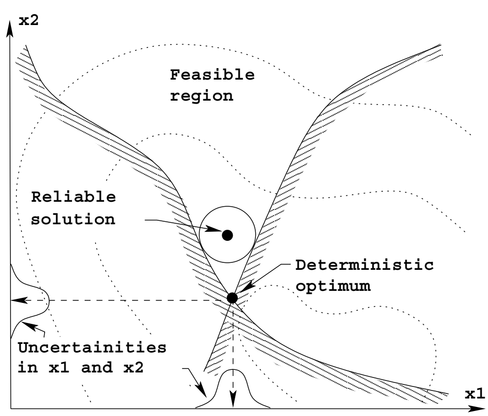

Fig. 16.2. The concept of reliability-based optimization.

region may be chosen. For a desired reliability measure R, it is then desired to find that feasible solution which will ensure that the probability of having an infeasible solution instance created through uncertainties from this solution is at most $(1 - R)$. To arrive at such a solution, a stochastic optimization problem can be converted to its deterministic equivalent (Birge and Louveaux, 1997; Romeijn *et al.*, 2006).

To handle such cases with a large reliability requirement, probabilistic methodologies involving a double loop, single loop and decoupled methods are used. For example, one can incorporate a decoupled method with an EMO procedure (Deb *et al.*, 2007b) to find reliable sets, instead of a sensitive Pareto optimal set, corresponding to a specified reliability value. More such studies are needed to make the approach computationally viable and applicable to practical multiobjective problem solving tasks.

Another issue involving uncertainty in solution evaluation comes from dealing with noisy environments, in which objective and constraint function evaluations introduce inherent noise. Although this issue has received a lot of attention in the context of single objective evolutionary algorithms (see a survey by Jin and Branke (2005)), some attempts have recently been made in EMO as well (Bui *et al.*, 2005; Hughes, 2001; Teich, 2001). Clearly, more studies are needed to fully understand the effect of noise in solution evaluation procedures in multiobjective optimization.

16.5 Conclusions

In this chapter, we have discussed some ideas for future research in the context of multiobjective optimization and decision making. However, plenty of research is still needed in many aspects of decision making. In this respect, some of the future directions mentioned by Miettinen (1999) still stand as relevant challenges. Let us hope that the future years will bring new light in them and many other fruitful and rewarding topics.

An important challenge is to increase awareness of the possibilities and potential of multiobjective optimization because there still are many application fields where multiobjective optimization is not used at all or is used in a very simplistic way even though the problems solved clearly involve multiple conflicting objectives. Often, the existence of decision support tools is simply not known to many researchers. Here, the importance of strong and encouraging case studies cannot be emphasized enough. For people dealing with applications, case studies give a possibility to see the benefits obtainable in a concrete and understandable way. A necessary and natural step of bringing multiobjective optimization tools closer to real DMs is the important challenge of designing user-friendly software for decision support. We need software that is easily accessible (like the WWW-NIMBUS® system (Miettinen and Mäkelä, 2000, 2006) operating via the Internet) and this certainly is a field needing more attention.

In many applications, multiple objectives are hidden and simplified in the modeling phase in order to produce a problem that seems to be solvable. Increasing awareness of the existence of multiobjective optimization methods and tools also encourages questioning the existing models in order to avoid simplifications that blur the possibility of studying the interdependencies between the conflicting objectives in real problems.

The possibilities and importance of interactive methods have been emphasized a lot in this book because interactive methods give the DM a possibility to learn about the problem considered. If the problem is complex and function evaluations take a lot of time, the interactive nature of the solution process may suffer because the DM has to wait for new and improved solutions. This sets requirements and challenges on the computational efficiency of the methods used. Besides using meta-modeling (see Chapter 10) and optimization techniques with increased accuracy (as the solution process proceeds), new approaches and ideas are needed. For example, ideas related to learning are discussed in Chapter 15.

One aspect has clearly emerged from the chapters of this book: multiobjective optimization using evolutionary algorithms or otherwise and decision making aids must be put together synergistically, computationally efficiently, and, above all, interactively for a DM to examine possible candidate solutions and choose one particular preferred solution at the end. Such a task requires one to first know both optimization and decision making literature well. This book has shown a number of possibilities of such mergers from various points of view. This chapter has also suggested a number of avenues for moving forward in this direction. With collaborative efforts from various research groups involving multiobjective optimization and decision making, we should witness more holistic approaches, interactive algorithms and software systems to be developed for practical use in the coming years.

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